NOTES ON INTEGRATION, WELFARE
AND PROJECT VALUATION
Louis Lefeber

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Santiago, Chile
1970
Published in August 1970

(Published in Spanish: December 1969)

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Mr. Louis Lefebre, the distinguished United States Economist, at present holds the Hecht Chair of International Economics at the Brandeis University, Waltham, Massachusetts. This study represents part of his contribution to the research on criteria for evaluating integration projects carried out by the Latin American Institute for Economic and Social Planning.
Introduction

The experience in Latin America and elsewhere shows no close correspondence between development planning and actual project selection. In some extreme cases, governments have found themselves embarrassed by their inability to generate sufficient projects to exhaust foreign aid provided in the form of programme finance. But even when there was no shortage of potential projects, it may not have always been possible to demonstrate that particular projects selected within the framework of a development programme were, of all the available alternatives, those best suited for furthering the purposes of that programme. This naturally could be the case if the costs and benefits of particular projects are not readily quantifiable — as in the fields of health or education — because selection might then have to be based on judgment unsupported by quantitative analysis. In other instances, however, when numerical valuation would be possible, the selection of projects may still not demonstrably correspond to the relative priorities of the over-all plan or programme, because the methods of project valuation may not be properly understood or applied.

It is the gap or lack of correspondence between plan purposes and project selection which has motivated several recent efforts to study and evolve applicable criteria for project valuation. If rigorous means for computing a project's social profitability can be designed, and if all potential projects can be valued accordingly, then from the alternatives those projects with the highest social profits can be selected for implementation.

Measurement of social profitability requires a quantifiable description of the connection between social welfare and the diverse activities affecting it. Given that total welfare can be represented by a relationship in which all activities of direct welfare interest are entered with suitable weights — i.e., that there is a well defined welfare function — and that the purpose of planning is to maximize welfare subject to the diverse technical, political and market constraints, the value of each added project or activity is determined by its contribution to the value of the welfare function.

Such an approach to project selection may, however, be more readily applicable to planning in an individual nation — be it an open or a closed economy — than in the context of an economically integrated group of countries. In a single nation which has a degree of social and political homogeneity (a willingness to accept and to further social goals which may conflict with and override particular individual interests) it may be legitimate

1/ See, for example, the forthcoming works by I.D. Little and James Mirrlees (sponsored by OECD) and by S.A. Marglin, A.K. Sen et al. (Sponsored by UNIDO). These are not yet available in final form. For published sources see references at the end of this paper.
and feasible to assume the existence of a dominant national concept of welfare which can be summarized, at least in schematic form, in a unique welfare function. On the other hand, in the case of economic integration which is a loose federation of otherwise politically independent nations, social and political homogeneity and the existence of a unique concept of welfare could not be expected. It is one thing to reach agreement on, say, domestic income redistribution and another to agree on international redistribution of incomes. In fact, unless economic integration is also accompanied by the formation of a political or governmental union among the participating countries, a unique welfare function for the integrated Region may not be definable. Hence, a unique set of criteria for the valuation of "integration projects," i.e., projects specially conducive to promoting, strengthening or building economic interrelationships among Latin American countries, may not exist either. For this reason, where project valuation is concerned, much depends on the nature and purpose of integration.

Economic and political integration may be wanted per se. This may be for sentimental reasons, i.e., because of a desire to realize Latin unity, or for rational considerations, such as the expectation that an integrated Region could carry greater weight in international political and economic negotiations than the sum of its components. I shall not further consider these possibilities, not because they are irrelevant—they may have important implications for economic development—but because this is not the place to discuss them. In any case, to the extent that integration is desired per se, in the valuation of integration projects a positive weight must be attached to the fulfillment of this objective as well.

For the purposes of this discussion, I shall assume that integration can be justified by the expectation of a favorable effect on the economic development of the Region as a whole. This would be in response to the subsequent increase in inter-regional trade and factor movements which in turn would bring about improved resource utilization due to production according to comparative advantage, exploitation of scale economies and other favorable static and dynamic changes. The measurable gains from integration would consist of an outward shift of the production possibility frontier of the Region as a whole. This increase in the total productivity of the integrated Region could not be automatically equated, however, with a net welfare gain in all participating countries or even for the Region as a whole. The reason is that integration would necessarily have to result in changes in the national and international distribution of income which—if not compensated for—may not be consistent with what is considered socially just by the affected individual societies (governments) or by the Latin American community as a whole.

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2/ The term "region" throughout this study relates to Latin America as a whole.

3/ Integration requires that instead of the quasi-bilateral economic relationships Latin American countries now individually maintain with hard currency blocs, the integrating countries should develop their resources on a regionally multilateral basis and that trading relationships with the rest of the world should be maintained by the Region as a whole. This necessarily would bring about trade diversion some of which might be harmful to the Region or the rest of the world or both. This I disregard in the discussion.
The traditional concept of international trade is based on the notion that well-defined national boundaries exist which prevent the international movement of productive factors but do not hinder the movement of commodities. The motivation to trade is provided by the unequal absolute and relative resource endowments of the trading nations (absolute and comparative advantage) and by the increasing returns to scale in those industries which due to the limitations of domestic markets cannot attain a minimum efficient size. In the case of trade motivated by absolute or comparative advantage, the changes in the structure of production and factor markets can be brought about by competitive market forces; however, the exploitation of scale economies can take place only in a monopolistic setting so that in the absence of government intervention, an a priori expectation about the direction of specialization and market adjustment may not be had. In either case, within the limits imposed by natural and artificial trade barriers, trade tends to close the gap between the price ratios (terms of trade) prevailing in the individual national markets. Since real factor prices are also affected in the process, trade results in a change in the functional distribution of income, i.e., that distribution which is determined by competitive market forces in response to the relative scarcities prevailing in the factor markets.

The gains from trade consist of an increased supply of goods for domestic use, i.e., more can be had of some or all commodities than would be attainable in autarky. In the case of trade motivated by increasing returns to scale, the source of this gain is obvious. However, to realize it, an adequate government control of monopolies, i.e., of pricing and resource allocation, may be necessary. Under competitive conditions the gains from trade are derived from a more efficient utilization of the unequally distributed productive resources among the trading nations. Furthermore, if competition prevails, some gains from trade are assured for each trading nation. This, of course, does not mean that every individual or group necessarily profits from trade. However, a welfare gain can be assured if those who within any one nation are adversely affected by trade can be compensated from the gains from trade.

Thus the above concept of international trade describes a world in which a series of spatially separated markets are linked through commodity movements. The international movement of productive factors is not permitted, but a more efficient resource utilization is nonetheless possible; commodity flows substitute, as it were, for the international reallocation of resources.

