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SIMULATION OF THE EFFECT OF MORTALITY DIFFERENTIALS BY PARITY ON PROPORTIONS ORPHANED USING DATA FROM THE HAGUE 1870-1880

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INDEX

Introduction

The current paper was written as a voluntary contribution to the Seminar on Orphanhood and Adult Mortality in the past, to be held by the Latin American Centre for Demography (CELADE) in December 1984. It is a by-product of a study undertaken together with Frans van Poppel , which has the objective of comparing indirect estimates of adult mortality based on proportions orphaned among newly weds according to their marriage registers, with those drawn up on basis of recorded deaths and census data for the period.

The purpose of this document is to simulate the effect of mortality differentials by parity on the proportions orphaned in a stable population calculated with the fertility and mortality schedules pertaining to The Hague in the decade between 1870 and 1880. Details on the data as well as a more elaborate account of the sources of variability of values estimated with the orphanhood approach to indirect mortality versus 'true' mortality levels, are to be found in the empirical study referred to above. One of the assumptions underlying the method was selected for this simulation, notably that there is no relation between the mortality of parents and their parity. The assumption is required to deal with the potential bias due to the fact that the probability of a parent's mortality experience being recorded in the sample is proportional to his or her number of children.

The reason for selecting this particular assumption for scrutiny is that, though overall fertility is moderate, legitimate fertility is quite high in The Hague around the end of the XIX'th century. The "late and non-universal marriage" mechanism of fertility regulation is functioning in a rather exemplary fashion. It's reassuring to see reality behave according to expectations, based, in this case, on the empirical generalisations that have found their way into theory since Hajnal's pioneering work on European nuptiality-fertility relations (J.Hajnal,1965). The point is that legitimate fertility being 'natural',for the group of women exposed to the risk of childbearing, the incidence of high parity will not be exceptional, specially in the later phases of the reproductive cycle.The presence of clear mortality differentials by marital status led us to consider the hypothesis that a bias might be caused by this combination of factors. The simulation is limited to the female sex.

Proportions not-orphaned are calculated using mortality and fertility measures, referring to the total population, the ever-married subgroup and encorporating the simulated effect of parity specific mortality. These results were compared among each other and with the observed proportions not-orphaned.

All calculations were carried out on a programmable hand calculator, except for a small linear programming exercise that was run on the computing facility of the Catholic University of Tilburg. Part of the work was done between other research activities, during office hours. On both accounts I acknowledge my gratitude to Prof. G.A.B.Frinking for the opportunity to carry out this piece of work.

Orphanhood

The calculation of the proportions not-orphaned by 5-year age groups, x,x+5, referred to as ${}_5PNO_x$, departs from the familiar (Hill-Trussel,1977) expression for all parities:

$$
{}_5PNO_x = \frac{\int_N^{N+5}\int_y^z e^{-r(a+x)} f(a)l(a+x)\, da\,dx}{\int_N^{N+5}\int_y^z e^{-r(a+x)} f(a)l(a)\, da\,dx} \qquad \ldots(1)
$$

The beginning of the reproductive period is indicated by $\underline{y}$, the end by $\underline{z}$. This continuous-form expression is approximated in discrete terms by:

$$ {}_5PNO_x = \frac{\sum_{y}^{z-5} e^{-r(a+x+5)} \; {}_5f_a * {}_5L_{a+x+2.5}}{\sum_{y}^{z-5} e^{-r(a+x+5)} \; {}_5f_a * {}_5L_a} \qquad \ldots(2) $$

