Mechanisms of default risk transmission and economic policy coordination

Karlo Marques Junior and Fernando Motta Correia

Abstract

This paper analyses the coordination between monetary and fiscal policy in an emerging economy with an inflation-targeting monetary regime, in a context in which default risk shocks can lead to macroeconomic imbalances. It develops a macrodynamic model in order to capture the mechanisms of default risk transmission and its effects on the definition of reaction functions for the monetary and fiscal authorities. The main findings of the model point to the existence of new mechanisms of default risk transmission associated with price and fiscal stability.

Keywords

Economic policy, fiscal policy, inflation, monetary policy, macroeconomics, external debt, emerging markets

JEL classification

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I. Introduction

Given the consensus that low, stable inflation is desirable (Carlin and Soskice, 2006), many countries opt for inflation-targeting regimes. Their experiences have shown that the operating framework of economic policy includes not only measures of a monetary nature, but also the quest for equilibrium in the public accounts.

Among the authors who draw attention to the relationship between fiscal and monetary policy are Sargent and Wallace (1981), Woodford (1995, 1996 and 2001), Leeper (1991 and 2009) and a number who examine the case of Brazil, such as Favero and Giavazzi (2003) and Blanchard (2004).

The main question is whether, on the one hand, the adverse behaviour of some fiscal variables may prevent monetary policy from functioning properly under an inflation-targeting regime with a monetary policy rule similar to Taylor’s (1993). On the other hand, the functioning of monetary policy also affects the performance of fiscal policy.

In an emerging economy, risk perception can affect and be affected by both monetary and fiscal policy. The risk attributed to each country depends largely on its sovereign bond solvency. When a rising interest rate pushes up the cost of debt servicing, default risk may be expected —all else remaining constant— to increase as well. This creates a cyclical effect, since financial investors will accordingly demand a higher premium to buy sovereign bonds.

It is important to look at some aspects of the effect that exogenous economic shocks —such as risk shocks— have on public debt, because inflation-targeting monetary regimes use the nominal interest rate to control the shocks an economy is exposed to. So the effect of monetary policy on public debt is passed through variables such as public debt composition and crises of confidence.

Initially, the effects of monetary policy on debt are associated with shocks that can push inflation off target and thus induce the monetary authority to raise interest rates to cancel out the effects of the shock. Since the monetary authority is free to change the interest rate, debt rises as a function of inflationary volatility. In addition, confidence shocks can occur at times of great uncertainty, when investors tend to flee from higher risk or demand higher yields in compensation, which again pushes up the cost of debt servicing.

In both cases, then, monetary policy affects debt, which can subsequently push up default risk even more and thus trigger the vicious cycle mentioned above. This is why a better understanding is needed of the relationship between the conduct of monetary policy and the behaviour of public debt in settings where sovereign risk can lead to macroeconomic disequilibria.

By the same token, following Sargent and Wallace (1981), if the fiscal authority does not respect intertemporal budgetary constraints, the monetary authority can find itself in difficulty, having to finance debt servicing through inflation tax, thus losing control over inflation.

This article analyses how the coordination of monetary and fiscal policies should be managed in an emerging economy with an inflation-targeting regime, in a context in which default risk shocks can lead to macroeconomic disequilibria. The main contributions of this work are: (i) suggesting a monetary policy rule that includes a fiscal variable, specifically the deviation of public debt from a target, and (ii) establishing a model in which risk is a factor that can lead to macroeconomic disequilibria through its effects on the exchange rate, on the interest rate and, indirectly, on public debt and inflation. In other words, the article aims to answer the following question: what considerations should the monetary authority bear in mind regarding its policies’ impact on the variables targeted by the fiscal authority, and vice versa, in an economic setting such as that described?

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1 In this work, macroeconomic disequilibrium is defined as a situation in which the interest rate, the debt-to-GDP ratio and default risk do not converge over time towards a stable equilibrium.
The article seeks to understand how macroeconomic policies should adapt to exogenous shocks in order to maintain stable equilibrium in different economic circumstances. This involves developing a monetary policy model that could be adopted to minimize the impact of shocks on debt risk, both when the interest rate is used to guide inflation towards set targets, and when fiscal policy targets a particular balance on the public accounts.