4/ For a discussion of the pure theory of trade see e.g., Kindleberger[1].
If trade is not profitable, nonparticipation or the option of autarkic production is automatically open to any one nation. This much is assured by competitive market responses. The latter provide protection, however, only against a reduction of the domestic consumption capacity below its autarkic level and not against a potential welfare loss caused by trade induced adverse changes in the distribution of income. If compensation from the gains from trade is not feasible for those who are adversely affected, the possibility of a welfare loss cannot be excluded.

In contrast to the case of free trade, the option of nonparticipation is by definition not given to member countries in a fully integrated Region. Integration means that in addition to free commodity trade, the movement of productive factors is also unhindered within the Region. It is to be expected that upon integration commodity flows as well as factor movements (migration and capital flows) will take place in a pattern which is determined by the supply and demand conditions in the different participating countries. The motivation for trade among the participants is based on Regional price differences, and factor movements are induced by the prevailing rent or wage differentials. Subject to the cost of migration or capital transfer, in the absence of other barriers, factor movements will continue until real factor returns are equalized everywhere within the integrated Region. As a consequence, in contrast to the earlier case of international trade, it is possible that one or more of the Regions could be deprived of productive factors. In fact, as long as production is more profitable in some Regions than in others, productive effort may become concentrated in one or a few efficient Regions, leaving the less efficient Regions with diminished productive activity.

Actually, herein lies the fundamental difference between trade and integration. With trade there is choice between autarky and participation in international markets and, in either case, the original resource endowments can be retained by the individual nations. In a state of integration the loss of some resources by one country to some other member country may be inevitable and in extreme cases integration could lead to a reduction in the level of economic activity in a particular member country. There is reason to believe, however, that in a world in which various frictions may slow down the process of adjustment, the difference between international trade and integration may be only a matter of degree. At least in the short run, the transition from an imperfect international trading relationship to a state of integration may not drastically change trade flows and the pattern of Regional resource distribution. Nonetheless, in a dynamic world, it is likely that in the long run both trade and factor movements can more readily develop through integration.

Though integration may adversely affect the domestic production capacity of a less developed or less efficient member nation, it is nonetheless possible that a corresponding welfare loss could be avoided. Specifically, if nonconstant returns to scale prevail in productive activities or if the initial factor endowments are skewed - e.g., there is surplus land or labor in any participating country - integration makes it possible

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5/ See Chapter 8 in Lefebert[3] and Chapter 6 to 8 in Lefeber[4].

6/ This should be particularly the case in Latin America where the cultural background of the participants is reasonably homogeneous. One of the most important current trade barriers, the lack of an efficient intraregional transport and communications system, could be readily overcome by a vigorous integration policy.
through the concomitant factor movements to produce more of some or all commodities within the integrated Region than would be feasible if all countries were producing in autarky or trading without international reallocation of resources. In other words, integration increases the production possibilities for the Region as a whole; hence, the compensation of adversely affected member countries is also possible.

3

Institutional Requirements

Thus the compensation principle can be invoked to justify not only free trade but also integration.7/ The problem is, however, that the existence of the means for the implementation of the principle can not be taken for granted. This is so because the feasibility of compensation depends: a) on a clear knowledge of and political agreement on where social justice lies and b) on the pragmatic issue whether the necessary institutional requirements for carrying out a suitable scheme of compensation are available. The first requirement assumes that there exist adequate institutions and processes for obtaining national and international (Regional) consensus with regard to income distribution. The second requirement implies that there is a legal and organizational capacity for bringing about the desired national and international distribution of incomes.

This is not an academic question. Domestic consensus in any one nation on the desirability of participation and international agreement on the form of integration may not be readily forthcoming unless there is also some assurance that the national and Regional productivity gains can be translated into welfare gains.8/

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7/ It is a well-known theorem of international trade theory that under certain conditions free trade combined with the use of neutral (lump sum) taxes and subsidies for the redistribution of benefits, leads to a higher level of utility in each trading country than any level of utility attainable in autarky or with restricted trade. For a rigorous argument see Samuelson[6]. The policy relevance of the theorem depends not only on its assumptions - e.g., competition, etc. - but also on whether there exists an operational procedure for the requisite redistribution of the benefits.

8/ To be sure, even the productivity gains could not be secured without a gradual and arduous process of political and economic reorientation from economic nationalism and bilateral outward orientation to inter-regional cooperation. This demands not only political willingness to act accordingly, but also the creation of a whole set of new institutions needed to cope with the problems of political and economic reorientation. Among the latter the most important ones would be a payments agreement and a nonpolitical forum for arbitration among the integrating nations. A payments agreement would be necessary to protect the participants against a real or imagined loss in foreign exchange earning capacity and, perhaps, to facilitate a coordinated discrimination against scarce currencies. A nonpolitical forum for arbitration, on the other hand, would make it possible to remove or separate the negotiations of interregional conflicts from the day to day national politics of the member countries. Of prime importance is to deal in a non-nationalistic manner with those interregional allocation problems which competitive market processes cannot resolve, such as market sharing agreements between suppliers of goods produced under conditions of increasing returns to scale, incentive pricing for agriculture and other problems affecting trade policy.
If it is assumed that in one form or another free trade or integration can be accomplished, what institutional means are available which could ensure a socially just redistribution of the gains? As far as internal redistribution is concerned, the existing means vary according to the established institutions from country to country. However, to the extent that redistribution depends on progressive taxation and direct income transfers, it is clear that the currently available means are at best marginal in most Latin American countries. As far as international redistribution among the nations of the Region is concerned, the means are for all practical purposes nonexistent. But assuming that effective means for redistribution could be devised, criteria according to which the gains should be redistributed nationally and internationally would also have to be determined. In this respect not much can be said about international redistribution because in the absence of a unique supranational concept of welfare it is a matter for international agreement and the corresponding criteria could be specified only accordingly. In contrast, criteria for domestic income redistribution within each particular nation can be derived from the stated goals of development if these can be specified in the form of a national welfare function.

4

The Goals of Development

If it is agreed upon that the immediate purpose of integration is to attain greater productivity through improved factor utilization, there remains the more fundamental question: what social purposes is this improved efficiency supposed to serve? It is evident that each country, depending on its own social priorities, would want to pursue somewhat different or differently weighted purposes. Nonetheless, in all parts of Latin America there is broad agreement on the highest political level that priorities should be consistent with and conducive to social and economic development.

This, then, is the light in which integration must be considered. Since it is meant to serve the development interests of the participating countries, its value, and hence the value of activities and projects undertaken for advancing integration, must also be established in terms of the development interests of the same countries.

The implication is that since integration projects by definition must have welfare consequences in more than one country, their valuation requires a knowledge of not only the goals of development but also their relative weighting in all affected Latin American countries. This is a tall order considering that in spite of the not dissimilar cultural heritage there is nonetheless a wide variation of physical and social condition within the Region. Furthermore, the purposes of development are so complex that their complete description, let alone relative weighting, would be impossible even for a single country.