In order to encorporate the effect of parity, the mortality/fertility factor in (2) is weighted:

$$ {}_5PNO_x = \frac{\sum_{y}^{z-5} e^{-r(a+x+5)} \sum_{i=1}^{8} {}_5Prop'^{\,i}_a * {}_5f^{\,i}_a * {}_5L^{\,i}_{a+x+2.5}}{\sum_{y}^{z-5} e^{-r(a+x+5)} \sum_{i=1}^{8} {}_5Prop'^{\,i}_a * {}_5f^{\,i}_a * {}_5L^{\,i}_a} $$

$$ \ldots.(3) $$

where ${}_5Prop'^{\,i}_a$ is the proportion of the women of ages a,a+5 with parity i, who had children in the interval and ${}_5f^{\,i}_a$ refers to the birth-order specific fertility rate for children of both sexes. ${}_5L_a\,i$ is the life table survivorship function expressing the mortality history of women transgressing lapses with parities up to <u>i</u>. The assumption implied in the weighting procedure is that the parity specific mortality level at a particular age is identical for all women of a given parity regardless of the spacing of births over their previous reproductive history. There will, in reality, be variability around an average level, but it is assumed that the effects are negligible. The dash in the proportions refers to the fact that only women who had children are included. That is, there are no ${}_5\mathrm{Prop}'^{0}_{a}$ here (in contrast to the proportions used below). It is understood that the mortality of women with no children, including unmarried women, is not included.

Mortality

Parity specific life tables were calculated by assuming that the level difference in mortality between ever-married and never-married subpopulations over the reproductive age span is caused by the childbearing process. This is a plausible assumption considering the fact that female mortality in the reproductive years is observed to be higher in the life table for married women than in that for the total population, in contrast to mortality after the age of 50. The differential for males also points in the opposite direction: married men have lower mortality than bachelors. Table 1 testifies to the statements with respect to the female population.

The life table for never-married women was calculated from ever-married and total life tables through:

$$l^{tot}(x) = (Prop.ev.mar) * l^{ev.mar}(x) + (Prop.nev.mar) * l^{nev.mar.}(x)$$

therefore,

$$l^{nev.mar}(x) = \{l^{tot.}(x) - Prop.ev.mar * l^{ev.mar}(x)\} / (Prop.nev.mar) \quad ..(4)$$

The proportions ever-married were estimated by fitting the Coale nuptiality schedule (A.Coale,1971) to the proportions single in the 25 to 30 and the 45 to 50 age ranges, available to the author on basis of empirical data. The level difference between the mortality of ever-married versus never-married women is formulated as:

$$Y^{i}(x) = \frac{i * a}{P(x)} + b * Y^{nev.mar.}(x) \quad ......(5)$$

where <u>a</u> and <u>b</u> refer to the intercept and slope of the least squares linear regression of the logit of the survivor function of the ever-married against the never-married populations (with logit l(x) indicated as Y(x)) and P(x) refers to an indicator of the average parity of the women at the age in question. The basic assumption is, then, that <u>a</u> is proportional to parity during the reproductive years in the Brass relational mortality system (W.Brass,1968). The parity indicator used was the average number of children borne per married woman from the mean age at marriage in the stable population up to age x. This may require some explanation. It is clear that overall parity is not a useful measure in this context, because there are women in the denominator, who are not exposed to the risk of childbearing, due to the fact that they are not married. On the other hand, taking marital fertility as the yardstick will not do either, since the implicit assumption would be that all women involved were married from the beginning of the reproductive period. The measure of parity used was, therefore, cumulated marital fertility from the mean age at which the women in the age group in question were married in the stable population.

For ages above 50 hybrid values were calculated for the survivorship of women of parity p , $l(x)^{p}$, as:

$$l(x)^{p} = \{l(50)^{p} / l(50)^{ev.mar.}\} * l(x)^{ev.mar.} \quad \ldots(6)$$

Fertility

The estimation of the birth order specific fertility rates and the proportions of women according to parity departs from the assumption that all births are generated at exact time periods 5-folds of years before time t, such as t-50,t-45 etc, where t refers to the year in which the orphanhood data were gathered. A cohort approach is applied, following women as they enter marriage and are subjected to the risk of having children over consecutive 5-year periods. Proportions of women according to parity and numbers of births by order are calculated for a particular cohort and assumed constant over cohorts. Legitimate fertility rates were used, assuming no illegitimacy.(Though about 5% of births are illigitimate in The Hague around 1880 many of the women in question susequently marry). The task at hand is to create a distribution of births by order, which is plausible and consistent with the total number of births in the stable population and which also yields a distribution of women according to parity that adds up to the total number of married females.

