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# PRASC



**Project for the Regional  
Advancement of Statistics  
in the Caribbean**

**Projet régional pour  
l'avancement de la statistique  
dans les Caraïbes**

Funded by the  
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# Project for the Regional Advancement of Statistics in the Caribbean

## PRASC

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**Population Estimates**  
**Component: Household Survey Infrastructure**

Funded by the  
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# **Module 4.0**

# **Estimates by Age and Sex and the Cohort-Component Method**

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# The Component Method

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$$\text{Population}_{(t+x)} = \text{Population}_{(t)} + \text{Components of pop. change}_{(t, t+x)}$$

where  $t$  is the beginning of the reference period  
 $t+x$  is the end of the reference period  
 $t, t+x$  is the reference period (usually a year)  
 $x$  is the duration of the reference period



# The **Cohort**-Component Method

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- The same method can be applied to produce population estimates by age and sex.
- Instead of doing it directly for the total population, we do it for each age cohorts (& sex)
  - People not born at BORY (born during reference year)
  - People who were aged 0 at the BORY (born a year ago)
  - People who were aged 1 at the BORY (born 2 years ago)
  - People who were aged 2 at the BORY (born 3 years ago)
  - etc...



# The **Cohort**-Component Method

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- Method used to produce estimates by age and sex, whereby
  - the population is aged from year to year and;
  - the components of population change are organised according to birth cohorts by sex
- Data required for this method include events related to deaths & migration that will be grouped by birth cohorts and sex



# Two production processes?

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- Estimates of total population and Estimates by age and sex will be produced simultaneously but independently
- The Total population will be estimated by summing and subtracting the total number of events for each component.
- The estimates by age and sex will be produced by doing the same calculations but by cohort and sex.



# Two production processes?

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- Because the two types of estimates are calculated independently, it might occur that the sum of estimates by age and sex will not add up to the estimates of total population
- In this case, the priority is given to the estimates of the total population
- A note will accompany the output table stating that because of roundings, the sum of ages might not add up to the total



# What is a cohort?

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- In statistics, marketing or demography, a cohort is a group of persons who share a defining characteristic
  - Typically persons who experienced a common event in a given time period, such as birth or graduation
- In our specific case, a cohort is a group of persons born during the same reference year (birth cohort)
  - If they are born in the same reference year, they will always have the same age at the beginning of a reference period (BORY)



# Age at event versus age at BORY

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- In order to produce population estimates, we will have to place each event (death, migration) in the right cohort.
- We are not interested at the age of the person at the moment of an event but at his age at the beginning of the reference year (BORY).
- The age at an event (death or migration) is not sufficient to place an event into the right cohort
- This is the reason we need the DOB of deceased or migrant in order to place his event in the right cohort



# Exercice #1

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- A person dies at age 50 in the year 2000...
- In what birth cohort does he belong or in, other words, in what year was he born?



# Exercice #1

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- A person dies at age 50 in the year 2000...
- In what birth cohort does he belong or in, other words, in what year was he born?
  
- He was born in 1949 if he died before his birthday
- He was born in 1950 if he had his birthday before his death



## Exercice #2

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- If the person out-migrates in October 1, 2015, what is her age at the moment of the event and what is her age at the beginning of the calendar year 2015?
- If her birthday is August,1 2010?
- If her birthday is December,1 2011?



## Exercice #2

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- If the person out-migrates in October 1, 2015, what is her age at the moment of the event and what is her age at the beginning of the calendar year 2015?
- If her birthday is August,1 2010?
  - Age at event 5
  - Age at 01/01/2015 4
- If her birthday is December,1 2011?
  - Age at event 3
  - Age at 01/01/2015 3



# Dealing with mid-year to mid-year reference period

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- When we are dealing with calendar years, the placement of events into the right cohort is easier
- As we play with mid-year population estimates and mid-year to mid-year reference years, it gets a bit more complicated
- This is when the use of the Lexis diagram is helpful.



