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ANNUAL SURVEY OF GENERAL ECONOMIC THEORY: THE PROBLEM OF INDEX NUMBERS *

by

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ANNUAL SURVEY OF GENERAL ECONOMIC THEORY

THE PROBLEM OF INDEX NUMBERS [x)]

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1. Introduction.

The problem of how to construct an index number is as much one of economic theory as of statistical technique. Indeed, all discussions about the "best" index formula, the "most correct" weights, etc., must be vague and indeterminate so long as the meaning of the index is not exactly defined. Such a definition cannot be given on empirical grounds only, but requires theoretical considerations. It seems fitting, therefore, to include a survey of this subject in the ECONOMETRICA surveys of general economic theory.

The index-number problem arises whenever we want a quantitative expression for a complex that is made up of individual measurements for which no common physical unit exist. The desire to unite such measurements and the fact that this cannot be done by using physical or technical principles of comparison only, constitute the essence of the index-number problem and all the difficulties center here.

Constructions of this sort may be attempted for many different kinds of measurements: prices and quantities of economic goods, traffic intensities, fertility of the soil, etc. Each kind of index-number may be considered separately or in connection with other kinds, as, for instance, when a price-index and a quantity index are used as elements in Irving Fisher's equation of exchange [1/] or, more generally, when an index-number is constructed for each of several factors that constribute to a joint result, as in the case considered by Wisniewski. [2/] This general aspect of the problem will not be treated here. The survey will be confined

1/ The Purchasing Power of Money, first edition, New York, 1911.
2/ Journ. Am. Stat. Ass., March, 1931

x/ Reprint from ECONOMETRICA, Journal of the Econometric Society, Vol. 4, No. 1, January, 1936 Colorado Springs, Colorado U.S.A.

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to the problem of price index numbers only.

Even within this narrow field several interpretations are possible. The variety of purposes is well discussed by Wesley C. Mitchell who says, amongst other things:[1]

"For example, some one should compile a special series for forecasting changes in business conditions. The compiler might select those commodities whose prices in the past have given earliest and most regular indications of changes that subsequently occured in the larger index-numbers, he might weight these series in accordance with their past reability as price, barometers, and he might use whatever method of averaging the fluctuations gave the best results for his porpose. Such a series probably would not be a reliable measure of variations in the purchasing power of money, but it probably would be better adapted to its special purpose..."

The present survey will not discuss index-numbers of this sort but be confined to those whose object is to measure some sort of purchasing power. Only the fundamental theoretical problems will be considered. We shall not take up practical questions connected with the reliability or fullness of the data nor discuss questions arising out of the practical necessity of using a limited number of representative articles.

The main contributions in the field, as thus circumscribed, are to be found in the works of W.Stanley Jevons,[2] F.Y. Edgeworth,[3] C.H. Walsh,[4] Irving Fisher,[5] Wesley C. Mitchell,[6] A.C. Figou,[7] Carrado Gini[8] François Divisia,[9] René Roy,[10] J.M. Keynes,[11] A.L. Bowley[12]

1/ The Making and Using of Index Numbers, U.S. Bureau of Labor Statistics bulletin No. 284, October 1921, page 24.
2/ Papers reprinted in Investigations in Currency and Finance, London 1909
3/ Various papers, most of which are reproduced in Vol. 1 of Papers Relating to Political Economy, London, 1925.
4/ The Measurements of General Exchange Value, New York, 1901. Also Quar.J Ec;, 1924.
5/ Loc. cit. and The Making of Index Numbers, first edition, Boston, 1922.
6/ Loc cit.
7/ Wealth and Welfare, London 1912, Chap. III, second edition, 1924, Chap.V.
8/ Metron, July, 1924, and Feb., 1931.
9/ "L'indice monetaire et la théorie de la monnaie." Separately and in Revue d'Economie Politique, 1925
10/ Révue d'Economie Politique, 1927.
11/ A Treatise on Money, London, 1930, Vol. 1, Book II.
12/ Jour. Roy. Stat. Soc., 1919, 1921, and 1926, Econ.Journ., 1923 and 1928

Gottfried Haberler, 1/ L.V. Bortkiewicz, 2/ A.A. Konüs, 3/ R.G.D. Allen 4/ and Hans Staehle. 5/ I may perhaps also add a reference to some of my own papers. 6/

The standard older work is Edgeworth's. Among the more recent contributions, Staehle's seems the most original and constructive, although perhaps lacking somewhat in simplicity and perspicuity.