The general approach is to look at fertility experience in each 5-year age-group in turn, and after having done so, to link it up with fertility in the cohort up to the beginning of the age group under consideration to obtain measures from the beginning of the reproductive period.

Use ${}_5s_{x-5}$ to refer to $\frac{{}_5L_x}{{}_5L_{x-5}}$, then:

$$ {}_5P_x^{p} = \sum_{i=0}^{p} {}_5P_{x-5}^{i} * {}_5\mathrm{Prop}_x^{p-i\;\;i} * {}_5s_{x-5} \quad \ldots(7) $$

where ${}_5\mathrm{Prop}_x^{p-i\;\;i}$ is defined as the proportion of women who start the current leap with parity i and have p-i children in the interval.

${}_5P_x^{p}$ is the number of women of parity p at the end of age interval x,x+5. The women who start a particular leap as

${}_5P_{x-5}^{0}$ should be treated slightly differently since they consist of those who left the previous round without children plus those who entered the "pool" by marriage. Therefore, strictly speaking (7) should be written as:

$$
{}_5P_x^{p} = \{ \sum_{i=1}^{p} {}_5P_{x-5}^{i} * {}_5^{p-i}\text{Prop}_x^{i} * {}_5s_{x-5} \} +
$$

$$
\{ [{}_5P_{x-5}^{0} * {}_5s_{x-5} + ({}_5N_x^{m} - {}_5N_{x-5}^{m}) * {}_5s_{x-5})] \; {}_5^{p}\text{Prop}_x^{0} \} \qquad \ldots \; 7(a)
$$

where ${}_5N_x^{m}$ refers to the number of married women $x, x+5$ years of age. This is, in fact, the procedure that was followed in the calculations. For brevity's sake the form of (7) will be maintained, it being implied that ${}_5P_{x-5}^{0}$ includes new entries due to marriage.

It follows from the definition of ${}_5^{l}\text{Prop}_x^{k}$ that we may write:

$$
\sum_{l=0}^{w} {}_5^{l}\text{Prop}_{x-5}^{k} = 1 \quad \forall k \quad \ldots\ldots (8)
$$

where *w* is the terminal parity, taken as 8 in the calculations. Also:

$$
\sum_{p=0}^{w} {}_5P_x^{p} = {}_5N_x^{m} \quad \ldots\ldots\ldots (9)
$$

substitute (7) in (9) :

$$ {}_5N^{m}_{x} = \sum_{p=0}^{w} \sum_{i=0}^{p} {}_5P^{i}_{x-5} * {}_5\mathrm{Prop}^{p-i}_{x} * {}_5s^{i}_{x-5} \quad \ldots(10) $$

The total amount of births these women had are their numbers times the number of children they had in the interval:

$$ {}_5B_{x} = \sum_{p=0}^{w} \sum_{i=0}^{p} {}_5P^{i}_{x-5} * {}_5\mathrm{Prop}^{p-i}_{x} * {}_5s^{i}_{x-5} *(p-i) \ldots(11) $$

If ${}^{q}_{5}B^{p}_{x}$ is defined as the number of births by women who start the interval with parity p and have q children during the leap, or

$$ {}^{q}_{5}B^{p}_{x} = {}^{q}_{5}\mathrm{Prop}^{p}_{x} * {}_5P^{p}_{x-5} * {}_5s^{p}_{x-5} * q \quad \ldots(12) $$

,then,

$$ {}_5B^{m}_{x} = \sum_{p=0}^{m-1} \sum_{q=m-p}^{w-p} \{ {}^{q}_{5}B^{p}_{x} / q \} \quad ..(13) $$

with ${}_5B^{m}_{x}$ referring to the number of children of birth order <u>m</u>.

If the ${}^{q}_{5}\mathrm{Prop}^{p}_{x}$ can be estimated such that (8) and (11) are also satisfied, the number of women by parity can be estimated with (7) and the number of births by order with expression (13).