# A special cohort: Age -1

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- What if a person is born in October 2015 and then leaves MNI in January 2016?
  - What is his/her age at the moment of the event?
  - What is his/her age (cohort) at the beginning of 2015-2016?
- For events related to people not born at the beginning of the year, we will place them in the cohort -1



# Preparing the event files for population estimates purposes

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- For both files related to deaths and migration, we need to transform the age at the moment of the event to the age at the beginning of the year (aka birth cohort)
- After this transformation, each event will be linked to a specific cohort and we can do the component method for each cohort



## Ex.#4: Placing the events in the right cohort

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Name	Date of event	Date of birth	Age at event	Age at BORY 2015-2016 (June 30, 2015)
Jack	23/07/2015	29/10/1963	51	
Jane	12/11/2015	15/08/2000	15	
Bill	15/02/2016	10/08/2015	0	
Belle	18/04/2016	20/04/2010	5	



## Ex.#4: Placing the events in the right cohort

---

Name	Date of event	Date of birth	Age at event	Age at BORY 2015-2016 (June 30, 2015)
Jack	23/07/2015	29/10/1963	51	51
Jane	12/11/2015	15/08/2000	15	14
Bill	15/02/2016	10/08/2015	0	-1
Belle	18/04/2016	20/04/2010	5	5



# Estimating the population for each cohort

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- The general formula is the following:

$$\text{Pop}_{(t+1, \text{age } x+1)} = \text{Pop}_{(t, \text{age } x)} - D_{(t,t+1, \text{cohort } x)} + \text{NIM}_{(t, t+1, \text{cohort } x)}$$

$$\text{Pop}_{(\text{MY}2016, \text{age } 45)} = \text{Pop}_{(\text{MY}2015, \text{age } 44)} - D_{(2015-16, \text{cohort } 44)} + \text{NIM}_{(2015-16, \text{cohort } 44)}$$

- There are two cohorts for which the formula is a bit different (see next slide)

# Two special cohorts

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- Cohort -1 that will lead to the estimates for age 0 at the end of the reference year

$$\text{Pop}_{(\text{MY2016, age 0})} = \text{B}_{(2015-16)} - \text{D}_{(2015-16, \text{cohort -1})} + \text{NIM}_{(2015-16, \text{cohort -1})}$$

- Cohort for the last open age group (here 90+)

$$\text{Pop}_{(\text{MY2016, age 90+})} = \text{Pop}_{(\text{MY2015, age 89+})} - \text{D}_{(2015-16, \text{cohort 89+})} + \text{NIM}_{(2015-16, \text{cohort 89+})}$$

where 89+ is the sum of population aged 89 and 90+



# Edit & Imputation

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- When receiving a file with only age, the user must convert the age at event to an age at the BORY
- For that, we need the DOB variable
- If DOB is missing but age is given,
  - 50% of the time, the cohort = age at event
  - 50% of the time, the cohort = age at event -1
- If age is missing, impute the age by using the modal age at BORY of the previous year



# Data needed by the CCM

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- Base Population by Age and sex
- Births by sex
- Deaths by DOB and sex
  - If DOB is missing, E&I will be done on age
- Migration by DOB and sex
  - If DOB is missing, E&I will be done on age



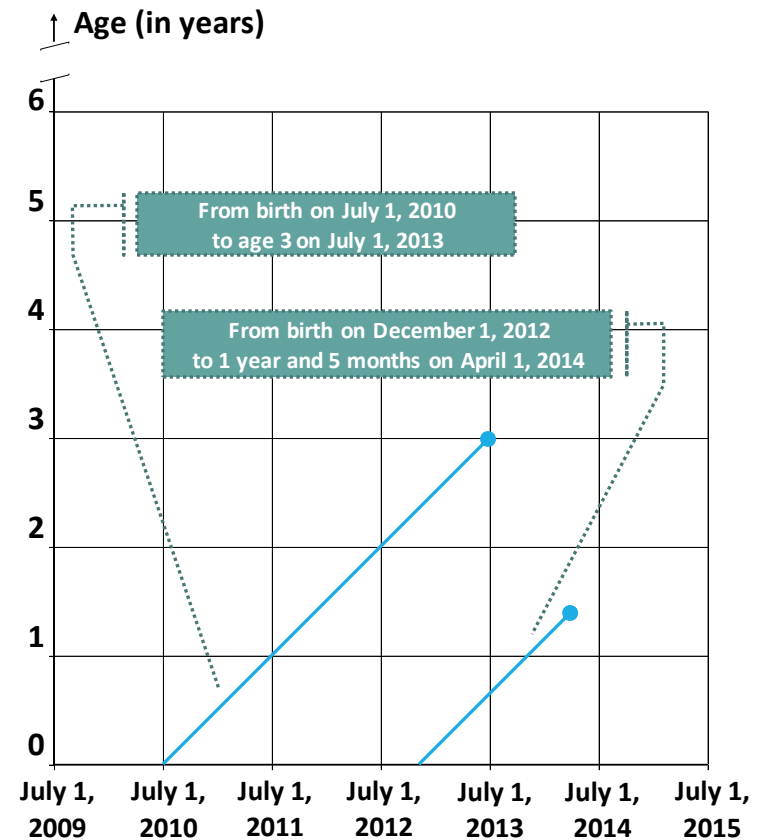
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# Appendix: The Lexis Diagram