I shall now summarize the salient features of the progress made in this field by these authors, and on certain points shall also try to extend the analysis a little further.

2. The Atomistic Approach.

There are two fundamentally different ways in which the problem of price index numbers may be approached. We term the atomistic and the functional approach. In the atomistic approach the prices p^1 , p^2 ,....,p^N, and the quantities q^1, q^2, ...,q^N, of the various goods are considered – at least in the main – as two sets of independent variables. And the desideratum is to define a certain function of these 2N variables which will give a plausible expression for the "general movement" of prices. In the functional approach certain characteristic relations are assumed to exist between prices and quantities. This changes the whole nature of the problem. While in the atomistic approach a logical and unique definition of the index number is impossible, such a definition becomes, as we shall see, possible in the functional approach.

Consider first the atomistic approach. A typical example is the way in which Irving Fisher lets a mechanical balance illustrate the two sides of the equation of exchange 7/ On one side are hung at different distances from the fulcrum a loaf, a coal scuttle, and a roll of cloth,

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1/ Der Sinn der Indexzahlen, Tübingen, 1927, Also Weltw. Arch. 1929.
2/ Nordic Statistical Journal , 1923, 1924, 1932
3/ Economic Bulletin. Conjuncture- Institute of Moscow. n. 9/10,1924. (Russian)
4/ Economica, May 1933.
5/ Archiv. f. sozialw. u. Sozialpol.,1932. International Comparisons of Food Costs, 1934 (In studies and reports on the International Labour Office. Series N., No. 20) ECONOMETRICA. 1934, page 59 and The Review of Economic Studies, June, 1935.
6/ New Methods of Measuring Marginal Utility, Tubingen, 1932. Section 9. Also Journ. Am. Stat. Ass., Dec., 1930.
7/ The Purchasing Power of Money, Chap. II, Section 3.

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the three being kept in balance by a purse suspended on the other side. The weights represent quantities and the"arms" (distances from fulcrum) prices. Alternative magnitudes of the weights and arms are discussed, illustrating the conception of prices and quantities as independent variables. The averaging of the prices is illustrated by hanging all three items in one average point. The arm of this point - if the momentum is to be the same - is, of course, uniquely determined, thus giving the impression that the conception of an average price is well defined. This latter part of the example is dangerously misleading. Indeed, that feature of the example which entails the unique determination if the average arm is the physical commensurability of the weights, one pound of bread being - from the viewpoint of the mechanical balance - equal to one pound of coal, etc. But it is precisely the absence of this phisical commensurability that constitutes the index-number problem.

The indeterminateness created by physical incommensurability is, in fact, very troublesome in the atomistic approach. Various attempts have been made to overcome it.

First we have what Edgeworth called the indefinite standard approach,1/ which may more appropriately be called the stochastic approach. Here the assumption is made that any change that takes place in the "price level" ought, so to speak, to manifest itself as a proportional change of all prices. Whatever deviation there is from this strict proportionality must be looked upon as due to other causes than those we think of when we speak of the price level change. How in concreto these two sets of causes are distinguished is not essential for the conceptual existence of a proportionality factor, but as a matter of fact the distinction is usually - more or less explicitly - taken to be that between monetary and non-monetary causes.