This article has six sections. After this Introduction, the second section reviews the theory on which the models used are based. The third section sets out the hypothesis and the comparative statics analysis of the model, in order to analyse the short-term effects of risk on important economic policy variables. The fourth section studies the model’s dynamic stability in a configuration consisting of a budgetary rule set by the fiscal authority in coordination with the monetary authority. In the fifth section, the model is extended by introducing risk perception as an explanatory variable for the exchange rate, with a view to capturing the effects of risk shocks on macroeconomic equilibrium more precisely. Here, default risk is part of the function that determines — and can thus have undesired effect on — the exchange rate. Lastly, the sixth section sets forth the article’s conclusions, which summarize the theoretical corollaries suggested in the model.

II. Coordination between monetary and fiscal policies: a literature review

A number of works draw attention to the link between fiscal policy and monetary policy and indicate that the central bank cannot ignore the influence of fiscal variables on price levels in an inflation-targeting regime that seeks a monetary reaction in line with Taylor (1993). Since, moreover, the functioning of monetary policy affects fiscal policy, coordination between the two is necessary. In setting the interest rate, then, the central bank must keep in mind the trajectory of debt over time.

According to Blanchard, Dell’Ariccia and Mauro (2010), in the 1960s and 1970s monetary and fiscal policies were equally important and were generally treated as two instruments for achieving the twofold objective of domestic and external equilibrium. In the intervening decades, however, fiscal policy tended to take a back seat as policymakers’ attention turned mainly to monetary matters. Nevertheless, several authors signalled the limitations of monetary policy, especially in hostile fiscal conditions. Some of these works are mentioned below.

Sargent and Wallace (1981) state that monetary policy could become ineffective to control inflation if the fiscal authority disregards the government’s long-term intertemporal budget constraint. This would occur because of the need to finance public deficits through seignorage, which generates inflation tax. Depending on the interaction of economic policies, the authors distinguish two situations: monetary dominance and fiscal dominance.

Monetary dominance obtains when the fiscal authorities’ spending is limited by the bond demand function, such that a fiscal surplus is needed to keep a constant ratio between public sector debt and GDP. In this situation, the monetary authorities determine the money supply and government spending is constrained by that decision. With this type of policy coordination, the monetary authorities control inflation through the supply of base money, thus setting up a scenario of monetary dominance in which monetary policy is said to be active and fiscal policy, passive.

In conditions of fiscal dominance, the fiscal authority sets its budgets independently, announcing all current and future deficits and surpluses and thus determining the amount of revenue that must be raised through bond sales and seignorage (Sargent and Wallace, 1981). Here, the fiscal authorities take no account of the need for a large enough surplus to keep the debt-to-GDP ratio under control.
This second type of coordination produces what Sargent and Wallace term “unpleasant arithmetic.” The monetary authorities become passive and lose control over inflation, because they are forced to increase seignorage revenue to keep the government solvent. This is particularly prejudicial when investors’ demand for bonds, balanced with the government’s demand for liquid resources, entails an interest rate higher than the economic growth rate. Nevertheless, although the inflation is generated by a fiscal imbalance, it remains a monetary phenomenon.

The work of Leeper (1991) is another important reference in the literature on coordination between monetary and fiscal policies. Leeper defines four situations in which monetary policy may be considered active or passive, depending on its responsiveness to public debt shocks.

In Leeper’s model (1991), policy authorities can set a control variable actively, while an intertemporally balanced government budget requires that at least one authority sets its control variable passively. When both policies are passive, policy is incompletely specified and the price function is indeterminate. If both policies are active, independent variations are allowed, violating the government budget constraint.

The notion of inflation as a purely monetary phenomenon was later questioned by economists whose ideas became formally grouped under what is known as the fiscal theory of the price level, among them Cochrane (1998 and 2001) and Woodford (1994, 1995 and 2001). According to Kocherlakota and Phelan (1999), under this theory, the definition of price growth as simply the difference over time between growth in the money supply and growth in output is severely flawed. The point they make is that the amount of money agents wish to keep in the present depends fundamentally on their expectations of inflation in the future. This opens up possibility of multiple equilibria in the trajectory of inflation, beyond the relationship between the money supply and the quantity of goods produced. Accordingly, Taylor-type monetary policy rules (1993) alone are not enough to control price levels.

According to Basseto (2008), “the fiscal theory of the price level (FTPL) describes fiscal and monetary policy rules such that the price level is determined by government debt and fiscal policy alone, with monetary policy playing at best an indirect role.” In that case, prices are determined by the fiscal authority through the government budget constraint. As part of this theory, Basseto (2008) also argues that the role of the monetary authority in pricing becomes evident when the interest rate affects the evolution of nominal public debt.