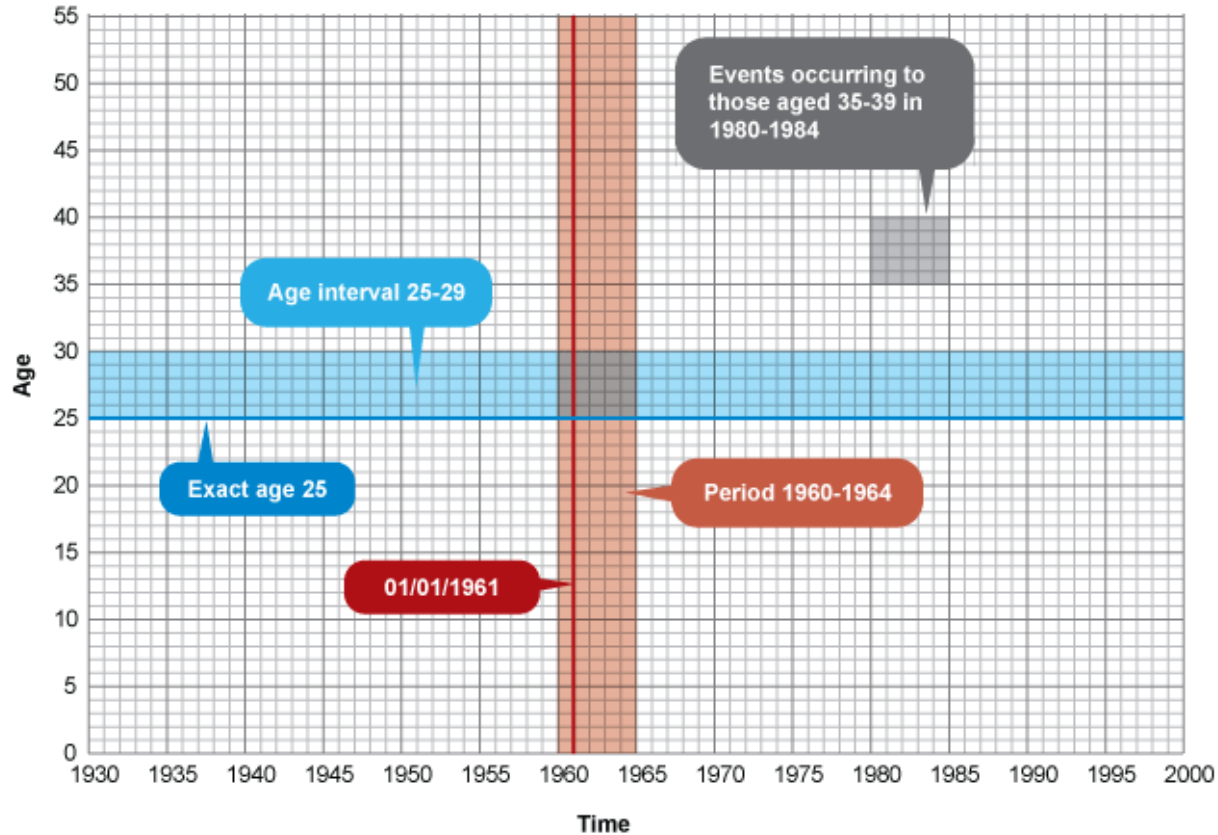
# Introduction to the Lexis diagram

- A Lexis diagram is a two dimensional diagram that is used to represent events (such as births or deaths) that occur to individuals belonging to different cohorts.
- Time is represented on the horizontal axis, while age is represented on the vertical axis.
- A person's life is represented in the Lexis diagram by a straight line called "life line"
  - The line begins on the time axis at the time of the person's birth
  - Continues diagonally upwards
- Life lines and events can be considered from a cohort or period perspective