Fortunately, however, certain fruitful simplifications as well as generalizations are possible. The necessity of an adequate rate of economic growth combined-with
improvements in the standard of living of the lower income groups was recognized by all signatories of the Charter of Punta del Este. From this it follows that when social goals are set, the primary attention must be focused on the problem of how to balance the welfare of current and future generations. In other words, it is the socially desirable time path of consumption (private and public) - i.e., its current level and its rate of growth - together with its distribution which must be determined.

If there exists this common denominator for the consideration of social goals, there also exists the possibility of obtaining a cohesive strategy for Regional development which is homogeneous of purpose and consistent with particular national interests. Industrial development -stimulated by free trade or integration- would have to be combined with a massive effort to strengthen the demand for unskilled labor which in turn would also require the development of the Region's agricultural and other primary resources. Such an approach would be quite in keeping with the stated purposes of Latin American development, because in addition to motivating growth it would, in the process, also provide the means for effective income redistribution.

5

Models for Project Valuation

It should be clear by now that project valuation is inherently an empirical task which required not only quantitative estimates of production relationships and demand and supply levels but also a knowledge of policy parameters including relative weights on social goals. The information may or may not be available. If it is, so much the better.

2/ Primary production is lagging and to varying degrees consciously neglected in many Latin American countries because demand is believed to be inelastic and because of the occasionally adverse historical experience with the behavior of the terms of trade between industrial and primary products. These concerns may be relevant only as long as the countries of the Region maintain bilateral international trade relationship with hard currency areas. If the Region were united so that the vertical integration of primary and manufacturing production would be possible on an intraregional basis, the advantages of the processing of primary products and related manufacturing would accrue to the producers in the Region. Moreover, the adverse income elasticity estimates of the demand for food and other agricultural outputs may be of lesser concern to a development strategy which aims to change the distribution of income. The point is that the large population segments in the lowest income groups necessarily have a higher income elasticity for food and staples than the higher income groups, with the result that income redistribution would boost the Regional demand for staples even if total Regional income were to remain constant. Population growth within the Region and in the rest of the world can also be expected to add to demand. Finally, the upgrading of production from staples to generally income elastic agricultural outputs, i.e., fruits and vegetables, is also feasible if sufficient Regional demand can be generated.

10/ The low income groups of the Region consist of the rural underemployed -the subsistence farmer and landless farm worker- and the unskilled migrants living on the fringes of the urban industrial or commercial economy. The size of these groups varies from country to country, but it is a significant proportion of the active population almost everywhere within the Region. Since the institutional means for income redistribution on the requisite massive scale do not exist in any country in Latin America, the only effective means would be to increase the rate of employment at minimum acceptable wage levels. Then, if the demand for and hence the relative scarcity of unskilled labor is increased, the market mechanism itself is made to contribute to effective redistribution. At the same time, given a strategy aimed at increasing agricultural production, the level of wage consumption - a constraint on the rate of employment in countries where labor productivity is low - can be raised pari passu with the growth of the employed labor force.
But if it is not, the implication is not that a rigorous analytical approach can be neglected and that some common sense method can be adopted. Here a very strong warning is in order. As is perhaps unrealized even by their proponents, common sense approaches are also based on some underlying sets of assumptions the consistency and validity of which cannot be tested as long as they remain implicit. Common sense can frequently be misleading and the less empirical information there is available the more important it is to be meticulous in the analytical approach.

Analytical project valuation relies on model building the purpose of which is to approximate the conditions surrounding a particular situation or problem. Thus even the simplest models from which criteria for project valuation are to be derived must be descriptive of conditions prevailing in the country where they are to be applied.

The simplest model corresponds to an analogue of a competitive free market system and its significance lies in the fact that the widely used and frequently recommended methods of valuation based on commercial profitability are related to it. These consist of the measurement of a project's private profitability over time accounted in terms of existing or estimated market prices and discounted by the market rate of interest (frequently -and wrongly- by the arbitrary rate paid to a lending agency). If the present discounted value of the market benefits exceeds the costs, the project is considered profitable. Ideally, several or many projects should be evaluated in order to select, in descending order, those which show the most favorable cost-benefit relationships. In reality, however, there seems to be a paucity of relevant alternatives, and the method is frequently used to justify single projects without comparisons.

Though variations of this method are widely practiced and are believed to be the "hard headed" approach to project selection by those who would equate commercial profitability with social welfare, it can be relevant only under certain very limited conditions. The reasons are important to analyze because they provide an insight into the most basic problems of project valuation.

The objection against reliance on the free market model is the implicit assumption that the satisfaction of private preferences (maximization of individual utilities) necessarily leads to the highest social good. This may be the case only if the conditions for pure competition prevail (in the absence of increasing returns to scale and certain externalities), if social preferences coincide with private ones and if the existing distribution of income is socially optimal. These conditions patently do not hold for a number of reasons, one of the most important being the inequality of income between rent earners, wage earners and chronically unemployed or underemployed labor to which reference was made in the previous section.
Commercial Profit Maximization as Basis for Valuation

Only under particular circumstances can commercial profit maximization and the use of free market prices lead to socially optimal project valuation. This would be the case if in an unemployment economy the social purpose were to attain the highest possible rate of economic growth -or the fastest route to full employment- without regard to current welfare. Then -if pure competition prevailed- the function of profits would be to generate investable surpluses and that of consumption to sustain the labor force needed for production. However, if profits were not reinvested but squandered on conspicuous consumption or moved abroad, the relevance of the model would be destroyed.

In any event such a model depicts a rather stark world in which the different social groups forego current satisfactions in the interest of rapid development. It is unlikely that close historical examples exist, and such as there are would be confined to limited periods in 19th Century England, the Soviet Union before the war and perhaps to postwar Japan. Capitalists -private or state- must be bent on accumulation and labor must accept a wage rate which is defined by some concept of a subsistence minimum (e.g. sueldo vital). Then, the highest growth rate consistent with that wage and the level of employment are determined by competitive free market hiring.\(^{11}\)

Thus from a social welfare point of view the uses of free market prices for project valuation in an economy where underemployment is chronically significant, implies the sacrifice of current welfare for the welfare of future generations. Its very starkness is such that no humanistically oriented society could readily accept it. Certainly, the implied puritanic form of entrepreneurship and the requisite political control of labor would be alien to Latin societies. There is nonetheless, one very important reason why in the context of project valuation the lessons of the free market model should be kept in mind. If sufficient institutional means for income redistribution in favor of the underemployed do not exist -which, as mentioned above, is the case in Latin America- the rapid creation of new employment opportunities over time may be the only effective

\(^{11}\) See Lefeber and Chakravarty[5]. To the extent that labor does have the political power to force wages upward (which is the case in many parts of Latin America) the choice may be between higher current consumption for the employed at the expense of current total employment. In any case, the rate of economic growth would be diminished and the rate of absorbing unemployment would be correspondingly slowed down. If labor does not save, a higher wage rate necessarily means an increase in the demand for consumer good production and a cut in the rate of investment; hence, the induced change in the current level of employment is ultimately determined by the relative factor intensities in investment and consumer good production. This points up the importance of considering the choice of technology: if investment is more labor intensive than consumer goods -which can be the case if, for example, construction and earthworks are important components of investment- an increase in the wage rate may decrease current employment. However, if relative intensities are reversed, the opposite may be the case. For rigorous discussion see Lefeber[2].
long run means for improving social justice. However, if a more humanistically oriented set of social preferences is selected, there still remains the immediate conflict between current welfare (redistributed consumption) and the capacity to increase the rate of growth of employment. This can be analyzed only by scrutinizing the relationship between wages, technology, employment, consumption, and growth.

