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PRODUCTIVITY CHANGES AND THE EDUCATION OF THE LABOUR FORCE*

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I

In references (1) to (4), the relationship between the education of the labour force and the growth of national income is studied. The object of these studies is to determine the enrollment required in the different levels of the educational system in order to sustain economic growth. These analyses are made under the assumption that the productivity of the labour force is constant in time. This assumption does not hold true in reality, especially for the long periods that have to be considered in the analysis of the educational processes. In this paper the influence of productivity changes on the educational structure of the labour force and on enrollment in the educational system is studied. The study is made with the aid of a simple model. This model is a modification of the ones used in references (1) to (4). The model is presented in Chapter II.

In the preparation of this paper, special emphasis was put upon the collection of statistical data. These data and their analysis by means of the model are presented in Chapter III.

II

II.1 The variables appearing explicitly* in the model are:

N^i : labour force with educational level i.

i = 1 more than zero, up to and including six years of education.

i = 2 more than six, up to and including twelve.

i = 3 more than twelve.

m^i : those among N^i who entered the labour force less than 6 years before.

n^i : enrollment in educational level i.

A total of 9 variables appear in the model. Although the gross product v does not appear explicitly as a variable in the model, it can be said that it is implicit in the model because it determines the rates of growth of

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* The meaning of explicit and implicit variables is explained below.

/the N^i.

the N^i. This explains the meaning of explicit and implicit variables. A subscript t will denote time. Time lags of 6 years will be used in the model. The equations in the model are presented below.

$$(1)\ N_t^1 = N_o^1 (1 + r^1)^t$$

$$(2)\ N_t^2 = N_o^2 (1 + r^2)^t + \Pi^1 n_t^1$$

$$(3)\ N_t^3 = N_o^3 (1 + r^3)^t + \Pi^2 n_t^2 + \Pi^3 n_t^3$$

where Π^i are the teachers/students ratios for educational level i. Equations (1) to (3) describe the demand for labour. The demand for labour is composed of two terms. One is a function of time and represents the demand for labour for the production processes. The other is the demand for qualified personnel to work as teachers.

$$(4\text{-}5\text{-}6)\ N_t^i = (1 - \lambda^i) N_{t-1}^i + m_t^i \quad i = 1, 2, 3$$

where λ^i are the percentage of death and retirements of the labour force. Equations (4) to (6) describe the supply of labour having different levels of education. The supply consists of the remainder of the labour force from the previous period, plus the additions coming from the educational system.

$$(7)\ (1 - \eta^1) n_{t-1}^1 = n_t^2 + m_t^1$$

$$(8)\ (1 - \eta^2) n_{t-1}^2 = n_t^3 + m_t^2$$

$$(9)\ (1 - \eta^3) n_{t-1}^3 = m_t^3$$

where η means the percentage of persons enrolled in educational level i who do not become - after graduation - members of the labour force. Equations (7) to (9) present the relationship between the demand for labour and the educational system.

II.2 Comparing the models in references (1) to (4) with the model presented in section II.1, the following observations can be made. For simplicity, the model in references (1) to (4) will be called model A, and the model presented in section II.1, model B. In model A equations (1) to (3), the terms referring to the demand of labour for the production processes are expressed as a constant multiplied by gross domestic product. That is

$$N_o^i (1 + r^i)^t = \gamma^i v_t$$

/where γ^i

where γ^i are workers with educational level i per unit of product. The coefficients γ^i are assumed to be constant, and this implies that no changes in productivity are assumed.

The gross domestic product v_t is assumed to grow at a given rate. This is equivalent to assuming that

(10) $r^1 = r^2 = r^3 = \text{constant}$.

In the model B, the relation (10) may or may not hold true. According to the statistical data presented in Chapter III, relation (10) does not hold true. This fact can be considered in model B but cannot in model A.