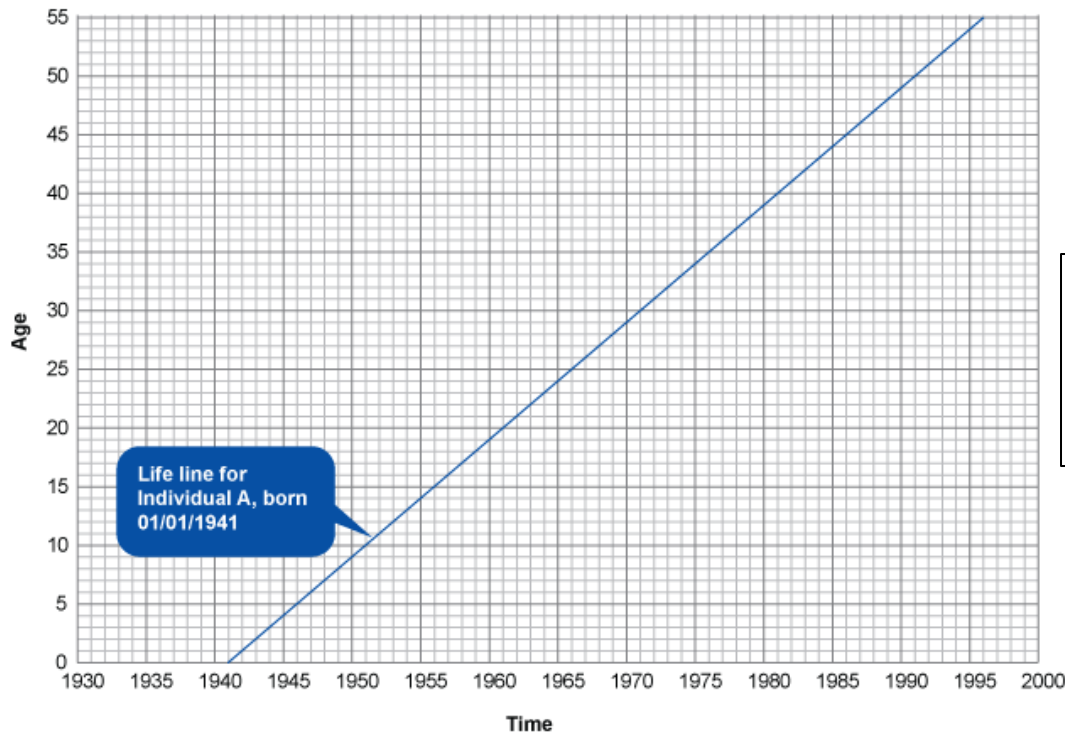
# Lexis diagram: period perspective, step by step



**Source:** Population Analysis for Policies & Programmes, PAPP101: Introduction to Demographic Analysis, S02: How to measure demographic events, International Union for the Scientific Study of Population, online course, [http://papp.iussp.org/sessions/papp101\\_s02/PAPP101\\_s02\\_010\\_010.html](http://papp.iussp.org/sessions/papp101_s02/PAPP101_s02_010_010.html)