At the beginning of the fertile period ${}_5P_x^{i}=0\ \forall i$, leaving only the ${}_5^{p-i}\mathrm{Prop}_x^{p}$ unknown. After the first round of calculations the ${}_5P_{15}^{i}$ are known, making it possible to solve for the ${}_5^{p-i}\mathrm{Prop}_{20}^{i}$ and so forth. The calculation procedure used was linear programming, where (10) was minimized under constraints (8) and (11), plus a set of conditions for each proportion to be estimated, which restrict the permitted values to a plausible range.

Results

The life tables used in the simulation exercise were calculated by Frans van Poppel with the U.S. Dept. of Commerce, Bureau of the Census, CPDA (Arriaga cs., 1975) package on basis of data from the population censuses of the period in combination with deaths from the civil registration system. Data quality checks corroborated our confidence in the reliability of the material. Examination of table 1 reveals that there are, as yet, no spectacular mortality declines in The Hague over the three decades under observation. The differentials between the life tables for the total population versus the ever-married populations are interesting, in the sense that mortality is higher in the childbearing years among married women, but lower at more advanced ages.

The fertility regime calculated from official stock and flow data was converted into legitimate rates through the intermediary of a model nuptiality schedule. After the calculations were finished Van Poppel informed me that direct estimates of marital fertility exist. The rates in question are lower over the major part of the reproductive range. Their use in the simulation would have led to a distribution with fewer women in the high parity categories. The issue at hand is that the procedure followed would tend to exaggerate mortality differentials, rather than underestimate them. In any case it is evident that those females that participate in the childbearing process, do so according to a schedule with high intensity and late localisation.

A comparison of the l(25+N)/l(25) estimated from the simulated proportions-not-orphaned through the Hill-Trussel regression coefficients with those calculated directly from the various life tables shows that our crude approximation leads to acceptable agreement (Table 2). The proportions-not-orphaned, as well as the l(25+N)/l(25) are similar for the ever married population to those for the total female populations, for the same reason: the denominator is smaller in the ever married group, while the deficits due to higher mortality in the reproductive years are partly (or totally, as in the 1870-80 tables) made up for by lower mortality thereafter.

That is to say that the pattern of divergence of the mortality levels of the two groups is such that compensatory effects cause the broad measures of adult mortality to be more alike than we would expect from face value examination of their life tables. Thus, taking the proportions-not-orphaned derived from the ever-married population as an approximation of those from the total population, as is actually done in the estimation procedure, would lead to estimates of survival that are close. This is clear upon comparing the 1(25+N)/1(25) derived from the ever-married proportion-not-orphaned with the observed values in the total population. Furthermore, the direction of the bias is not necessarily such, that the survival estimates based on the ever- married population are underestimates. In the calculations that follow the life tables for the 1870-79 period were used, a period for which the bias would be in the contrary direction.

Comparing the proportions-not-orphaned calculated with those recorded in the marriage registers of the period (also taken up in table 2) we observe a degree of consistency in the order of magnitude of the mortality levels implied which inspires confidence in the method. No disconcerting irregularities have appeared as yet. Proceeding now, to the last item in the paper, we present the simulated levels of mortality and the terminal distribution of women by parity in table 3. It needs no elaboration that the conditions simulated are not to be considered as moderate: there are clear mortality differentials and high proportions of women in the high parity categories. If the simulated proportions-not-orphaned encorporating the effect of parity specific mortality (Table 4) are compared with the proportions-not-orphaned calculated directly from the 1870-79 life tables the bias that results is not at all alarming. The direction of the bias is to underestimate mortality slightly.

Besides the factors mentioned above to account for the comfortably small differences between proportions-not-orphaned in the ever-married and the total female populations, the height of the mortality level plays a role here. However large the differential may be in mortality levels in the later phases of the fertile period, and however large the proportion of high parity women may be, the number of women who survive to the ages in question is no more than around half of the birth cohort. If overall fertility rates at these ages are low, so are birth order specific rates, even though a large proportion of all births are of orders over 4. These factors contribute to the fact that the final weight that fertility of very high order receives is small.