7

Weighting of Current Welfare as well as Growth

If the free market valuation aimed at attaining the highest possible growth rate is too extreme -and there is little doubt that this is so- the level of current as well as future redistributed consumption must be included in the social welfare function with a positive weight. In countries with a high rate of underemployment this must logically manifest itself by higher levels of current consumption by the lowest income groups. Hence, the output of consumer goods must be increased relative to new investment and the increment in consumption output must be made available to the low income earners.

If current income and hence consumption redistribution can be accomplished only by increasing the current rate of employment, a free market allocation of resources may not be equal to the task. Assuming that the wage rate is maintained at an institutional minimum level, the market -determined employment of labor would have to be such as to equate labor's marginal product to that wage rate. Hence, as long as the stock of capital and other resources remains constant and the wage rate remains the same, employment in the economy cannot be increased above the free market rate without a loss in profits which private owners of capital would not want to incur.

Thus if the social priorities indicate a positive weight on current consumption combined with redistribution -as one can infer from political pronouncements to be the case in Latin America- government intervention may be needed to decrease the capital-labor ratio, as the unaided free market cannot be expected to bring about such a change. Even if the elasticity to change the capital-labor ratio in existing installations is limited,

12/ But, in any case, this argument in favor of the competitive free market approach is destroyed if consumption as a function of profits is significant.

13/ Consumption, in this context, is understood to include also such social services as health care and education.

14/ See Lefeber[2]. The argument could be made that such an approach is relevant only in countries in which unemployment or underemployment prevails. This is true. However, the Region in its totality has sizeable labor reserves in the form of agricultural underemployment which could be made available to specific labor shortage areas through migration. Furthermore, the labor force participation rates are generally low throughout the Region and these would automatically grow if the demand for labor would increase. In particular, increases in female employment could augment the total participation rates by as much as thirty percent or more.
public and private entrepreneurs would have to be motivated to lower the capital-labor ratios at least in new undertakings.

Since the preceding arguments have a bearing on project design as well as on valuation, it is useful to bring up the question of commercial efficiency. It is well-known that Latin American manufacturing enterprises are frequently inefficient and unable to break into international markets. This, of course, can have many causes, such as a lack of rigorous managerial practices or insufficient technological preparedness. But assuming that these can be overcome, the question still remains: What is the proper technology for a Latin American enterprise? As suggested above, those who favor criteria based on commercial profitability would recommend a profit maximizing choice of technology. But profit maximization in terms of what prices? It is clear that private enterprise in general will not want to sacrifice profits for some intangible concept of social welfare. Furthermore, production decisions and project valuation in a free market system are naturally carried out in terms of actual prices. And yet, should a private entrepreneur be permitted or even encouraged to select a capital intensive mode of technology in a country with an incipient abundance of labor -if that happens to be the profit maximizing choice of technology- because the institutional minimum wage rate is in excess of labor's marginal product for more labor intensive technologies? This is not an easy question to answer. The social good may be best served if current unemployment is diminished by labor intensive patterns of production. At the same time, profitability and the need to compete in free markets may require -given the actual prices confronting the entrepreneur- a more capital intensive technology. The problem is then, could the price system be so adjusted as to coordinate social interests and private profitability and could that same price system be used also to reward private entrepreneurs for being both commercially efficient and contributing to the social good? The answer is yes if the difference between free market prices and the socially optimal price structure can be determined.

Consider the difference between the private and social cost of, or private and social benefit from hiring a unit of labor. The private cost of labor is the wage rate the employer must pay and the private benefit is the increment in output (revenue) the employer obtains from hiring the last unit of labor. The social cost of the last unit of labor is the value of the added resources needed to sustain it and the social benefit is the sum of the increment in social welfare due to the increase in redistributed consumption and of the private benefit which accrues to the employer. Thus, a social valuation must include along with the private benefits also the value of redistributed consumption or must subtract the latter from the private cost of hiring labor. The greater the weight placed on the current levels of employment and consumption as opposed to their rates of growth, the greater the gap between the social and free market valuation of a project's profitability because of the difference in the valuation of the added demand for employment created by the project.

The social valuation of redistributed consumption and hence the social valuation of employment must naturally vary from country to country depending on economic conditions and social priorities. But one thing can be stated with certainty: as long as there is an abundance of low productivity labor and current welfare as well as growth is positively valued, the choice of projects and technology must be directed toward more employment creation and higher labor intensities than would be the case if commercial efficiency in a free market were the basis for valuation. The implication is that the employment
consequences of alternative projects should be scrutinized and that engineering plans for each project should present alternative technologies. Then, from the alternatives, those projects and technologies can be selected which in terms of the particular country's or countries priorities appear to be most desirable.

Accordingly, if publicly sponsored integration projects are to be designed to reflect social preferences as to redistributed consumption, the capacity to create new employment opportunities -be it through a higher labor-capital ratio or through the opening up of frontier regions for new employment or through other means- must be prominent among the selection criteria. Furthermore, the induced secondary, tertiary, etc., net increments in employment must also be accounted for along with all other private and public benefits.

8

Shadow Pricing when the Rate of Employment
is Limited by the Supply of Wage Goods 15/

The social valuation of all goods and services -including labor's contribution to welfare- can be obtained only through shadow pricing. If in a purely competitive free market economy the allocation of resources happens to be socially optimal, then the shadow prices correspond to the free market prices. More generally, however, shadow prices have a meaning if and only if there is a clearly articulated and properly weighted set of social goals -a welfare function which is to be maximized subject to prevailing resource limitations and technological, political and other constraints. Then, the shadow price of an activity -output or service- can be defined as the change or increment in the value of the welfare function obtained in response to a small (unit) increase in that same activity. Any reference to shadow pricing outside such a framework is meaningless.

A simple and empirically applicable welfare function is of a type where a constant proportion of the socially valued income is spent on consumption and investment, the constant of proportionality being a politically determined parameter. 16/ The latter is equivalent to a relative weight on current consumption and investment. If the scale of the welfare function is so specified that the increments in welfare are measurable in money units, then total welfare can be expressed in terms of national income and the constant of proportionality represents the social rate of saving. Current consumption and investment -the latter representing growth- being the two arguments of the welfare function, the shadow prices of current consumption and investment are determined by

15/ See the Appendix for an illustrative model in which consumption constrains the rate of employment.
16/ The proportion, of course, can be adjusted over time. See relationship (1) in Appendix.
their respective marginal contributions to the value of the welfare function. In other words, the welfare value of consumption is the ratio of the increment in total welfare to an increment in consumer good input. The welfare valuation of investment is analogous.