# Lexis diagram: cohort perspective, step by step (1)

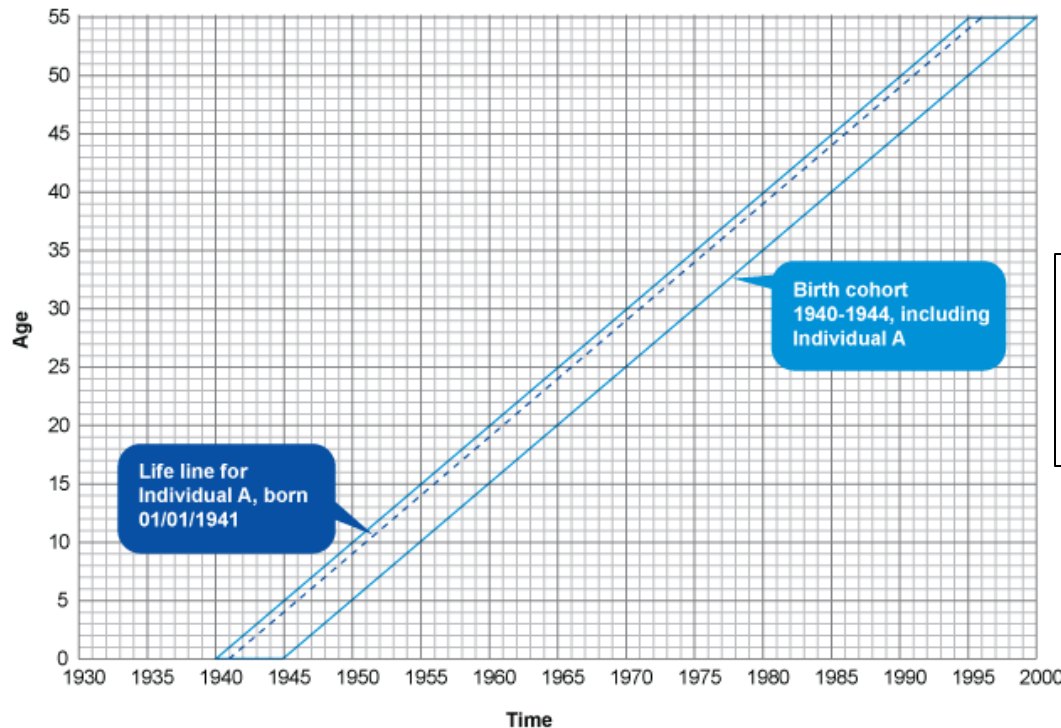


**Source:** Population Analysis for Policies & Programmes, PAPP101: Introduction to Demographic Analysis, S02: How to measure demographic events, International Union for the Scientific Study of Population, online course, [http://papp.iussp.org/sessions/papp101\\_s02/PAPP101\\_s02\\_010\\_010.html](http://papp.iussp.org/sessions/papp101_s02/PAPP101_s02_010_010.html)

Cohorts are composed of individuals with a shared demographic event. The lives of the individual members of a cohort are shown on a Lexis diagram as a life line. A life line is a 45° diagonal line. This line shows that as each interval of calendar time passes, an individual will increase in age by the same amount.



# Lexis diagram: cohort perspective, step by step (2)

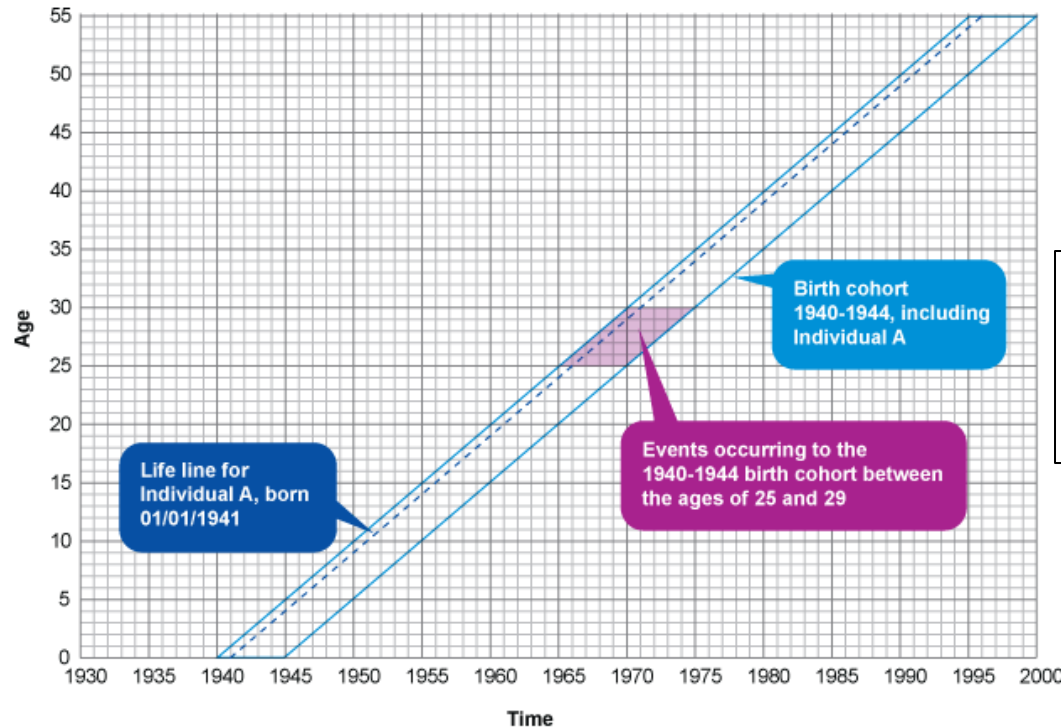


**Source:** Population Analysis for Policies & Programmes, PAPP101: Introduction to Demographic Analysis, S02: How to measure demographic events, International Union for the Scientific Study of Population, online course, [http://papp.iussp.org/sessions/papp101\\_s02/PAPP101\\_s02\\_010\\_010.html](http://papp.iussp.org/sessions/papp101_s02/PAPP101_s02_010_010.html)

In the same way that horizontal and vertical bands represent age or time intervals, a diagonal band represents a birth cohort.