According to this conception, the deviation of the individual price changes from proportionality must be considered more or less as errors of observation. But then the application of the theory of errors should enable us to determine the underlying proportionality factor. If we compare the two price situations, 0 and 1 (denoted by subscripts), any

1/Papers, Vol I, pages 196 and 235

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of the individual price ratios $p_1^k | p_o^k$ (k=1,2,....,N)may in a first approximation be taken as just as good an estimate of the price level change as any other of these ratios. Consequently, their simple average will give an estimate of the price level change between 0 and 1. If weights are to be applied at all in this averaging, they should express the precisions of the individual observations; these need not be proportional to the economic importance of the goods, as measured, for instance, by the quantities, q^k, or the values,1/ $p^k q^k$. Refining this type of analysis, one is led to study the statistical distribution of the individual ratios $p_1^k | p_o^k$. Criteria may be developed for testing the normality of the distribution, the independence of the observations, and so on. Considerations of this sort lead to the adoption of averages different from the arithmetic, in particular - in the case of skew distributions - the geometric average 2/ and - if the observations are noticeably dependent - the weighted median 3/ (Laplace's "Method of Situation").

Thus, the notion of a "price level" here becomes essentially stochastic. We may make probability statements about it, but not "exact" statements like those we make about other magnitudes in an economic theoretical scheme. In consequence, the price level has a meaning only when a great number of individual goods enter into it. As Edgeworth says: 4/ "To me the conception appears somewhat indefinite as applied to two or a few articles and without relation to the theory of averages."

This conclusively rules out the possiblity of using the above concept as the definition of the price level. At least for many kinds of economic analysis it will be an impossible concept, for instance, in cases where it is desired to build up a hierarchical order of indices, each index being itself a composite of lower order indices. Furthermore the logical basis of the whole concept seems untenable. We cannot assume that the "monetary factor" will manifest itself as a proportional change of all prices. I am, therefore, in agreement with Keynes when he

1/ Edgeworth, Vol 1, page 243. " Assuming...accidental deviations..."
2/ Edgeworth, Vol I, page 238
3/ Edgeworth, Vol I, page 249
4/ Econ. Journal, 1923, page 343

/ vigorously criticizes

vigorously criticizes the stochastic definiton of the price level as being "root-and-branch erroneous",[1] and with Gini, who says, "qu'on ne peut arriver á résoudre le probleme de la manière envisagée."[2]

I am here speaking of the exact definiton of the price level concept. The study of price ratio distributions and similar questions is in itself highly significant from other different viewpoints, for instance, as a means of describing concretely the various goods according to their price behaviour, etc., as done by Frederick C. Mills [3] and others, or as a means of elucidating the nature of various index number formula that have been suggested heuristically as approximations.

Another attempt to escape indeterminateness - while still employing the atomistic viewpoint - is the test-approach. It consists in formulating certain formal tests regarding the function that expresses the price level change from one situation to another. The exponent of this approach is Irving Fisher. Let P_{01} be the index number that expresses the ratio between the price level in point 1, the "object point", and the price level in point 0, the "base point", P_{01} is assumed to depend on the prices $p_0^1 \ldots p_0^N$, $p_1^1 \ldots p_1^N$, and the quantities $q_0^1 \ldots q_0^N$, $q_1^1 \ldots q_1^N$. Some of the more important tests are:

The Identity test: $P_{00} = 1$

The point reversal test ("time"reversal test): $P_{01}\,P_{10} = 1$

The Circular test: $P_{01}\,P_{12} = P_{02}$.

The Commensurability test: P_{01} shall not change by changing the unit of measurement for any of the individual goods.

The Determinateness test: P_{01} shall not become zero, infinite or indeterminate, if an individual price or quantity becomes zero.

The Proportionality test: If all individual prices have changed in the same proportion from 0 to 1, P_{01} shall be equal to the common factor of proportionality.

Sauerbeck's index, i.e., the simple arithmetic mean of the price ratios,

$$P_{01}^{Sau} = \frac{1}{N} \sum \frac{p_1}{p_0} \tag{2.1}$$

1/ A Treatise on Money, Vol.I page 85.
2/ Metron, 1924, page 21
3/ The Behavior of Prices, New York, 1927.