A number of works on fiscal dominance focus on the role of default risk as a mechanism by which the monetary authority of an emerging country running an inflation-targeting regime can lose control of price levels. Two major works in this line refer to the Brazilian economy in the period around the 2002 elections: Favero and Giavazzi (2003) and Blanchard (2004).

Favero and Giavazzi (2003) emphasize Brazil’s highly volatile country risk between 2002 and 2003, and the way in which some economic variables, especially the exchange rate, fluctuated in parallel with risk. This pointed to a vicious cycle propagated in the Brazilian economy by default risk. The authors found that a rise in default risk led —because of the interruption in capital flows— to exchange-rate devaluation and a rise in the debt-to-GDP ratio, which was heavily indexed to the dollar at that time. Devaluation and rising debt pushed up expectations of inflation and thus the monetary policy interest rate as well.

Blanchard (2004) examines the effects of tight monetary policy in an inflation-targeting regime in the context of a high debt-to-GDP ratio, strong indexation of the public debt to foreign currencies and a high degree of risk aversion on the part of international investors.

In these circumstances, an inflation shock leads to a rise in the interest rate and thus in the ratio between net government debt and GDP. This sequence will likely increase risk perception, triggering a capital flight that produces exchange-rate devaluation and, ultimately upward pressure on inflation.
Thus the situation is one of fiscal dominance similar to that described by Favero and Giavazzi (2003), whereby monetary policy is ineffective in controlling inflation in a situation of fiscal imbalance amid high risk aversion. Emerging economies under an inflation-targeting regime may be particularly prone to finding themselves in this perverse situation, insofar as investors consider their bonds a high-risk portfolio choice.

In this scenario, monetary policy loses control over inflation and becomes dominated by expectations regarding fiscal conditions. Blanchard (2004) suggests empirically that the Brazilian economy exhibited such fiscal dominance between 1999 and 2004 and that default risk was the factor that triggered these economic policy interactions.

It may be deduced from these reflections on the combination of macroeconomic policies that the Taylor rule has limitations in terms of ensuring long-term economic stability, especially in emerging economies.

It is important to establish the nature of the default risk transmission mechanism in emerging economies, particularly given their exposure to exchange-rate shocks. Considering this mechanism, the original Taylor rule seems limited in its ability to guide the action of the monetary authority.

III. Risk, interest and debt: identifying default risk transmission mechanisms

Following the discussion above with respect to macroeconomic policy coordination, a macroeconomic model is presented below that seeks to represent hypotheses for an emerging economy with an inflation-targeting regime, in which economic policy coordination is aimed at minimizing the adverse effects of risk shocks.

A few additional considerations are called for before presenting the basic structure of the model. Since economies that rely on inflation-targeting regimes may be exposed to exchange-rate shocks, insofar as exchange-rate flexibility is a basic condition for such a monetary regime, many authors include the exchange rate in their analysis as a variable in the reaction function of the central bank. Under the assumption of an open economy with an inflation-targeting monetary regime, Ball (1999) suggests that optimal monetary policy should incorporate a monetary conditions index comprising the interest rate, the exchange rate and a measure for the inflation target.

However, Ball's analysis (1999) has some limitations when it comes to emerging economies, where—as noted earlier—high risk perceptions can compromise the objective of monetary policy. In such economies, the risk rating is positively correlated with the exchange rate. So, in emerging economies such as Brazil, a reaction function such as the Taylor rule could increase risk perception through interest rate rises, and lead to frequent exchange-rate shocks, as suggested by Favero and Giavazzi (2003) and Blanchard (2004).

Accordingly, in the case of an economy in which the fragility of certain fiscal variables could affect the conduct of monetary policy because of the link between the risk premium and the exchange rate, there are grounds for suggesting that central banks react to agents' risk perception.

The magnitude of the risk premium thus incorporates the uncertainties built into the commitment to remunerate the public debt until its maturity. It must always be recalled that the shocks to which

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2 Svensson (2000) and Ball (1999) assume that in industrialized countries the risk premium follows a random walk that does not affect the conduct of monetary policy, whereas in developing countries there is a strong link between risk perception and capital flight, which has effects on the exchange rate and on inflation. This argument complements the definition of emerging economy given earlier.
the structure is exposed through the maturity structure of the interest rate make the risk premium component subject to expectations shocks, bearing in mind that, where fiscal policy is not committed to a stable debt-to-GDP ratio, agents can demand a high rate of return to compensate for the high risk of assuming a debt with great default probability.