# Lexis diagram: cohort perspective, step by step (3)

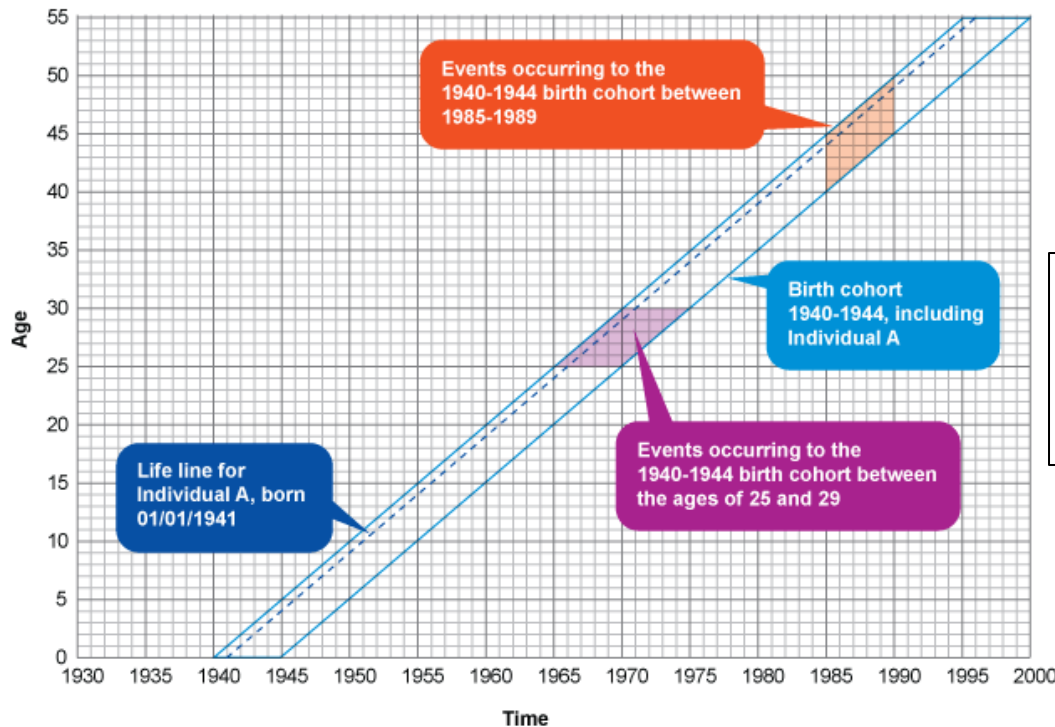


**Source:** Population Analysis for Policies & Programmes, PAPP101: Introduction to Demographic Analysis, S02: How to measure demographic events, International Union for the Scientific Study of Population, online course, [http://papp.iussp.org/sessions/papp101\\_s02/PAPP101\\_s02\\_010\\_010.html](http://papp.iussp.org/sessions/papp101_s02/PAPP101_s02_010_010.html)

A cohort at a particular age (cohort-age measures) can be seen as a "lozenge" shape which shows that the demographic events we are interested in are happening to a specific birth cohort in a specific age interval. For example, the 1940-1944 birth cohort at ages 25-29 is depicted by the purple "lozenge" shape. Notice how the shape is bounded by horizontal lines which depict age.



# Lexis diagram: cohort perspective, step by step (4)



**Source:** Population Analysis for Policies & Programmes, PAPP101: Introduction to Demographic Analysis, S02: How to measure demographic events, International Union for the Scientific Study of Population, online course, [http://papp.iussp.org/sessions/papp101\\_s02/PAPP101\\_s02\\_010\\_010.html](http://papp.iussp.org/sessions/papp101_s02/PAPP101_s02_010_010.html)

A cohort at a particular time (cohort-time measure) also can be seen as a "lozenge shape" which shows that the demographic events we are interested in are happening to a specific birth cohort in a specific time interval. For example, the 1940-44 birth cohort during the period 1985-1990 is shown as the orange "lozenge" shape. Notice how the shape is bounded by vertical lines which depict a time period.