II.3 Model B in equations (1) to (9) is a simple system of non-homegeneous difference equations. The solution has two parts: one is the solution of the homogeneous part and shows the oscilations that enrollment might have due to the values of the parameters and the initial values. In reference (1) it is shown that these oscilations are due more to a defect in the construction of the model than to characteristics existing in reality. Since for long term planning these oscilations are of no particular interest, no attention will be paid to the solution of the homogeneous part. The second component of the solution is the particular solution that depends upon the rates of growth r^i, of the functions of time appearing in equations (1) to (9). This particular solution is of special interest in determining the trends of the educational structure of the labour force when the trends of productivity and of gross domestic product are known. In reference (5) it is shown that this particular solution takes the form of

(11) $\alpha(1 + r^1)^t + \beta(1 + r^2)^t + \gamma(1 + r^3)^t$

where α, β and γ are constants that can be expressed in terms of the coefficients in equations (1) to (9), and are different for each of the 9 variables appearing in the model. It is not necessary to find explicitly the values of α, β and γ in terms of coefficients of the equations. The point of interest is that the rates of growth of all the variables in the model are linear combinations of the rates of growth of each of the levels of education of the labour force. For instance, the growth of enrollment in the universities <u>is not</u> independent of the rate of growth of educational level 1. The importance of this dependence varies according to circumstances.

/The main

The main conclusion that can be obtained from the analysis of equation (11) is that in the long run, all the variables in the model tend - for large t - to grow at a rate r^* defined by

(12) $r^* = \max (r^1, r^2, r^3)$

where max (...) means the maximum of the quantities in parenthesis. In particular, this is true for the rate of growth of enrollment in all the levels of the educational system.

III

III.1 In Chapter II it was shown that the rate of growth of enrollment in all the levels of the educational system tends to be equal to the maximum value among r^i $i = 1, 2, 3$. In this Chapter some data are presented and used to estimate r^i as a function of the changes in labour productivity.

No attempt is made below to test the hypothesis that education of the labour force is required to promote and sustain economic growth. A test of this hypothesis is presented in reference (1). It will be assumed below that the education of the labour force is required to sustain economic growth.

The changes in productivity will be taken as "independent" variables, while the education of the labour force is the dependent variable. This arrangement is more fruitful in the present paper.

An additional notation:

$i = 0$ denotes lack of formal education. Persons who do not have any formal education will be called analphabets.

r rate of growth of the total labour force.

rate of growth of gross domestic product.

III.2 The basic statistical data are presented in Table 1. Below, an explanation is given of the way in which the figures were obtained.

In "La Estructura Demográfica de las Naciones Unidas", the following data is presented: population by age and sex classified in active or inactive population and by years of education completed. The classfication of active and inactive population is not crossed with the educational classification. For each interval of sex and age it was assumed that the persons in the

/Table 1

Table 1

STRUCTURE OF THE LABOR FORCE BY LEVEL OF EDUCATION NUMBER OF WORKERS PER MILLION DOLLARS OF PRODUCTION

Country \ Level of education		Analfabets	0 - 6	6 - 12	12 +	Total	V millions of dollars
Brasil (1950)	N^1	10 178 619	6 023 249	759 845	318 769	17 280 482	10 140
	$\frac{N^1}{V}$	1 003 809	594 009	74 935	31 436	1 704 189	
Colombia (1951)	N^1	1 514 367	1 937 736	267 547	35 959	3 755 609	3 001.0
	$\frac{N^1}{V}$	504 621	645 697	89 153	11 982	1 251 453	
Costa Rica (1950)	N^1	51 128	199 115	17 212	4 199	271 654	173.3
	$\frac{N^1}{V}$	295 026	1 148 961	99 319	24 230	1 567 536	
Cuba (1953)	N^1	493 432	1 224 610	208 263	45 961	1 972 266	2 190.8
	$\frac{N^1}{V}$	225 229	558 978	95 063	20 979	900 249	
Chile (1952)	N^1	452 224	1 215 659	424 358	56 118	2 148 359	1 985.0
	$\frac{N^1}{V}$	227 821	612 423	213 782	28 271	1 082 297	
El Salvador (1950)	N^1	401 973	226 164	21 466	3 275	652 878	282.0
	$\frac{N^1}{V}$	1 425 436	802 000	76 121	11 613	2 315 170	
Guatemala (1950)	N^1	635 587	289 436	23 262	5 087	953 372	421.0
	$\frac{N^1}{V}$	1 509 708	687 496	55 254	12 083	2 264 541	
Nicaragua (1950)	N^1	207 701	109 596	9 952	2 270	329 526	165.3
	$\frac{N^1}{V}$	1 256 509	663 013	60 206	13 733	1 993 503	
Panama (1950)	N^1	81 808	143 122	34 710	4 495	264 135	259.0
	$\frac{N^1}{V}$	315 861	552 595	134 015	17 355	1 019 826	
United States (1950)	N^1	3 267 371	10 064 334	38 001 086	8 310 199	59 642 990	285 283
	$\frac{N^1}{V}$	11 453	35 278	133 205	29 130	209 066	
Venezuela (1950)	N^1	841 000	774 526	64 688	22 467	1 702 681	3 379.0
	$\frac{N^1}{V}$	248 890	229 218	19 144	6 649	503 901	