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(which may be looked upon as the result obtained by the simplest possible form of the stochastic approach), satisfies only the identity, commensurability, and proportionality tests. The well known formulae of Laspeyre and Paasche,

(2.2) $$P_{01}^{La} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$$

(2.3) $$P_{01}^{Pa} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$$

satisfy the commensurability, determinateness, and porportionality tests, but not the point reversal test, and, a fortiori, not the circular test. The crossing between them,

(2.4) $$P_{01}^{Ide} = \sqrt{P_{01}^{La} \cdot P_{01}^{Pa}},$$

considered by Bowley, recommended by Walsh and Pigou, and called by Fisher the "ideal" formula, satisfies the point reversal test but not the circular test. The same is true of Edgeworth's formula,

(2.5) $$P_{01}^{Ed} = \frac{\Sigma p_1 (q_0 + q_1)}{\Sigma p_0 (q_0 + q_1)}$$

On the other hand, the arithmetic average with constant weights,

(2.6) $$P_{01}^{Ari.c.w.} \quad \frac{\Sigma p_1 \, q}{\Sigma p_0 \, q}$$ (the q's independent of the point 0 and 1),

as well as the geometric average with constant weights,

(2.7) $$P_{01}^{Ge.c.w.} = \frac{\Pi p_1^{\,a}}{\Pi p_0^{\,a}} = \frac{(p_1^{\,1})^{a^1} \ldots (p_1^{\,N})^{a^N}}{(p_0^{\,1})^{a^1} \ldots (p_0^{\,N})^{a^N}}$$ ($\Sigma a = 1$ and the a's independent of the point 0 and 1)

satisfy the circular test (for any set of three points for which the q's or the a's are the same). In addition, (2.6) will satisfy the other tests mentioned. For any comparison where the quantities, q, can be assumed sensibly constant, (2.6) gives, therefore, a satisfactory solution. This is, however, only a trivial case. The fundamental difficulty is that,

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in most cases, particularly for geographical comparisons or comparisons between remote points of time, it is absurd to assume constant q's. In any such case, we must let the formula depend on the actual q_0's and q_1's, which brings us back to formulae of types (2.1)-(2.5).

The difficulties here discussed are unavoidable so long as we maintain the atomistic viewpoint and consider the p's and q's as independent variables. On this assumption (and assuming certain continuity properties of the index-number formula), I have indeed proved that three such fundamental tests as the commensurability, determinateness, and circular tests cannot be satisfied at the same time.[1/] And, even if some of the tests are abandoned (Fisher is, for instance, willing to give up the circular test), the remaining ones do not lead to a unique formula.

Although the test approach cannot lead to one particular formula that may be taken as <u>the</u> definiton of the price level, it is however, a convenient tool for judging the comparative merits of various formulae that suggest themselves heuristically as approximations to a price level defined by some other means.

A special tool that deserves mention in connection with the atomistic approach is the <u>chain method</u> originally introduced by Alfred Marshall.[2/] This method is adopted to data where the points are ordered in a unique <u>sequence</u>, which, in practice, means time series, but not geographical data. Let P_{01} be any index formula for the direct comparison between two points, for instance, one of the formulae defined by (2.1)-(2.7). The <u>chain</u> index $\bar{P}_{st}$, between any two points s and t, is then defined by

(2.8)
$$\bar{P}_{st} = \frac{P_{01}P_{12}\cdots P_{t-1,t}}{P_{01}P_{12}\cdots P_{s-1,s}}$$

0 being the first point existing in the data. The definition (2.8) obviously applies as well for $s<t$ as for $s\geq t$. If $s<t$, (2.8) reduces to

(2.9)
$$\bar{P}_{st} = P_{s,s+1}P_{s+1,s+2}\cdots P_{t-1,t}.$$

1/ <u>Journ. Am.Stat.Ass., Dec., 1930</u>
2/ <u>Contemporary Review, March, 1887</u>

/ Any chain

Any chain index, $\bar{P}_{st}$, satisfies the point reversal test and the circular test no matter on what kind ofelementary index it is built. If the elementary index meets the circular test, there is no difference between the chain index and the direct index computed by the same formula.