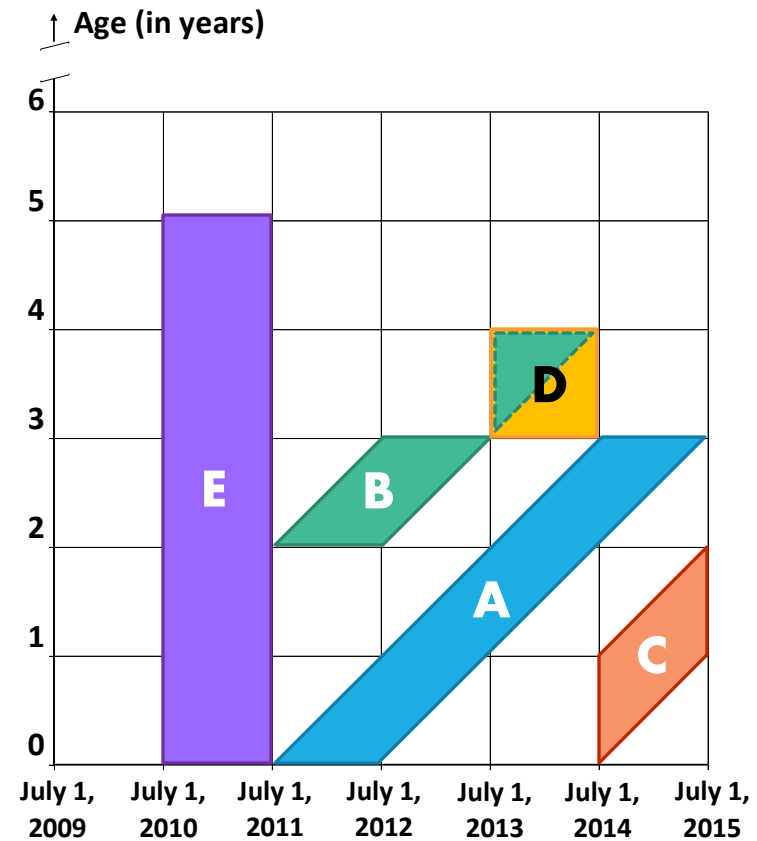


# Lexis diagram: a recap

- The Lexis diagram can show the experience of a cohort as they move through life between (parallelogram A)
- It can also show the experience of a cohort in a particular age interval (parallelogram B) or in a particular time interval (parallelogram C)
- A square includes the demographic events from two different cohorts (square D)
- A rectangle for a time interval shows experience of all age groups of interest during that specific time period (rectangle E)

Cohort data

Period data

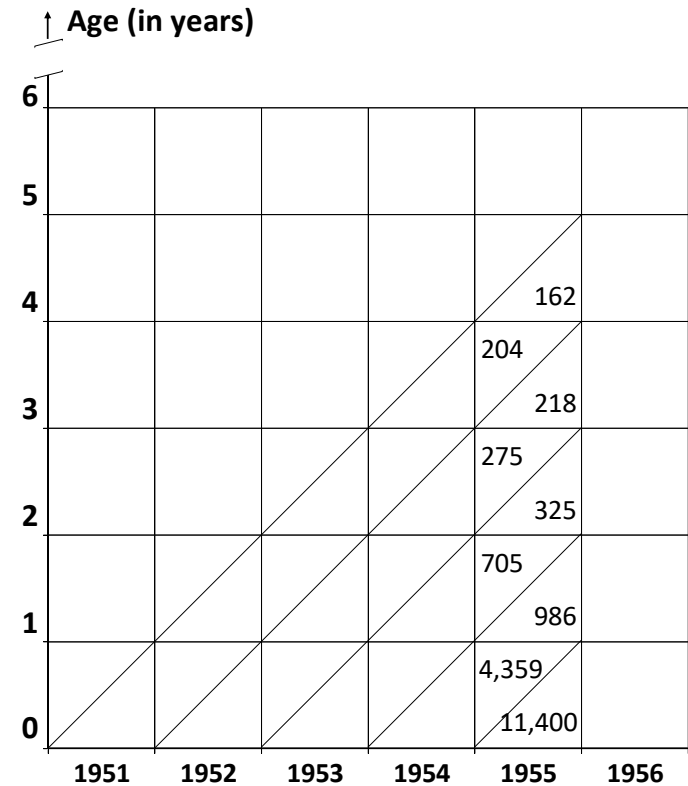




# Lexis diagram: an example

**Male Deaths, by Year of Birth and by Age, France, 1955**

Year of birth	Age in years	Number of deaths
1955	0	11,400
1954	0	4,359
1954	1	986
1953	1	705
1953	2	325
1952	2	275
1952	3	218
1951	3	204
1951	4	162



**Source:** Pressat R. Demographic Analysis: Methods, Results, and Applications. 1972.

# Using birth cohort information to measure demographic events: technical details

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- Because the annual period of interest overlaps two calendar years (July to June), the birth cohort will be named accordingly.
- Hence, birth cohort  $x/x+1$  will consist of persons born between July 1, Year  $x$  and June 30, Year  $x+1$ .

**Example:** Persons born between July 1, 2014 and June 30, 2015 will belong to the 2014/2015 birth cohort.

- Persons that were not born yet at the beginning of the reference period could be categorized as aged “-1”.

**Example 1:** A child is born on **August 1, 2014** (birth cohort **2014/2015**). This child is emigrating on **January 1, 2015**. For classification purposes, this will result in one emigrant aged -1. (Nobody from birth cohort 2014/2015 was born at beginning of reference period.)