In the context of an underemployment economy, where the real wage rate paid to employed labor directly translates itself into consumption, one can think of consumer good production as fulfilling two functions: first, as above, of contributing directly to current welfare to the extent of the social weight placed on current consumption and second, of providing sustenance to the labor force qua input into the production process.

Since the welfare valuation of the consumer good as defined above accounts only for its direct contribution to current welfare, there must be an added component to the total valuation of the consumer good for its use as input in current production, including that of investment undertaken for future welfare. The two component shadow prices - the welfare valuation and the input valuation - add up to the total shadow price or optimal market valuation of the consumer good, i.e., that price which is needed to bring forth the last unit of the optimal consumer good output. It is clear that the greater the relative weight on current consumption in the welfare function, the greater must be the welfare valuation relative to the input valuation of the consumer good and vice versa.

If society places no weight on current consumption per se, then the welfare valuation of current consumption is zero and the market price of the consumer good coincides with its input valuation. This gives the clue to the shadow valuation of labor, i.e., the shadow wage rate. Specifically, when focus is on growth alone, then the real wage - labor's consumption - is valued in terms of the competitive free market price of the consumer good; hence, competitive free market pricing brings about socially desirable results and - given that other required market conditions are met - commercial efficiency coincides with social efficiency. On the other hand, when some positive weight is placed on current redistributed consumption per se, then by the above argument, the input valuation of consumer goods must be smaller than the market price. In that case the shadow wage rate is smaller than the market wage rate; hence, the corresponding socially desirable capital-labor ratio is smaller and total employment is greater than what free market efficiency would imply. Under these conditions free market allocation or commercial efficiency is inconsistent with social efficiency; hence government intervention is needed to motivate profit maximizing producers to use resources in a socially desirable manner. In such cases, short of direct controls, payroll subsidies or other appropriate means are indicated for increasing the private returns to added employment and hence, the labor-capital ratio.

The previous analysis focuses on the balance between current and future welfare or current redistributed consumption and investment. Let there be no question about it: the determination of the appropriate rate of social time preference is the heart of the planning problem in all fully or partially planned economies. The analysis, which has already yielded a basic operational concept (the shadow wage rate with its implications for project selection), also provides the means for relating the social time preference to project selection. Specifically, it yields a concept of the social rate of discount which is needed to compare the costs and benefits accruing to different projects over time.

There is a school of thought according to which the social rate of discount is naturally zero because a homogeneous society with a continuous existence naturally places
the same weight on tomorrow's consumption as on today's. However, a decision on the balancing of current consumption and investment in itself reveals preferences concerning the trade-off between the different welfare levels at different time periods. Correspondingly, the model outlined above yields a discount rate which is not independent of the rate of growth indicated by social priorities.

As was the case with the shadow wage rate, the social rate of discount could coincide with the private free market rate if in the interest of growth no weight were to be placed on current welfare. However, if current redistributed consumption is positively weighted in the welfare function, the social rate of discount must necessarily be less than what the free market rate would be. Hence, the free market rate of interest is relevant for discounting only if all the weight is on growth. Otherwise -which would be the normal case in a humanistically oriented society- the relevant rate of discount is smaller than the free market rate.

The social rate of discount and the shadow wage rate are the two primary expressions of social preferences. In addition, of course, as many shadow prices can be derived as desired degrees of sectoral disaggregation. In general, however, shadow prices can be applied to project valuation only if they refer to highly specific activities, e.g., the use or earning of foreign exchange. The corresponding shadow rate can readily be interpreted as the measure of the relative social scarcity of foreign exchange.

Secondary Social Goals

There are certain activities which are so strategic to the attainment of basic welfare goals that they can be thought of as if they were goals. These can be referred to as secondary goals. Improvement in the efficiency of industry and agriculture, improvement in the balance of payments and hedging against future contingencies in a world of uncertainty —to mention a few— come under this heading.

As was the case in the previous section the need for shadow pricing may arise because of the divergence between the free market and socially desirable outcomes in an otherwise perfect world. But if markets are imperfect, if prices are distorted and production is inefficient, shadow pricing can also be used as the basis for correcting the existing price structure.

The primary sources of distortion are feudal or other non-competitive agricultural practices, monopolistic market forms and certain government policies. Of the latter, the overvaluation of the domestic currencies may be the single most important cause of distortion in resource allocation. Even if the exchange rate is set to clear the market, if protection is needed to maintain clearing, the rate may not correspond to its social scarcity. If due to inflationary or other reasons the controlled rate gets out of line with what is maintainable in the long run, the substitution of a shadow rate for the
official rate becomes mandatory in project valuation. To illustrate the case, one has only to refer to projects which on the basis of official—hence, commercial—valuation turn from profitable to unprofitable (or vice versa) at the stroke of midnight on the day of devaluation. In contrast, the social profitability computed on the basis of shadow rates is insensitive to arbitrary rate changes.  

Improvements in economic efficiency in Latin America are sufficiently important so as to include efficiency per se among the secondary goals. To the extent that a project has an important impact on efficiency in industry or agriculture, the value of the estimated net improvements (the induced decreases in the marginal costs of production) must be accounted for along with the primary benefits. Since integration projects may logically be of that type which can significantly add to the general level of efficiency, this must be kept in mind as it may make the difference between the acceptance or rejection of the project in question. Transportation and communication projects particularly need to be scrutinized from this point of view.

There are certain activities—either of a purely political nature or relating to nonquantifiable aspects of social welfare—whose valuation in terms of their quantitative contribution to social goals may not be feasible. The nonquantifiable benefits must not be neglected in project valuation, though it also should be kept in mind that nonquantifiable social purposes may be improperly used to justify projects which aside from short term expediency provide no significant benefits.

Goals which are difficult to weigh or evaluate are numerous and only a few will be mentioned. As mentioned earlier, Latin American integration may have a political value per se which could lend added importance to projects leading to integration. It is not possible to say, however, how a qualitative weight translates in a measurable magnitude.

A more tangible but from the point of view of quantification still difficult problem relates to hedging against foreign exchange losses. Reliance on a single or a few foreign exchange sources may be inadvisable. At the same time, diversification has its costs in the sense that the variance in foreign exchange earnings may be reduced only by accepting a lower expected value of foreign exchange income and even national income. However, the trade-offs cannot be quantified without a knowledge of probability distributions and specific social risk preferences.