The chain method has been given an elegant logical justification by Divisia. His derivation is in essence as follows. The problem is to split the total value pq into a product of two factors,

(2.10) $$PQ = \sum pq,$$

of which the first P may be taken as representing the "general level of prices" and the second Q as "the total physical volume." In order to do so, Divisia considers the historical path in the 2N dimensional space whose coordinates are $p^1 \ldots p^N$, $q^1 \ldots q^N$. Along the historical path we have

(2.11) $$PdQ + QdP = \sum pdq + \sum qdp,$$

and dividing this by (2.10) we get

(2.12) $$d \log P + d \log Q = \sum ad \log p + \sum ad \log q,$$

where

(2.13) $$a^1 = \frac{p^1 q^1}{\sum pq} \ldots a^N = \frac{p^N q^N}{\sum pq}$$

The formula (2.12) holds good whatever the definiton of P and Q, provided only that (2.10) is fulfilled. The formal analogy between the terms on the two sides of (2.12) very naturally suggest accomplishing the definition of P and Q by equating the terms of(2.12) <u>separately</u>, i.e., by putting

(2.14) $$d \log P = \sum ad \log p,$$

(2.15) $$d \log Q = \sum ad \log q.$$

The equality (2.14) is a differential definition of the price index and (2.15) a similar definition of the quantity index. If (2.14) is integrated numerically, we are merely led to a chain index of the form (2.9), or, more generally, (2.8). As the elementary formula of the chaining, we may get Laspeyre's or Paasche's or Edgeworth's or nearly any other formula, according as we choose the approximation principle for the steps of the numerical integration. If the weights (2.13) are constant

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over the integration period, the result will sinply be - as pointed out by Roy[1] - the geometric average (2.7). Since (2.7) satisfies the circular test, there is in this special case no difference between the chain index and the direct index.

The divergency which exists between a chain index and the corresponding direct index (when the latter does not satisfy the circular test) will often take the form of a systematic drifting. This means that, with increasing t. the ratio $\bar{P}_{st}$ P_{st} $(t > s)$ departs more and more from unity (upwards or downwards as the case may be). To understand this, consider the triangle divergency

(2.16) $$D_{rst} = P_{rs}P_{st} \,|\, P_{rt},$$

In terms of D, we have

(2.17) $$\frac{\bar{P}_{0t}}{P_{0t}} = D_{012}D_{023}D_{034}\cdots D_{0,t-1,t}.$$

If the circular tests holds, D= 1. Otherwise it may be highly probable that D is say, larger than 1. As an example, consider Sauerbeck's index. Here

(2.18) $$D^{Sau}_{0,s+1,s+2} = \frac{\frac{1}{N}\sum\frac{p_{s+1}}{p_0}\cdot\frac{1}{N}\sum\frac{p_{s+2}}{p_{s+1}}}{\frac{1}{N}\sum\frac{p_{s+1}}{p_0}\cdot\frac{p_{s+2}}{p_{s+1}}}$$

Formula (2.18) can be transformed by means of

(2.19) $$\frac{1}{N}\sum xy = \bar{x}\cdot\bar{y} + \sigma_x \sigma_y r_{xy},$$

where x and y are any two variables, $\bar{x}$ and $\bar{y}$ their means over N values, σ_x and σ_y their standard deviations, and r_{xy} their correlation coefficient. The relation (2.19) is verified simply by writing the formula for the correlation coefficient and rearranging the terms. (The formula also holds if all summations are taken with weights.) Equation (2.19) shows that the average of a product is larger or smaller than the product of the averages according as the two variables are positively or negatively correlated. Putting in (2.18)

1/ Revue d'Economie Politique, 1927

/ we get

we get

(2.20)

$$x = \frac{p_{s+1}}{p_0}, \qquad y = \frac{p_{s+2}}{p_{s+1}}$$

$$D^{Sau}_{0,s+1,s+2} = \frac{1}{1 + uv.\, r_{xy}},$$

where $u = \sigma_x \mid \bar{x}$, $v = \sigma_y \mid \bar{y}$ are essentially positive. On the whole, those prices that have changed less than the average from 0 to s + 1, will change more than the average from s + 1 to s + 2; they will "catch up"; hence r_{xy} negative and (2.20) larger than 1. Sauerbeck's index used for chaining will, therefore, drift upwards. A similar argument shows that Laspeyres index will drift upwards, Paasche's downwards. Even in the crossing, (2.4), some downward drifting is left, as shown experimentally by Persons. 1/ The word "drifting" must not be taken to mean that the direct index is right and the chain wrong. This cannot be decided by the above formal considerations.