In this context, this work develops a model with three non-homogenous, simultaneous first-order linear differential equations, in order to study the coordination between fiscal and monetary policies. Its long-term variables are the risk premium, the nominal interest rate and the behaviour of the debt-to-GDP ratio. Thus, an analysis is made of the long-run behaviour of the variables, that is, whether or not they converge over time with their steady state, i.e. if the equilibrium is dynamically stable. It should be recalled that long-run risk stability is the desired outcome in order to guarantee macroeconomic stability and monetary policy effectiveness in an inflation-targeting regime.

1. Model of default risk transmission mechanisms

According to the maturity structure of the interest rate, the rate of return on a bond at time $t$ depends on the average short-term interest rate during its term $n$, plus a risk premium given by the market conditions for that bond. Thus, the relationship between short- and long-term interest rates may be rendered as follows:

$$ r_{nt} = r_t + r_{t+1}^e + r_{t+2}^e + r_{t+3}^e + \ldots + r_{t+n-1}^e + R_{nt} $$

where $r_{nt}$ denotes the bond’s real long-term interest rate at maturity, $r_t$ the real short-term interest rate for period $t$ and $r_t^e$ the real interest rate expected for period $t$.

The maturity structure of the interest on the public debt security can be simplified as follows:

$$ r = r^e + R $$

Equation (1) decomposes the return on government bonds into two components: first, the expectations of short-term real interest rates up to maturity ($r^e$) and, second, $R$, the risk premium to which bond buyers are exposed.

In equation (1), $R$ is a measure of default risk that captures the uncertainties relating to the commitment to pay the return on the bond until its maturity. In general, the longer the maturity, the higher both yield and risk. Agents’ perceptions of the magnitude of default risk variation depend on the comparison of a bond that pays rate $r$ with respect to a risk-free security, in this case denoted $\bar{r}$. Since $\bar{r}$ represents the nominal rate on a risk-free security, it may be assumed that the variation in risk $R$ over time is reflected in the difference between those two rates, that is, the difference between rate $r$ and rate $\bar{r}$. The idea is that this gap arises from the risk compensation demanded by agents, so that, over the long term, the larger this difference, the larger the intertemporal variation in default risk, as shown in the following differential equation (2):

$$ \dot{R} = \sigma(r - \bar{r}), \quad \sigma > 0 $$

Thus, coefficient $\sigma$ captures the sensitivity of default risk $R$ to the spread between rates of return on positive-risk bonds and risk-free bonds. Analogously, $\sigma$ captures economic agents’ risk aversion. This is expected to show a direct relation to the debt-to-GDP ratio.

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3 In general, United States Treasury bonds are considered risk-free bonds for international investors.
The nominal interest rate in the economy in question is defined by real interest rate \( r \) plus the inflation rate \( \pi \), as in the relation represented by equation (3) below, similar to Fisher’s rule:

\[
i = r + \pi
\]  

(3)

This equation suggests that the nominal interest rate can vary as a result of changes in both the real interest rate and in the inflation rate.

It is assumed that the short-term nominal interest rate is set by the central bank \( i^* \), as the main monetary policy tool for guiding inflation towards the desired target, and that it is differentiated from \( i \) as being the base rate desired by the monetary authority, that is:

\[
i = i^*
\]  

(4)

Non-Ricardian public debt behaviour can also cause the monetary authority to lose control over inflation. Accordingly, the success of an inflation-targeting regime may also require the fiscal authority to set a target for the proportion of net debt over GDP to ensure debt solvency in the long run. The central bank’s reaction function can thus take into account fiscal shocks in the economy.

Three factors thus influence the central bank’s decision to set the nominal interest rate intertemporally in an emerging economy: on the one hand, when inflation \( \pi \) diverges from the preset target \( \pi^* \), the monetary authority reacts to contain that deviation. On the other hand, since that rate also determines the return on government bonds, as in equation (3), it is assumed that the interest rate must react to deviations in public debt \( b \) from a target \( b^* \) set according to economic policy guidelines aimed at maintaining public debt sustainability, that is, to equalize government expenditures and revenues at present values.\(^4\) In that sense, a component of present policy coordination is presupposed in the reaction function of the monetary authority.