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17/ The valuation of projects requiring or producing large amounts of foreign exchange illustrates the difference between primary and secondary goals. The contribution of a project toward the primary goals is valued in terms of the relative weight placed on the goal itself. On the other hand, the contribution of the project to a secondary goal—such as foreign exchange earnings—is valued in terms of the social scarcity of foreign exchange expressed by the shadow rate.
Shadow Pricing of Multi-national Investments

The previous analysis assumes that the investment for a particular project is located in a single country. However, integration projects - such as transportation and communication works - may characteristically require investments or services which are distributed over several countries. What are then the relevant principles of project valuation? This is an important problem since the benefits accruing to a country may not be proportionate to the costs of the project incurred within the same country. Hence, there is the question whether it is possible to devise an equitable plan for the financing of and loan repayment on multi-national projects.

The answer depends on the relevant concept of integration. In the case of a complete political-economic union characterized by a unique supranational welfare function and by free movement of labor and other factors, a single set of shadow prices would be relevant for all countries. On the other hand, if welfare is maximized on the assumption that within the context of integration each particular country maintains its own distinct welfare function, the shadow wage rate, the discount rate and - depending on the payments and clearing relationships - the shadow rates of foreign exchange are necessarily different in each country. Furthermore, while with increasing Regional trade the prices of most goods and services might in the long run converge toward a unique level (except for differentials caused by transportation and other frictions) as long as social welfare priorities differ, the social discount rates and the shadow wage rates will have to remain different from country to country.

It is reasonable to assume that for the foreseeable future, the latter will be the case. This means that the net benefits accruing to each country from a particular multi-national project would have to be valued differently and that the burden of financing would have to be allocated according to a cost-benefit computation which is based on a correspondingly different set of shadow prices in each affected country. The significance of such a procedure is readily illustrated by any multi-national project with potentially high balance of payments effects on the participants. If in each country the project's direct and indirect contribution to the balance of payments is not valued according to the social scarcity of the foreign exchange - which will be different in each participating country - the burden of debt service and repayment will be unjustly distributed among the participants.
Though the qualitative use of shadow prices in itself provides useful insights into the principles of project valuation (e.g., the relationship between social goals and choice of technology), actual valuation requires quantitative estimates. The information from which such estimates can be derived is at best weak and this part of the exercise must be handled with caution. Apart from the insufficiency of data, in countries with steady inflationary pressures actual market prices, wages, rents and interests cannot be made use of without suitable deflation for estimating the real magnitudes needed for valuation. The problem is compounded by the fact that there is no readily available method to identify and separate out the different causes of distortions in the observed price structure attributable to inflation, monopolistic influences and government intervention.

The estimation of shadow prices is, perhaps, the least problematical if the emphasis is totally on growth. In that case, as argued above, the relevant rate of discount is the market rate of interest and the relevant wage rate is the institutional rate. Furthermore, the shadow rate of foreign exchange would correspond to the market clearing rate in a competitive free market. There still remains, however, the problem of eliminating the inflationary effects so as to obtain that price structure which characterizes the real growth path. The money rate of interest less the percentage rate of change in the price level yields the estimate of the discount rate, and the shadow wage rate is the real value of the increment in total consumption due to hiring and added unit of labor.

There must be no ambiguity, however, about the meaning of the market rate of interest. In any Latin American country there is bound to be a variety of observed interest rates ranging from government subsidized development loans to rates on unsecured consumption loans. Which one is appropriate? In all probability none. Instead this is a matter for estimation. The Fisherian concept of the marginal rate of return over cost or the rate of return on new investment gives the relevant magnitude. These can be estimated by well-established methods.

If the social welfare function positively weights current redistributed consumption, then the shadow wage rate and the social discount rate will be smaller than the free market rate. Their estimation may require a complete econometric analysis of the structure of the economy. Another method, based on revealed preferences, for

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18/ A quantitative general equilibrium analysis is given in Lefeber [2] and a demonstration model of similar structure is provided in the Appendix. If such frameworks are used for estimating the discount rate, consumption by the owners of capital has to be explicitly introduced. This is not at all difficult if the not unrealistic assumption is made that capitalists' consumption varies with the stock of capital and not with profits.
estimating a relevant range of discount rates has been suggested by Marglin. If planners can identify alternative feasible time paths for aggregate consumption, then the relevant range of social discount rates is the one for which the present discounted value of the most preferred path is higher than for all other alternatives.

The estimation of the shadow rate of foreign exchange is not more straightforward, since there is no direct indication of what the hypothetical competitive free market scarcity would amount to. The gray or black market rate may provide an estimate but its relevance depends on the proportion of the total foreign exchange transactions which go through the market. In those countries where foreign exchange is controlled, that proportion may be quite small. Hence, the observed free market rate would not be indicative of the relative scarcity in the entire economy.

A more relevant approach consists of measuring the productivity of foreign exchange as input of production. Specifically, one can estimate the present discounted value in domestic currency of an income stream generated by, say, one hundred dollars' worth of imported capital goods. Then, the ratio of this discounted income stream to the dollar value of the imported capital goods gives an estimate of the shadow rate of foreign exchange. The rationale of this approach is that in an efficient market system the importation of capital goods would be carried to the point where such an activity is no longer profitable. Notice, however, that the method relies on a present discounted value estimate; hence, it requires the use of a discount rate. Furthermore, since imported capital goods seldom can be used by themselves to generate income, but must be incorporated into an investment scheme which includes buildings and other domestic components, an estimate of the imputed income shares is also necessary. As an alternative along similar lines, the present discounted value of the increment to the total consumption stream in response to a unit import would also yield an estimate.

Projects Affecting the Structure of the Economy

Some integration projects may be of such size and influence that they may importantly affect production relationships in one or more sectors or countries. This may be the case with very large multi-purpose projects of water control which may discontinuously change transportation and power capacities as well as the supply of cultivable land. In such cases the principles of shadow pricing, as discussed in the previous sections, become irrelevant. Shadow pricing is derived from a logical framework applicable to continuous marginal changes. If discontinuity is induced, if constraints are shifted by discreet intervals, then valuation must be based on some other principle. This is so because the discontinuous structural changes cause changes in the universe itself. In other words, the set of shadow prices which correspond to an economy described by a particular set of constraints cannot be used for the valuation of a particular project in the altered economy which is described by a different set of constraints.
This is not to say that projects of this character should not be evaluated. However, instead of relying on marginalist methods, the proper question is as follows: given that there are resources for the discontinuous displacement of one or more of the constraints on the economy, what are the alternatives? In other words, what must be searched for is not the best position on a given feasibility surface—which can be obtained at the margin with the help of shadow price based project valuation—but the socially most desirable direction of the discontinuous change in the shape of the feasibility surface itself. Obviously, if social welfare concepts are stable, that discontinuous pattern of shifts in the constraints is desirable which, given the resource limitations, permits the attainment of the highest level of social welfare. But here the standard statistical methods relevant to valuation fail (most price parameters are derived from the measurement of margins) and the only practicable means of selection may consist of submitting all relevant alternative projects capable of inducing discontinuous changes to a political body for selection.