The chain method has been generalized by Gini into a net-work method, 2/ applicable whether the data are ordered in a sequence or not. He proposes two formulae which may be termed Gini's aggregate crossing and two point crossing, respectively.

(2.21)

$$P^{Gi.a.c.}_{01} = \sqrt[M]{\prod_r \frac{p_1 q_r}{p_0 q_r}}$$

(2.22)

$$P^{Gi.t.p.c.}_{01} = \sqrt[M]{\prod_r P_{0r} P_{r1}}.$$

Here Π designates a product over all the M points occurring in the material. The quantity P_{0r} in (2.22) is any elementary index. Both (2.21) and (2.22) satisfy the circular test over the range to which the crossing is extended. For M =2, (2.21) reduces to Fisher's "ideal" formula. The inconvenience of the Gini crossing is that recomputations have to be made when more data are included. This will not occur frequently in geographical price comparisons, however, for which this method is primarily intended.

1/ Review of Economic Statistics, May, 1921

2/ Metron, Aug., 1931, page 10.

/ 3. The Functional Approach.

3. The Functional Approach.

In the functional approach, prices and quantities are looked upon as connected by certain typical - in point of principle, observable - relations. Here we do not - as in thestochastic approach - make the assumption that ideally the individual prices ought to change in the same proportion as we pass from one situation to another. We face the deviations from proportionality and take them merely as expressions for those systematic relations that serve to give an economic meaning to the index number. The resulting index will, in point of principle, appear as observable with the same sort of precision as the price of an individual commodity, provided the necessary data are available.

These data include something more than just a set of prices and a set of quantities associated with each situation, and in practice, of course, the complete data may be difficult to get. This leads to methods of approximations and limits in which one uses to a considerable extent formulae of the kind discussed in Section 2. But there is the fundamental difference that we now know the question to which an answer is sought and have, therefore, a basis for our judgement about the various formulae.

There are various alternative sets of data, each of which is sufficient for the functional definition of the index number. Subsequently, some of them will be mentioned. To start with, we shall indicate certain general properties which the data must have in order to make the definition possible.

Consider any two situations, 0 and 1. In the most general formulation of the problem, these situations may differ in any number of respects: different kinds of populations, different kinds of goods transacted or consumed or produced, etc. We assume that total money expenditure is well defined and quantitatively observable in each of the situations; let it be ρ_0 and ρ_1 respectively. It must, if the concepts of prices p_t and quantities q_t are defined, be equal to

(3.1) $$\rho_t = \sum p_t\, q_t.$$

If each of the situations, 0 and 1, is characterized by a given set of prices and quantities, then ρ_0 and ρ_1 will by (3.1) be two given numbers. In the functional approach they must not be looked upon as such, but as

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capable of a certain variation, i.e., the expenditure within situation 0 may assume different values, and similarly for 1.

Suppose that we dispose of some criterion by which it is possible to ascertain objectively whether or not a person who in 0 spends an amount ρ_0, is just "as well off" as a person who in 1 spends an amount ρ_1. If they are equally "well off", the two amounts may be called equivalent. In symbols, ρ_0 equiv. ρ_1. If such a criterion exists, we take the ratio

$$P_{01}^{Func} = \frac{\rho_1}{\rho_0} \qquad \text{(when } \rho_0 \text{ equiv. } \rho_1\text{)}, \tag{3.2}$$

as the definiton of the functional price index number between 0 and 1. So far the definition is, of course, only formal; its practical value will depend on the possibility of actually finding an objective equivalence criterion. Before turning to this, note that the idea of being equally "well off" enters in some form or another as an essential element into the theories of writers on index-numbers. For instance, Knut Wicksell[1/] says: "...müsste man zu diesem Zwecke die verbrauchten Warengattungen selbst und ihre relative Bedeutung für die wirtschaftenden Individuen anstellen..." Konüs takes "Konstanter Bedürfnisstand" as the criterion [2/] by which to define the true index. Gini [3/] presents an argument which is tantamount to defining equal "well being" by the equality of the "average level" of the marginal utilities of the individual goods. Bowley [4/] defines the cost-of-living index number by asking: "What change in expediture is necessary, after a change of prices, to obtain the same satisfaction as before? and Bortkiewecz [5/] requires that: "... der dem Arbeiter im