This nominal interest rate reaction occurs because the prospect of debt insolvency would force the monetary authority to resort to inflation tax, thus losing control of inflation. It also occurs because public debt produces an autonomous effect on the risk premium \( R \) when it diverges from a preset target, acting as a kind of thermometer for investors as to the risk of default on government bonds. It will be recalled that a rise in default risk can trigger capital flight and thus exchange-rate depreciation, which in turn produces inflationary pressure.

The third component of the monetary authority’s reaction function is the spread between the nominal interest rate in the domestic market \( i^* \) and the nominal interest rate in the external market \( i \). The larger this spread, the less the need for the monetary authority to raise its own domestic interest rate, bearing in mind that, if domestic interest rates remain constant, reduction in the external interest rate stimulates exchange-rate appreciation, which aids price stability. The idea, then, is to capture the indirect effect of the exchange rate on inflation, since the interest rate spread should determine that rate. The parameter \( \mu \) below thus refers to the central bank’s concern with exchange-rate variations. This dynamic is set forth in the following differential equation, which is an adaptation of Taylor’s rule (1993):

\[
\frac{di}{dt} = \beta (\pi - \pi^*) + \alpha (b - b^*) + \mu (i^* - i) , \beta > 0; \alpha > 0; \mu < 0
\]  

(5)

The suggested monetary rule described above is one of the main contributions of this work. If it is accepted that Taylor’s rule is not the best suited to an emerging economy, a fiscal variable must be

\(^4\) The fiscal authority could stipulate target \( b^* \) in coordination with the monetary authority to determine macroeconomic policies to reach the inflation target.
included in the monetary policy rule adopted by the central bank. Section V makes another important contribution, by identifying the channel through which default risk can destabilize the model presented.

In light of the above, it is necessary to establish the difference between the last two equations, given that equation (4) reflects the exogenous short-term nature of the interest rate in an inflation-targeting regime. Unlike equation (5), which specifies a monetary policy rule that will limit the monetary authority’s decisions on setting the base rate of interest over a given period, some fluctuations in the base rate can occur over short periods, as defined in (4).

Equation (6) shows the government’s intertemporal constraint:

$$\dot{b} = ib + g - t$$

where \( g \) is government expenditure and \( t \) its revenues. Considering a public debt indexed to the nominal interest rate, an increase in \( i \) will have an incremental effect on debt \( b \), and produce primary deficits in the public sector \((g-t > \theta)\).

The inflation rate is, a priori, determined by an expectations-augmented Phillips curve, to which a component representing the nominal exchange rate is added:\(^5\)

$$\pi = \tau (y - \bar{y}) + \pi^e + \beta (E) \quad , \tau > 0; \beta > 0$$

where \((y - \bar{y})\) represents the output gap, \(\pi^e\) expected inflation and \(E\) the nominal exchange rate.\(^6\)

In this case, the exchange-rate effect is included as well as the traditional effects on inflation represented by the Phillips curve. Empirical tests conducted by Goldfajn and Werlang (2000) and Correa and Minella (2010) found that the pass-through effect is sharper when: (i) the economy is in a rapid expansion cycle; (ii) exchange-rate volatility is low; (iii) the economy is very open; (iv) the initial rate of inflation is high, especially —according to the first work cited— in emerging economies, and (v) exchange-rate misalignment is severe.

However, the expectations component \((\pi^e)\) is determined by the deviations of expected output \((y^e)\) with respect to potential output \((\bar{y})\), and by the difference between the expected nominal exchange rate\(^7\) and its equilibrium level \((E^e - E^*)\), since, as noted earlier, the exchange rate affects price levels, especially when it is above equilibrium. Thus:

$$\pi^e = \phi (y^e - \bar{y}) + \chi (E^e - E^*) \quad , \phi > 0; \chi > 0$$

The idea behind the expectations component is that agents make their forecasts of inflation by observing the equilibrium between aggregate supply and demand. Their expectations also incorporate forecasts on the exchange-rate, which is an important variable in price composition. Under the hypothesis of rational expectations, agents forecast output behaviour expecting that, in the absence of exogenous shocks, observed output will be equal to potential output and the exchange rate will be equal to the equilibrium rate. Accordingly, assuming that \(y^e = \bar{y}\) and \(E = E^*\), (8) may be substituted into (7) to give the following Phillips curve:

$$\pi = \tau (y - \bar{y}) + \beta (E)$$

\(^5\) The effect of the nominal exchange rate on inflation was considered of negligible magnitude in early works with the Phillips curve. However, several works have estimated it empirically.