Conclusion

The simulation exercise undertaken departing from mortality and fertility schedules for the city of The Hague in the second half of the XIX'th century, in order to gain some insight into the potentially disruptive effect of mortality differentials according to parity, permits the conclusion that it is unlikely that the non-validity of the assumption of independance between mortality and parity is a significant cause of bias in the application of the orphanhood method of indirect mortality estimation in this particular case.

The application of indirect estimation of adult mortality to The Hague, was not intended to add to our knowledge about it's level of mortality, but about the performance of the method. It is therefore only interesting to know whether the non-validity of a particular assumption might have biased the results, if generalisation of the finding to other situations is permitted. The conditions prevailing are typical for those existing in historical and contemporary populations with defective statistics: high mortality and marital fertility, plus the existence of mortality differentials between the ever-married and single-populations.The fashion in which these factors were combined in the simulation of the effect of mortality differentials was designed to bring out any biassing potential which is consitent, within reason, with the stable parameters of our model population. The fact that no significant disturbances were generated is seen as an argument in favor of the robustness of the method from non-validity of the assumption studied.

Finally, the fact that the calculation of proportions-not-orphaned was done with a hand calculator, albeit a programmable one, and led to acceptable results, demonstrates that a certain degree of independance from 'mechanical' procedures is attainable, without the infrastructure that was applied in the process of generating these standard procedures. This might be useful in situations where the data are not amenable for handling with tabulated values and so forth, because they are grouped in inconventional age categories, or because their numbers are so low that random fluctuations exclude the use of five-year age groups.

Tables

Table 1a. l(x) and $_5q_x$ values in abridged life tables over 3 decades, late XIX'th century; Total female Population; The Hague

x	1850-1859		1860-1869		1870-1879	
	l(x)	q(x)	l(x)	q(x)	l(x)	q(x)
15	.63271	.02289	.62600	.02839	.62494	.02722
20	.61823	.02776	.60823	.02791	.60793	.03237
25	.60107	.03498	.59125	.03691	.58825	.03622
30	.58005	.04612	.56943	.03979	.56671	.04070
35	.55336	.04926	.54677	.05101	.54374	.04964
40	.52610	.05125	.51888	.05358	.51666	.04559
45	.49914	.06263	.49108	.05570	.49311	.05059
50	.46788	.08525	.46373	.07224	.46816	.06978
55	.42799	.10143	.43022	.02805	.43549	.08176
60	.38548	.13962	.38760	.13906	.39989	.12207
65	.33088	.20559	.33370	.19385	.35107	.19021
70	.26285	.30383	.26901	.31063	.28430	.30580
75	.18299	.43463	.18545	.41700	.19736	.43732
80	.10346	1.00000	.10812	1.00000	.11105	1.00000

Table 1b. l(x) and $_5q_x$ values in abridged life tables over 3 decades, late XIX'th century; Ever-Married female Population; The Hague

x	1850-1859		1860-1869		1870-1879	
	l(x)	q(x)	l(x)	q(x)	l(x)	q(x)
15	.63271	.02289	.62600	.02839	.62494	.02722
20	.61823	.04497	.60823	.03411	.60793	.03825
25	.59043	.04520	.58748	.05035	.58467	.04142
30	.56374	.04878	.55790	.04702	.56045	.04387
35	.53624	.05196	.53167	.05679	.53587	.05144
40	.50838	.05400	.50148	.05788	.50830	.04344
45	.48093	.05990	.47245	.05495	.48267	.04930
50	.45212	.08388	.44649	.07405	.46230	.06534
55	.41419	.09696	.41343	.09447	.43209	.08084
60	.37403	.13537	.37437	.13746	.39716	.11339
65	.32340	.20720	.32291	.19041	.35213	.19041
70	.25639	.30037	.26143	.31067	.28508	.30113
75	.17938	.43893	.18021	.41534	.19923	.42003
80	.10064	1.00000	.10536	1.00000	.11555	1.00000