**Example 2:** A child is born on **June 1, 2014** (birth cohort **2013/2014**). This child is emigrating on **January 1, 2015**. For classification purposes, this will result in one emigrant aged 0. (Everybody from birth cohort 2013/2014 was aged 0 at the beginning of reference period.)



# Using birth cohort information to measure demographic events

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## ■ Births

- Age does not apply for births. (everybody is born at age 0 !)
- Easiest component to derive birth cohort.
- All births occurring between July 1,  $x$  and June 30,  $x+1$  are part of the same birth cohort.

## ■ Deaths

- Age at death should not be used unless it is the only information available.
- Date of birth should be used to derive age at the beginning of the reference period (July 1,  $x$ ).

## ■ International migration

- Age at the time of migration should not be used unless it is the only information available.
- International departures and international arrivals should both continue to be calculated to measure net international migration.
- Date of birth should be used to derive age at the beginning of the reference period (July 1,  $x$ ).
  - Using birth cohort rather than age is particularly crucial for measuring net international migration. When using age, cases where departure/arrival are recorded for same the person could contain inaccuracies if birthdate is occurring during time of travel.



# Equations of the cohort-component approach in the Canadian context

Age 0

$$P_{(t+1)}^0 = B_{(t,t+1)} - D_{(t,t+1)}^{-1} + I_{(t,t+1)}^{-1} - (E_{(t,t+1)}^{-1} + \Delta TE_{(t,t+1)}^{-1}) + RE_{(t,t+1)}^{-1} + \Delta NPR_{(t,t+1)}^{-1} + \Delta N_{(t,t+1)}^{-1}$$

From 1 to 99

$$P_{(t+1)}^{(a+1)} = P_t^a - D_{(t,t+1)}^a + I_{(t,t+1)}^a - (E_{(t,t+1)}^a + \Delta TE_{(t,t+1)}^a) + RE_{(t,t+1)}^a + \Delta NPR_{(t,t+1)}^a + \Delta N_{(t,t+1)}^a$$

For 100 years and over

$$P_{(t+1)}^{100+} = P_t^{99+} - D_{(t,t+1)}^{99+} + I_{(t,t+1)}^{99+} - (E_{(t,t+1)}^{99+} + \Delta TE_{(t,t+1)}^{99+}) + RE_{(t,t+1)}^{99+} + \Delta NPR_{(t,t+1)}^{99+} + \Delta N_{(t,t+1)}^{99+}$$

where

- (t, t + 1) = interval between times t and t+1;
- a = age; ★ *At beginning of period*
- $P_{(t+1)}$  = estimate of the population at time t+1;
- $P_t$  = base population at time t
- B = number of births;
- D = number of deaths;

- I = number of immigrants;
- E = number of emigrants;
- $\Delta TE$  = net temporary emigration;
- RE = number of returning emigrants;
- $\Delta NPR$  = net non-permanent residents;
- $\Delta N$  = net interprovincial migration.

**Source:** *Population and Family Estimation Methods at Statistics Canada*, Catalogue number 91-528-X, Statistics Canada, 2015.  
<http://www.statcan.gc.ca/pub/91-528-x/91-528-x2015001-eng.htm>

# Equations of the cohort-component approach in the SDM context (a proposal)

**Age 0** 
$$P_{(t+1)}^0 = B_{(t,t+1)} - D_{(t,t+1)}^{-1} + (\text{In}_{(t,t+1)}^{-1} - \text{Out}_{(t,t+1)}^{-1})$$

**From 1 to 84** 
$$P_{(t+1)}^{(a+1)} = P_t^a - D_{(t,t+1)}^a + (\text{In}_{(t,t+1)}^a - \text{Out}_{(t,t+1)}^a)$$

**For 85 years and over** 
$$P_{(t+1)}^{85+} = P_t^{84+} - D_{(t,t+1)}^{84+} + (\text{In}_{(t,t+1)}^{84+} - \text{Out}_{(t,t+1)}^{84+})$$

## From 5 to 84:

- For dissemination purposes only, after calculating population estimates for single years, it could be considered to aggregate 5-year age groups.
  - Small numbers
  - Negative populations are possible

$$P_{(t+1)}^{(a,a+4)} = P_{(t+1)}^{(a)} + P_{(t+1)}^{(a+1)} + P_{(t+1)}^{(a+2)} + P_{(t+1)}^{(a+3)} + P_{(t+1)}^{(a+4)}$$



# Thank you!

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