Sources: a) Population, education and labor force figures:
La Estructura demográfica de las Naciones Americanas. Instituto Interamericano de Estadística. Unión Panamericana. United Nations, Demographic Yearbook 1957.
b) Gross Domestic Product:
Cuadros del GDP. ECLA Mimeograph paper 1962.
United Nations, Yearbook of National Account Statistics.

labour force had a distribution by level of education equal to the distribution by level of education of the total population in the same interval. That is, it was assumed that in each sex and age interval the proportions of persons having a given level of education were the same in the active and in the non-active population. With this assumption it was possible to estimate the labour force classified by level of education. Four levels were considered in each country. These levels are: analphabets, elementary, secondary and higher education.

The next step was to modify the data in such a way as to make comparable the figures for different countries. This step is required because the number of years of education included in each level changes from country to country. For example: Colombia has elementary education of 5 years, while Chile has 6, and the US 8.

For each country two tables were prepared: (a) one of the population classified by levels of education; (b) the other of the population with no education, from zero up to and including six years of education, with more than six and up to and including twelve, and with more than twelve. To pass from classification (a) to (b) it was necessary to add or subtract some quantities from each interval, according to the case. A proportion of these quantities was added to or subtracted from the labour force classified by level of education. The factor of proportionality was the labour force divided by total population.

This second step was not possible in the cases of Chile and Guatemala. The secondary general education in Chile extends for 6 years. The technical middle education might extend in some cases up to 8 years. Thus, the estimation presented by Chile of the labour force having 6 years to 12 years of education is higher than what it would have been if the correction in step 2 could have been made. However, this over estimation is not very important because all the population with secondary/technical education only reaches 2.4 percent of the total.

/In the

In the case of Guatemala secondary education extends to only 4 years. Thus the figure in the table under-estimates the number of workers having from 6 up to 12 years of education and over-estimates the number having more than 12. In this case also, the under-estimation is unimportant. because the population having college education is only 0.5 percent of the total.

The values of $\frac{N^i}{v}$ also appear in Table 1. A comparison of the values of $\frac{N^i}{v}$ of Venezuela with those of all the other countries shows that labour productivity in Venezuela is very high. The reasons for this are well known, and are not related to the problem under study. To avoid distortion in the estimation, the results for Venezuela were excluded from the following analysis.

III.3 To estimate the relationship between change in the product of the labour force and the education of its members, functions of the following form were assumed:

$$(13)\quad \left(\frac{N^i}{v}\right) = A^i \left(\frac{N}{v}\right)^{b^i} \quad , i = 0, \ldots, 3.$$

where A^i and b^i have to be estimated.

The b^i are the elasticities of the $\frac{N^i}{v}$ with respect to changes in $\frac{N}{v}$. A well-known property of the elasticities - proved for example in reference (6) - is that

$$(14)\quad \sum_{i=0}^{3} b^i \frac{N^i}{N} = 1$$

where $\frac{N^i}{N}$ are the proportion of the labour with educational level i in a given country. If property (14) is fulfilled, the estimations made of $\frac{N^i}{v}$ satisfy the following relationship

$$\sum_{i=0}^{3} \frac{N^i}{v} = \frac{N}{v}$$

/In order

In order to estimate A^i and b^i in (13) subject to constraint (14), the following procedure was adopted.