Fortunately, it is difficult to conceive of many projects which would have sudden discontinuous effects on the structure of the economies involved. Large projects which are executed over a long time period could result in gradual and continuous changes so that shadow pricing may still be adapted to the particular situation. The problem of dealing with increasing returns to scale projects still remains but as long as the scale effects are confined to the project itself, i.e., they are not transmitted to the relevant portions of the feasibility surface, social marginal cost pricing retains its validity. It should be clear, however, that under those conditions private enterprise can pursue a socially desirable course only if it is subsidized and otherwise controlled by the government.

13

The Choice of Technology

One of the recommendations which emerge from the discussion of social welfare oriented project valuation is the need to consider alternative technologies in the design stage. This is so because, as argued above, a positive weight on current welfare may imply a need for more labor intensive techniques than indicated by free market criteria.

In project design the choice of technology enters at two interconnected stages. First, the method of building or providing the required investment must be considered. Second, the technique for the manufacture of the output or provision of the service must be determined. As an example, consider the case of a transportation project. The construction of the roadbed may be undertaken by methods which can range from highly labor intensive to highly capital intensive techniques. The service of transportation—which relies on the use of the roadbed as well as on other investments in rolling stock and maintenance equipment—can also be provided with variable factor proportions.

It is a shortcoming of actual project design that insufficient attention is given to the choice of technology. In this respect, two basic biases can be singled out: a) the apparent belief that there is no variability in factor proportions, i.e., that the capital-
labor ratios are fixed and that b) the most desirable technology is the one which from the engineering point of view is most advanced. Apart from the fact that these two types of biases contradict each other, each taken by itself is erroneous.

It is true that in certain industries - particularly in hydro-electric power and certain petro-chemical processes - the adjustment of factor proportions is quite difficult and after the installation of the chosen type of investment it may even be impossible. But in most industries - even in heavy industrial manufacturing units such as steel plants - there is scope for variation. If nothing else, yard operations can always be designed to conform to a wide range of capital-labor ratios. In any case, some of the rigidity in capital-labor ratios is clearly due to institutional rather than technical conditions. For instance, even though a given piece of equipment at a given moment of time can only be used in combination with a single operator, if the rate of output is measured over a time interval it becomes evident that the same equipment may be profitably exploited with two operators in attendance. The speed of machine work which requires physical exertion or particular care must correspond to the capacity of a single operator who, if in continuous attendance over an eight or ten hour workday has to proceed at a sustainable rate. On the other hand, if two operators are present, one can be permitted to rest while the other is at work so that the rate of output of the active operator can be increased. 19/ Except in the case of full automation, any tool or machine from pick and shovel to semi-automatic equipment can be treated in this manner. Whether it is profitable to do so depends on the relative cost of labor and equipment; it is only an institutional bias (or puritanical belief) that a worker should not be permitted, let alone encouraged, to take frequent rest periods during paid working hours.

As to the natural propensity of designers to choose that technology which from the engineering point of view is the most advanced, it is likely to lead to higher capital-labor ratios than warranted by actual factor endowments. This is so because new techniques developed in already industrialized countries do not relate to Latin American economic conditions and may even have been developed for labor saving purposes. 20/

Finally the balance of payments implications of project design must also be considered. In general, the greater the capital intensity, the greater will be the proportion of imports in the total investment cost. If the project is evaluated in terms of overvalued official exchange rates and market wage rates, the true social cost of the capital intensive technology will not register. On the other hand if the project is valued on the basis of proper shadow wages and foreign exchange rates and if the secondary benefits are also accounted for, the case in favor of a more labor-intensive technology may be overwhelming.

19/ I am indebted to this example to Mr. M. Solomon, at one time with USAID in India.

20/ Less capital intensive plant design - e.g., steel plants built according to pre-Second World War technology instead of integrated continuous processes - may also be more suitable in terms of the availability of particular technical skills. Furthermore, the educational effects of suitably chosen technology on an unskilled or semi-skilled labor force must not be neglected. Intensive use of tools and semi-automatic machinery, as discussed above, not only increases the rate of employment and output but also results in the training of more semi-skilled or skilled machine operators.
Appendix

The text referred to analytical frameworks in which the rate of employment is constrained by the supply of consumer goods. Since the discussion was general rather than specific, in this Appendix a simplified but fully specified analytical framework is presented. It goes without saying that this analytical framework serves only illustrative purposes. Though it could be made applicable for deriving rough and ready estimates of maximum attainable growth rates and marginal rates of transformation between consumption and investment, the framework itself is too simple to be used for the econometric estimation of the parameters needed for project valuation. Furthermore, as will be evident, it provides no criteria for the selection of optimal capital labor ratios. It demonstrates, however, certain basic logical relationships which are relevant to the argument of the paper. Analysis of greater generality can readily be obtained in the sources cited at the end of this Appendix.

The framework to be presented is a fixed coefficient linear programming model. Fixed coefficients represent rigid technologies and unless more than one set of fixed coefficients is stipulated for each productive process, choice among techniques does not exist. However, the inclusion of multiple techniques could be readily accomplished.

The general form of the welfare function discussed in the text is as follows:

\[ Y = \alpha I^p C^{1-p}, \quad 0 \leq p \leq 1 \]

Here \( Y \) represents the level of utility and its scale is so chosen as to permit the cardinal measurement of social welfare in terms of income. The exponent \( p \) represents the proportion of investment in the national income and \((1 - p)\) is the proportion of consumption. The national income is, of course, measured in terms of welfare prices, i.e., shadow prices associated with a particular solution. \( I \) and \( C \) represent the level of net investment and consumption in either physical or constant money units. The function in the \( I, C \) plane can be represented by a family of hyperbolas which are skewed toward the \( C \) or \( I \) axis according to the chosen value of \( p \). Since the latter must be between zero and one, the utility curves in the limit approach a vertical or a horizontal line.

The fixed coefficient approximation of the welfare function described above is given by the following two relationships:

\[ p Y \leq I; \]
\[ (1 - p) Y \leq C \]
A common sense way of looking at these relationships is to think of income as if it were produced by two inputs: investment and consumption. In other words, the fixed coefficient utility function has the exact characteristics of a Leontief type production function.\(^{21}\)

The analytical framework which delineates the feasible region is given by the following six relationships:

\[
\begin{align*}
(4) & \quad a_1 I \leq L_1; \\
(5) & \quad b_1 I \leq K_1; \\
(6) & \quad a_2 C \leq L_2; \\
(7) & \quad b_2 C \leq K_2; \\
(8) & \quad w(L_1 + L_2) \leq C; \\
(9) & \quad K_1 + K_2 \leq K(t)
\end{align*}
\]

(4) and (5) together represent the fixed coefficient production function of investment and (6) and (7) describe the production function of the consumer good. The a - s and b - s represent the labor (L) and capital (K) requirements per unit of output. Subscript (1) denotes the coefficients and the inputs in the investment good industry and subscript (2) denotes the same in the consumer good industry.

