Calibration Theory

Starting point, Let Y (say Income for instance) be a variable of interest and X (some related auxiliary variable)

$$\hat{Y}_a = \sum_{i=1}^{n} a_{is}\, y_i$$

a_{is} are the non-response adjusted weights.

Consider the following distance function

$$f(w_{is}, a_{is}) = \sum_s (a_{is} - w_{is})^2 / 2a_{is}q_{is}$$

Where w_{is} will be the Calibrated weights.
Minimize $f(w_{is}, a_{is})$ under:

$$\sum_s w_{is}x_i = X = \sum_{i=1}^{N} x_i$$

To find the optimum form the lagrangian **L** :

$$L = \sum_s a_{is}\left(\frac{w_{is}}{a_{is}} - 1\right)^2 \frac{1}{2q_{is}} + \lambda\left(X - \sum_s w_{is}x_{is}\right)$$

Where q_{is} is an independant parameter to be specified. It follows that

$$\frac{\partial L}{\partial w_{is}} = \sum_s \left(\frac{w_{is}}{a_{is}} - 1 \right) \frac{1}{q_{is}} - \lambda x_{is} \text{ and that } w_{is} = a_{is} + \lambda x_{is} a_{is} q_{is}$$

Then $\lambda = \dfrac{\left(X - \sum_s a_{is} x_{is} \right)}{\sum_s a_{is} x_{is}^2 q_{is}}$ and $w_{is} = a_{is} \left(1 + \left(\dfrac{x - \hat{x}}{\sum_s a_{is} x_{is}^2 q_{is}} \right) x_{is} q_{is} \right)$

$$w_{is} = a_{is} \left(1 + \left(\frac{x - \hat{x}}{\sum_s a_{is} x_{is}^2 q_{is}} \right) x_{is} q_{is} \right) = a_{is} * g_{is}$$

Where g_{is} is known as the g-weight.

We can easily generalized to many X variables

$$\hat{Y}_w = \sum_{i=1}^{N} w_{is} \, y_i$$

- w_{is} is the calibrated weight

$$w_{is} = g_{is} \, a_{is}$$

- Now consider an auxiliary vector

$x_i = (x_{i1}, \ldots, x_{ip})$ (p variables)

Minimize

$$\sum_{i=1}^{n} f(w_{is}, a_{is})$$

Under

$$\begin{aligned} \sum_{i=1}^{n} w_{is} \, x_{i1} &= \sum_{i=1}^{N} x_{i1} \\ \vdots \quad \vdots \quad & \quad \vdots \\ \sum_{i=1}^{n} w_{is} \, x_{ip} &= \sum_{i=1}^{N} x_{ip} \end{aligned}$$

- $f(w_{is}, a_{is})$ need to be a convex function

  - Extensive number of choices

$$\frac{(w_{is} - a_{is})^2}{2a_{is}q_{is}}$$ [regression and ratio]

$$\frac{1}{q_{is}}\left[w_{is}\log\left(\frac{w_{is}}{a_{is}}\right) - w_{is} + a_{is}\right]$$ « Raking Ratio »

  - Could add boundaries i.e) $a \le w_{is} \le b$