Let

$$y_j^i = \log \left(\frac{N_j^i}{v_j} \right) \quad \begin{matrix} i = 0, \ldots, 3 \\ j = 1, \ldots, 10 \end{matrix}$$

$$x_j = \log \left(\frac{N_j}{v_j} \right) \text{ and}$$

$$a^i = \log A^i$$

where the different values of j refer to different countries.

The usual procedure to estimate A^i and b^i is to minimize

$$F^i = \sum_{j=1}^{10} (y_j^i - a^i - b^i x_j)^2$$

In this paper, the function

$$F = \sum_{i=0}^{3} F^i$$

subject to the constraint (14) was minimized. The proportions $\frac{N^i}{N}$ corresponding to the USA and Guatemala were used. In Table 2 the values of the b^i are presented.

/Table 2

Table 2

Elasticities of $\frac{N^i}{v}$ respect to $\frac{N}{v}$

	b^o	b^1	b^2	b^3
Without constraint (14)	2.015	1.236	- 0.342	- 0.303
With USA proportions in the constraint (14)	1.899	1.346	1.030	0.031
With Guatemala's proportions in constraint (14)	1.119	0.877	- 0.439	- 0.359

Source: Explained in the text.

III.4 Formula (13) with known values of A^i and b^i permits the evaluation of the influence on $\frac{N^i}{v}$ of changes in $\frac{N}{v}$.

The changes in $\frac{N}{v}$ are assumed to be autonomous. That is, it is assumed that

$$N = N_o (1 + r)^t$$

and $$v = v_o (1 + \rho)^t$$

The problem below is to express r^i as a function of r and ρ.

It can be easily verified that

$$(15)\quad N_t^i = A^i N_o^{b^i} v_o^{(1 - b^i)} \left[\frac{(1 + r)^{b^i}}{(1 + \rho)^{(1 - b^i)}} \right]^t$$

Thus, the following approximation can be used

$$r^i = b^i r + (1 - b^i) \rho.$$

Values of r^i for different values of r and ρ are presented in Tables 3 A and 3 B. In Table 3 A the values of b^i corresponding to the USA have been used; in 3 B those corresponding to Guatemala.

Some observations with respect to Tables 3 are presented below. In the USA the highest rate of growth corresponds to the part of the labour force having more than 12 years of education. This is not the case in Guatemala. There the highest rate of growth corresponds to the part of the labour force having 6 to twelve years of education. This observation indicates that in the USA the main requirement is a labour force with more than twelve years of education, while in Guatemala, a labour force with 6 to 12 years of education has more priority.

With the data of Tables 3 it is possible to compare the rates of growth of productivity and of per capita income with the rates of growth of the parts of the labour force having different levels of education. The difference (ρ - r) gives the rate of growth of productivity and an approximate value of the rate of growth of per capita income. In the USA only the labour force with more than 12 years of education grows faster than productivity and per capita income. In Guatemala it is the labour force with 6 to 12 and also that with more than 12. In both cases the magnitude of the difference is really important.