\(^6\) More precisely, inflation is a dependent function of the rate of exchange-rate devaluation. Equation (9), which denotes the parity of the interest rate and the exchange rate, should also be expressed in terms of exchange rate devaluation, assuming the form \(E^e/E = (i - i) + R\). However, the simplification used does not affect the model’s results.

\(^7\) The exchange rate presented is the ratio between the real and the dollar, i.e. national currency/foreign currency.
In determining the real exchange rate, \( e \) is assumed, for simplicity’s sake, to be the equilibrium between domestic and external prices \( p = p^* \). Consequently, the exchange rate is determined by interest rate parity, according to equation (9):

\[
E = E^* = e = \rho (i^* - \bar{i}) \quad \rho < 0
\]

In turn, aggregate demand is composed of the consumption function, the investment function, government spending and the trade balance, as expressed in equation (10), which denotes an IS curve for an open economy:

\[
y = c(y) + I(i) + g + x(E), c_y > 0, I_i < 0, x_e > 0
\]

2. Short-term analysis: a comparative statics study

A comparative statics analysis is now presented in order to deduce some short-term relations between the model’s key variables.

By substituting (4) into (10), we obtain:

\[
y = \left( \frac{I_i}{1 - c_y} \right) i^* + \left( \frac{1}{1 - c_y} \right) g + \left( \frac{X_e}{1 - c_y} \right) E
\]

(10.1)

As inflation is a function of income, equation (10.1) is substituted into (7.1), giving the following behaviour for inflation:

\[
\pi = \left( \frac{\tau I_i}{1 - c_y} \right) i^* + \left( \frac{\tau}{1 - c_y} \right) g + \left( \frac{\tau X_e}{1 - c_y} + \theta \right) E - \tau \bar{Y}
\]

(7.2)

Inserting equations (1) and (4) into (3), and the result into (7.2), then inserting (9) in the final result gives:

\[
\pi = \left[ \frac{\tau I_i}{1 - c_y} \right] + \frac{\left( \theta + \frac{\tau X_e}{1 - c_y} \right) g + \left( \theta + \frac{\tau X_e}{1 - c_y} \right) \rho}{v} \]

\[
\frac{\left( \theta + \frac{\tau X_e}{1 - c_y} \right) g + \left( \theta + \frac{\tau X_e}{1 - c_y} \right) \rho}{v}
\]

(7.3)

where \( v = 1 - \left( \frac{\tau I_i}{1 - c_y} + \theta + \frac{\tau X_e}{1 - c_y} \right) \rho > 0 \)

The derivative that indicates the short-term effect of risk on inflation can be obtained from equation (7.3):

\[
\frac{\partial \pi}{\partial R} = \left( \frac{\tau I_i}{1 - c_y} \right) + \frac{\left( \theta + \frac{\tau X_e}{1 - c_y} \right)}{v} \rho < 0
\]

(7.3.1)
It is concluded from this derivative that inflation and risk are negatively correlated in the short run. The risk variable is one of the components of the short-term interest rate, as shown in (1). Thus, rising default risk exerts increasing pressure on the interest rate, thereby compressing aggregate demand and, thus, inflation. Importantly, however, although this relationship is observed in the short run, risk can introduce macroeconomic instability in the long term, leading the monetary authority to lose control over the inflation target.

Diagram 1 illustrates the channel of risk pass-through to inflation. When risk rises, the interest rate on government bonds must also rise to sustain demand by international investors. Consequently, aggregate demand falls and, ultimately, so does inflation.

**Diagram 1**
Flow diagram 1: channel of risk pass-through to inflation

Source: Prepared by the authors.

In order to study the effect of risk changes on public debt, a function must be defined for that variable. Thus, (3) is inserted into (6), such that:

\[ g - t = (r + \pi)b \]  

(6.1)

When (1) is then substituted into (6.1), we obtain:

\[ g - t = (r^e + R + \pi)b \]  

(6.2)

Recalling that the behaviour of inflation was shown in (7.3), substituting this into (6.2) and isolating \( b \), gives the following behaviour for public debt:

\[ b = \left( \frac{\pi}{1 - c_j} + \frac{\theta X_e}{1 - c_j} \right) \left( \frac{R}{1} + \frac{\tau I_i}{v} + \frac{\theta + \frac{\tau X_e}{1 - c_j}}{1 - c_j} \right) \left( \frac{\tau + \theta X_e}{1 - c_j} \right) \]  