Relationship (8) shows the total demand for consumer goods as determined by the rate of employment. \(w\) is an arbitrarily stipulated real wage rate measured in the units of the consumer good. Since according to (8) the rate of employment cannot exceed that level which in terms of the stipulated minimum real wage rate can be supplied with consumer goods, it is the total supply of consumer goods which limits or constrains the rate of employment.

Relationship (9) shows the distribution of capital. \(K(t)\) is the total amount of capital available in the economy and in the initial period its quantity is given. Note that since the system described by the relationships from (2) to (9) is subject to the law of constant returns to scale and capital is the only scarce input, the exact amount of capital available in the economy is needed only for determining the absolute values of the variables. All relative magnitudes, such as the growth rate or the aggregate capital-labor ratio (total capital to total employment in the economy) can be derived with arbitrarily stipulated initial capital stock values.

\(^{21}\) This fixed coefficient approximation, instead of a set of price lines, is needed because in linear programming the feasibility region is delineated by linear segments. Linear price lines, if they coincide with a linear segment, result in indeterminacy over the entire length of that segment. Determinate solutions must all be confined to vertex points. In contrast, a fixed coefficient utility function permits a determinate solution not only at vertex points but anywhere in between them.
Instead of a formal linear programming solution, the entire feasibility surface can be derived by a process of simple substitutions. Specifically, by substituting (4) and (6) into (8) and (5) and (7) into (9), we obtain the following two overall constraints:

\[(10) \quad C > (w_{a1}/l - w_{a2}) I;\]
\[(11) \quad C < K(t)/b_2 - (b_1/b_2) I\]

These represent two linear relationships in \(I\) and \(C\) with parameters composed of the coefficients of the six relationships from (4) to (9).

The two overall constraints can readily be represented diagramatically as shown in the Figure. Relationship (11), denoted by the line AD is a simple constraint imposed by the scarcity of capital and requires no further interpretation. Relationship (10), denoted by OE indicates that, because of the wage demand, the output of the consumer good must exceed a given minimum amount for each rate of investment. The feasible minimum rate of consumption on the capital constraint is determined by the intersection of (10) and (11). It corresponds to the wage consumption of that rate of employment which is just needed if the highest feasible rate of investment is to be attained. The output combination corresponding to the maximum investment point is shown in the Figure at point B. Those outputs on relationship (11) which in the diagramatic representation are to the right of the intersection denoted by B, i.e., on segment BD, are
non-feasible. The feasible region corresponds to the area of the triangle denoted by OAB and the efficient combinations of outputs lie on the segment AB shown by the heavy line.

The outputs corresponding to the intersection itself are readily obtained by solving (10) and (11). They are given by the following two values:

\[ I = \frac{K(t)/b_2}{(w_1/1 - w_2) + (b_1/b_2)}; \]

\[ C = \frac{K(t)/b_2}{1 + (b_1/b_2) (1 - w_2)/w_1} \]

(12) provides the value of the maximum feasible rate of investment. (13) provides the total consumption which must be produced in the economy if the rate of investment is pushed to its maximum feasible level. Now the maximum rate of growth -a relative magnitude which does not depend on the initial value of the capital stock- can also be deduced. By taking the ratio of the maximum feasible rate of investment to the given initial capital stock, the maximal growth rate is obtained as follows:

\[ g = \frac{1 - w_2}{K(t)} = \frac{1}{w_1 b_2 + (1 - w_2) b_1} \]

Since the absolute value of K(t) enters both the numerator and the denominator of the ratio, it cancels out. Thus the growth rate is shown to depend only on the fixed technical coefficients and the wage rate and not on the initial capital stock.

Notice that as shown by constraint (10), the wage rate and the labor coefficient of the consumer good stand in a significant relationship to each other. Specifically, the product formed by the wage rate and the labor coefficient of the consumer good must be smaller than one. Since the labor coefficient represents the inverse of the average product of labor, the condition is that the real wage rate (defined in units of the consumer good) must be smaller than the average product of labor. The real wage rate represents the marginal demand for consumption in response to employing an added unit of labor and the labor coefficient is the marginal addition to the supply of consumer goods in response to employing one more unit of labor in the consumer good sector. If then the wage rate exceeds the latter, the implication is that a marketable surplus cannot be generated in the consumer good industry since not even its own labor can be supplied with sufficient consumption. Hence, at the given wage rate the economy is not viable. This manifests itself by the fact that the slope of (10) is negative so that there can be no consumption constraint which is binding in the positive quadrant (i.e., where all variables are positive). If the product is equal to one, the constraint is undefined. That would be the case in a subsistence economy where the entire food output is consumed on the farm. If and only if the product is smaller than one can we say that the system is viable. In that case the average product of labor in the consumer good industry exceeds the wage rate so that a marketable surplus can be generated to sustain employment also in the investment good sector.

This system operates on the assumption that labor consumes its entire real wages and that consumption out of profits is zero. Alternatively, one can assume that the
consumption by the owners of capital is subsumed under \( w \) in the form of a surplus value. Since profits are net of consumption, they represent savings. If all saving (net profits) are re-invested, it follows that the profit maximizing solution corresponds to that point on the feasibility surface where investment is at its maximum. Hence, given the assumptions of this model, profit maximization in a purely competitive free market economy automatically leads to the highest rate of investment and the fastest attainable rate of growth. If the two output variables are measured in terms of constant money units, national income \( (Y) \) in constant money terms can be obtained by the simple summation of \( C \) and \( I \). Then if the \( Y \) corresponding to the maximal growth solution is known, the corresponding \( p \) - the social average rate of saving - can also be deduced with the help of (2) and (3). A higher rate of saving is not feasible and an attempt to force it cannot increase the rate of investment or rate of growth. If, on the other hand, a welfare decision is reached to increase the rate of consumption at the expense of investment, \( p \) can be reduced and total consumption can be increased along the feasibility function shown by the line segment \( AB \) in the diagram. However, since it is the growth maximizing point \( (B) \) which corresponds to the competitive free market solution, other output combinations on the feasibility function can only be achieved either by direct government intervention or by so adjusting the price and the wage structure as to motivate profit maximizing entrepreneurs to transfer labor and capital from the production of investment goods into the production of consumer goods. The employment consequences of such a move depend, in this simple framework, on the relative size of the fixed capital and labor coefficients in the two lines of production. In any case, the implied increase in the supply of consumer goods can make the employment of a correspondingly larger active labor force possible.

If the intervention in the market is to be undertaken by decentralized means, i.e., by suitably adjusting the price structure, it is important to consider the shadow prices. Based on the well-known direct-dual correspondence of linear programming maximum and minimum problems, the relationships governing the price variables and the shadow prices themselves can readily be obtained. Their interpretation and implications for decentralized plan implementation are discussed in detail in [2], [5], [6] and [7] cited below.
References