1/Geldzins u. Guterpreise, Jena, 1898, page 12. Staehle (Intern.Comp. of Food Costs, page 4) takes Wicksell as one of the authors who has given up the attempt at defining a "true" index number. This is not correct, it seems, Wicksell realized the impossibility of doing it on an atomistic basis, but at the same time saw the funtional possibility, as indicated by the above quotation.

2/Quoted after Bortkiewicz, Nordisk Statistisk Tidskrift, Bd.11, page 18

3/Metron, 1924, Vol.IV. See in particular pages 16, 22, 140, and 144.

4/Economic Journal, 1928, page 223

5/Nordic Statistical Journ., 1932, page 17

/ Zeitraum 2

Zeitraum 2 zuzubilligende Geldlohn...ihm die gleiche Gesamtbefriedigunf sichert, wie der Geldlohn, der ihm Zeitraum 1 zustland, oder anders angedrückt, dass der Reallohn...gleich hoch bleibt."

Royal Meeker 1/ rallies to Bowley's view. Keynes 2/ says:
"Two collections of commodities are equivalent if they represent... the things which are purchased by... two persons of equal sensitiveness and possessed of equal real incomes of utility".

Haberler 3/ takes a similar position. Allen 4/ and Staehle 5/ define equivalence of expenditure by the fact that the two quantity combinations considered lie on the same indifference locus in a given indifference map.

A great number of other authors could also be quoted who more or less explicitly adopt the definition (3.2). This definition seems, indeed, to be the only plausible one. It is applicable not only to cost of living indices, but equally well to general indices of deferred payments, wholesale prices, etc.

How then can we obtain objective criteria for being equally "well off"? This requires, in the first place, that we segregate a certain group of individuals, the definitional group for the index number in question, for instance, working men's families in the case of a cost of living index, wholesale merchants (or perhaps retailers?) in the case of an index of wholesale prices, etc. We assume that the question of the definitional group is settled. In the second place, it must be possible to observe objectivly one or more parameters, $u, v, \ldots, \lambda$, that characterize the behaviour of a typical individual in the definitional group, and which may be taken as indicators of equal "well being". Let us call them the behavioristic parameters. Suppose that it is possible to observe objectively- within each of the situations considered the covariation between the money expenditure and the behavioristic parameters. Let this function for the situation t be

1/ Encyclopedia Britannica, article on "Cost of Living"
2/ A Treatise on Money, Vol I, page 97
3/ Der Sinn der Indexzahlen, 1927, pages 77-83
4/ Economica, May, 1933
5/ Review of Economic Studies, June, 1935

/ (3.3)

(3.3) $$\varphi_t = \varphi_t\,(u, v, \ldots, \lambda).$$

The functional price index between 0 and 1 - which now may be called a general parametric index - will then be

(3.4) $$P_{01}^{Par} = \frac{\varphi_1\,(u, v, \ldots, \lambda)}{\varphi_0\,(u, v, \ldots, \lambda)}$$

In general P_{01}^{Par} will depend on $u, v, \ldots, \lambda$. But, conceivably, the functions φ_1 and φ_0 may be such that (3.4) is independent of these parameters and depends only on the subscripts 0 and 1. If this is so for any situations 0 and 1, we shall say that the index satisfies the expenditure proportionality condition. In this case, a small 0 expenditure and a large 0 expenditure must be multiplied by the same number in order to get the equivalent 1 expenditure.

Formula (3.4) fulfils the circular test identically in $u, v, \ldots, \lambda$, and whatever the functions φ_t (provided only they are single-valued). Thus, by adopting an appropriate theoretical basis, we sarisfy a desire which is ineradicable on the part of most practical statisticians but which it is virtually impossible to satisfy with the usual formulae of the atomistic type. In this survey we shall consider various methods of constructing behavioristic parameters.

[illegible]