(6.3)

The effect of a marginal increase in risk on the behaviour of public debt can be analysed on the basis of the comparative statics outcomes. Below is the derivative of public debt with respect to risk, followed by some algebraic manipulations.

\[ \frac{\partial b}{\partial R} = \frac{(t - g) \left[ \frac{\tau I_i}{1 - c_j} + \left( \frac{\theta + \frac{\tau X_e}{1 - c_j}}{1 - c_j} \right) \left( \frac{\tau + \theta X_e}{1 - c_j} \right) \right]}{t^2} > 0 \]  

(6.3.1)

> 0 for \( t > g \) or, < 0 for \( t < g \).
By algebraic reorganization and treating the marginal propensity to save as $s$, the above derivative may be rewritten as:

$$\frac{\partial b}{\partial R} = (t-g) \left[ 1 + \frac{\tau \ell}{(s-\tau \ell)} + \frac{\theta + \tau X}{(s-\tau \ell)} \rho \right] \frac{1}{s}$$

(6.3.2)

Diagram 2 shows the mechanism of risk pass-through to public debt in the event of a primary surplus. An increase in risk generates a rise in the interest rate (owing to investor's requirements) and thus increases the public debt.

**Diagram 2**

Flow diagram 2: channel of risk pass-through to public debt

Source: Prepared by the authors.

The impact of the risk variable on public debt will depend on the primary balance in the government accounts. In the event of a surplus —whereby government revenues exceed expenditure— risk will have a positive effect on public debt. In the opposite case, i.e. a primary deficit, the risk impact on public debt will be negative. In addition to these considerations, the higher the marginal propensity to save and the higher the interest rate, the smaller the impact of risk on debt. These results are consistent with macroeconomic theory.

The transmission channel of the primary balance shows an ambiguous relation between risk and public debt, which calls for more detailed explanation: in the case of a primary surplus, a risk shock is followed by a nominal interest rate rise, as observed in equations (1) and (3). Consequently, as indicated in equation (6), the public debt increases.

Conversely, a primary deficit puts pressure on inflation. In this case, since inflation is already high because of the deficit, under the monetary policy rule (5), the interest rate is already high. Here, a risk shock will have a smaller effect on the interest rate and, thus, on public debt.

**IV. Long-run equilibrium and the fiscal policy rule**

The model’s long-run dynamics can be configured on the basis of the results of the short-term comparative statics. This section manipulates the model to obtain a primary surplus rule for the fiscal authority to follow. Initially, the model presents three differential equations, namely:

$$\dot{R} = \sigma (r - \bar{i}) \quad , \sigma > 0$$

(2)

$$\frac{di}{dt} = \beta (\pi - \pi^*) + \alpha (b - b^*) + \mu (\bar{i} - \bar{i}) \quad , \beta > 0; \alpha > 0; \mu < 0$$

(5)

$$\dot{b} = ib + g - t$$

(6)
By isolating $r$ in equation (3) and substituting the result into (2), we observe the differential equation that gives default risk over time. We thus obtain:

$$\dot{R} = \sigma \left( i^* - \pi - \bar{i} \right)$$

(2.1)

The model equilibrium initially requires a system. However, a steady state dynamic is assumed for public debt such that the movement of debt over time can be taken as nil, i.e. the fiscal authority follows a rule under which it is passive and avoids incurring fiscal deficits. Thus, we obtain:

$$ib = g - t$$

(6*)

In other words, we assume a fiscal policy rule that gives a large enough primary surplus to cover public debt service, thereby keeping the debt sustainable over time, i.e. $b = 0$. This manoeuvre allows the model, which previously had three differential functions, to be described as a system of two dynamic equations.