Table 2a. Proportions-not-orphaned, by 5 year age groups, calculated with female life tables for three decades, late XIX'th century, The Hague

Life Tables for Ever-Married and Total female Populations

x	1850-1859 tot	1850-1859 ev.mar	1860-1869 tot	1860-1869 ev.mar	1870-1879 tot	1870-1879 ev.mar
20	.75422	.74882	.76433	.75117	.78023	.78123
25	.67405	.67078	.68709	.67594	.70838	.71163
30	.57868	.57720	.59293	.58431	.61770	.62289
35	.46898	.46849	.48266	.47649	.50716	.51376
40	.35001	.34971	.36191	.35773	.38193	.38914

Table 2b. l(25+N)/l(25) derived from table 2a with Hill-Trussel regression coefficients for conditional survival probabilities for three decades, late XIX'th century, The Hague

N	1850-1859 tot	1850-1859 ev.mar	1860-1869 tot	1860-1869 ev.mar	1870-1879 tot	1870-1879 ev.mar
25	.77989	.77418	.79049	.77664	.80690	.80784
30	.71775	.71417	.73145	.71954	.75343	.75664
35	.64068	.63883	.65600	.64636	.68204	.68729
40	.54571	.54479	.56101	.55335	.58765	.59455
45	.43072	.42992	.44468	.43905	.46732	.47513

Table 2b. l(25+N)/l(25) calculated from table 1 for three decades, late XIX'th century, The Hague

N	1850-1859 tot	1850-1859 ev.mar	1860-1869 tot	1860-1869 ev.mar	1870-1879 tot	1870-1879 ev.mar
25	.77841	.76575	.78432	.76001	.79585	.79070
30	.71205	.70151	.72764	.70373	.74031	.73903
35	.63983	.63349	.65556	.63725	.67980	.67929
40	.55048	.54774	.56440	.54965	.59680	.60227
45	.43730	.43424	.45499	.44500	.48330	.48759

Table 2c. Proportions-not-orphaned, by 5 year age groups, observed in marriage registers; late XIX'th century, The Hague *)

x	1869-1870	1879-1880
20	.7831	.7599
25	.6637	.6764
30	.5831	.6016
35	.4451	.4743
40	.3014	.3626

* refer to Van Poppel and Bartlema's empirical contribution to this seminar for a more elaborate account

Table 3a. l(x) functions by parity of female, estimated for 1870-1880, for The Hague

x	parity 1	2	3	4	5	6	7	8
25	.59665	.58350	.57022	.55685	.54339	.54339	.54339	.54339
30	.58266	.57273	.56275	.55271	.54263	.54263	.54263	.54263
35	.56836	.56129	.55419	.54707	.53993	.53993	.53993	.53993
40	.54532	.53937	.53341	.52744	.52146	.51547	.50948	.50349
45	.51494	.50922	.50350	.49778	.49205	.48633	.48062	.47490
50	.48440	.47897	.47354	.46812	.46271	.45730	.45190	.44652
55	.45275	.44767	.44260	.43753	.43247	.42742	.42237	.41734
60	.41615	.41148	.40682	.40216	.39751	.39286	.38823	.38360
65	.36896	.36483	.36069	.35656	.35244	.34832	.34421	.34011
70	.29871	.29536	.29201	.28867	.28533	.28200	.27867	.27535
75	.20875	.20641	.20407	.20174	.19941	.19708	.19475	.19243
80	.12107	.11972	.11836	.11700	.11565	.11430	.11295	.11161

Table 3b. Proportion of married women with terminal parity indicated in simulation, late XIX'th century, The Hague

x	parity 1	2	3	4	5	6	7	8
50	.01	.01	.01	.02	.19	.21	.27	.25

Table 4. Proportions-not-orphaned, by 5 year age groups, calculated with simulated parity specific mortality for 1870-1880, The Hague

x	1870-1880
20	.7909
25	.7184
30	.6277
35	.5123
40	.3711

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