/Table 3 A

Table 3A

UNITED STATES: ANNUAL RATES OF GROWTH OF LABOR FORCE BY GROUPS HAVING DIFFERENT NUMBERS OF YEARS OF EDUCATION

r \ p	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
Rates of growth of the group having more than 12 years of education												
1.5	-	2.0	2.5	3.0	3.4	3.9	4.4	4.9	5.4	5.9	6.3	6.8
2.0	-	-	2.5	3.0	3.5	3.9	4.4	4.9	5.4	5.9	6.4	6.8
2.5	-	-	-	3.0	3.5	4.0	4.4	4.9	5.4	5.9	6.4	6.9
3.0	-	-	-	-	3.5	4.0	4.5	4.9	5.4	5.9	6.4	6.9
Rates of growth of the group having 6 to 12 years of education												
1.5	-	1.5	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.3
2.0	-	-	2.0	2.0	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.8
2.5	-	-	-	2.5	2.5	2.5	2.4	2.4	2.4	2.4	2.4	2.4
3.0	-	-	-	-	3.0	3.0	3.0	2.9	2.9	2.9	2.9	2.9
Rates of growth of the group having 0 to 6 years of education												
1.5	-	1.3	1.1	0.9	0.7	0.5	0.3	0.1	-0.1	-0.3	-0.5	-0.7
2.0	-	-	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2	0.0
2.5	-	-	-	2.3	2.1	1.9	1.7	1.5	1.3	1.1	0.9	0.7
3.0	-	-	-	-	2.8	2.6	2.9	2.2	2.0	1.8	1.6	1.4
Rates of growth of the group without education												
1.5	-	1.0	0.6	0.2	-0.3	-0.7	-1.2	-1.6	-2.1	-2.5	-2.9	-3.4
2.0	-	-	1.6	1.1	0.6	0.2	-0.2	-0.7	-1.1	-1.6	-2.0	-2.5
2.5	-	-	-	2.1	1.6	1.2	0.7	0.2	-0.2	-0.6	-1.1	-1.5
3.0	-	-	-	-	2.6	2.1	1.7	1.2	0.8	0.3	-0.1	-0.6

Source: Explained in the text.

Table 3B

GUATEMALA: ANNUAL RATES OF GROWTH OF THE LABOR FORCE BY GROUPS HAVING DIFFERENT NUMBERS OF YEARS OF EDUCATION

r \ p	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
					Rates of growth of group having more than 12 years of education							
1.5	-	2.2	2.9	3.5	4.2	4.9	5.6	6.3	6.9	7.6	8.3	9.0
2.0	-	-	2.7	3.4	4.0	4.7	5.4	6.1	6.8	7.4	8.1	8.8
2.5	-	-	-	3.2	3.9	4.5	5.2	5.9	6.6	7.3	7.9	8.6
3.0	-	-	-	-	3.7	4.4	5.0	5.7	6.4	7.1	7.8	8.4
					Rates of growth of group having 6 to 12 years of education							
1.5	-	2.2	2.9	3.7	4.4	5.1	5.8	6.5	7.3	8.0	8.7	9.4
2.0	-	-	2.7	3.4	4.2	4.9	5.6	6.3	7.0	7.8	8.5	9.2
2.5	-	-	-	3.2	3.9	4.7	5.4	6.1	6.8	7.5	8.3	9.0
3.0	-	-	-	-	3.7	4.4	5.2	5.9	6.6	7.3	8.0	8.8
					Rates of growth of group having 0 to 12 years of education							
1.5	-	1.6	1.6	1.7	1.7	1.8	1.9	1.9	2.0	2.1	2.1	2.2
2.0	-	-	2.1	2.1	2.2	2.2	2.3	2.4	2.4	2.5	2.6	2.6
2.5	-	-	-	2.6	2.6	2.7	2.7	2.8	2.9	2.9	3.0	3.1
3.0	-	-	-	-	3.1	3.1	3.2	3.2	3.3	3.4	3.4	3.5
					Rates of growth of group without education							
1.5	-	1.4	1.4	1.3	1.3	1.2	1.1	1.1	1.0	1.0	0.9	0.8
2.0	-	-	1.9	1.9	1.8	1.8	1.7	1.6	1.6	1.5	1.5	1.4
2.5	-	-	-	2.4	2.4	2.3	2.3	2.2	2.1	2.1	2.0	2.0
3.0	-	-	-	-	2.9	2.9	2.8	2.8	2.7	2.6	2.6	2.5

Source: Explained in the text.

III.5 From Tables 3, the limiting* rate of growth of the educational system when t increases can be read. In Section II.3 it was shown that the limiting rate of growth of enrollment in all levels of the educational system is given by formula (12). Thus, for example, if labour is growing at 2.5 percent per year, and gross domestic product at 5 percent per year, enrollment in all the levels of the educational system should tend to grow at 4.9 percent per year in the USA an at 6.3 percent per year in Guatemala. This is so while per capita income in the USA and Guatemala would be growing at 2.5 percent per year. This means that income elasticity of the demand for education should be about 2 in the USA and about 3 in Guatemala.

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* Limit is used here in the mathematical sense.

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