Accordingly, (2.1) and (5) may be recast as follows:

$$\dot{R} = \sigma \left[ i^* - \pi(R, i) \right]$$

(2*)

$$\frac{di}{dt} = \beta \left[ \pi(R, i) - \pi^* \right] + \alpha \left[ b(R, i) - b^* \right] + \mu \left[ i^* (R, i) - \bar{i} \right]$$

(5*)

The system thus becomes a (2x2) one, in which equilibrium is drawn from equations (2.1) and (5). Thus, at equilibrium (steady state), we obtain:

$$\dot{R} = 0 \Rightarrow \pi(R, i) = i^* - \bar{i}$$

$$\frac{di}{dt} = 0 \Rightarrow \pi(R, i) = \pi^* + \left\{ \frac{-\alpha \left[ b(R, i) - b^* \right] - \mu \left[ i(R, i) - \bar{i} \right]}{\beta} \right\}$$

When the system is linearized around the equilibrium using a Taylor expansion, we obtain:

$$\frac{\partial R}{\partial t} = \sigma \left\{ \frac{-\partial \pi}{\partial R} \right\} (R_* - R_0) + \sigma (i_* - i_0)$$

(12)

$$\frac{\partial i}{\partial t} = \left\{ \beta \frac{\partial \pi}{\partial R} + \alpha \frac{\partial b}{\partial R} \right\} (R_* - R_0) + \mu (i_* - i_0)$$

(13)

Casting the results in matrix notation:

$$\begin{bmatrix} \frac{\partial R}{\partial t} \\ \frac{\partial i}{\partial t} \end{bmatrix} = \begin{bmatrix} \sigma \left\{ \frac{-\partial \pi}{\partial R} \right\} & \sigma \\ \left\{ \beta \frac{\partial \pi}{\partial R} + \alpha \frac{\partial b}{\partial R} \right\} & \mu \end{bmatrix} \begin{bmatrix} (R_* - R_0) \\ (i_* - i_0) \end{bmatrix}$$

(14)
The necessary and sufficient condition for the equilibrium of a two-dimensional dynamical system to be asymptotically stable (where the two eigenvalues of the solution have negative real parts) is that the trace and determinant of the Jacobian matrix be negative and positive, respectively. It is observed that:

\[
\text{Trace} = \sigma \left( -\frac{\partial \pi}{\partial R} \right) + \mu = ?
\]

\[
\text{Det} = \sigma \left( -\frac{\partial \pi}{\partial R} \right) \mu - \sigma \left( \beta \frac{\partial \pi}{\partial R} + \alpha \frac{\partial b}{\partial R} \right) = ?
\]

To satisfy the stability of equilibrium conditions, necessarily \(|\mu| > \sigma \left( -\frac{\partial \pi}{\partial R} \right)\). Thus the trace will be negative.

In short, the first condition for stability is that, over time, the absolute value of the sensitivity of the nominal interest rate to the interest rate spread must be greater than the absolute value of the product of the impact of risk on inflation and the sensitivity of risk to the interest rate spread. There are thus two channels acting on macroeconomic stability: (i) the exchange-rate channel, which leads to stability, represented by \(\mu\), and (ii) the risk channel, which leads to instability, represented by \(\sigma \left( -\frac{\partial \pi}{\partial R} \right)\).

This relationship is expected, given that, on the one hand, a reduction in the external interest rate pushes up the exchange rate, containing inflationary pressures occurring through that channel and thereby reducing the need to raise interest rates in the short term to meet the inflation target set by the monetary authority, in accordance with equation (5). On the other hand, a reduction in the external interest rate produces an effect on risk measured by \(\sigma\), as given in equation (2). If risk is then high, this will require a significant rise in the domestic interest rate, as may be observed in (1), which will destabilize \(i\). It should also be considered that if the negative impact of risk on inflation is very large, then the rise in the interest rate may also be significant. As a consequence, for the trace \(\sigma \left( -\frac{\partial \pi}{\partial R} \right) + \mu\) to be negative, necessarily \(|\mu| > \sigma \left( -\frac{\partial \pi}{\partial R} \right)\).

Another way of looking at this relationship is to admit the hypothesis that debt is directly correlated with GDP and \(\sigma\), bearing in mind that this coefficient measures creditor mistrust in the government’s ability to pay. Thus, public debt forms one of the channels through which an explosive effect on risk could be propagated, because the rise in interest pushes up the cost of debt servicing, increasing creditor mistrust of the government’s ability to pay. A fiscal policy that takes public debt solvency into consideration could contribute to economic stability.

Given that fiscal policy was underlined as a possible instrument for achieving stability in the model, it is important to analyze possible channels of transmission with respect to the variables whose stability is studied. A surplus-targeting policy prevents the public debt from rising over time, thus stabilizing the default probability and, thus, the long-term interest rate (see diagram 3).

**Diagram 3**

*Flow diagram 3: channel 1 of fiscal policy transmission*

Source: Prepared by the authors.

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8 See more detail in Gandolfo (1997).

9 It should be recalled that, according to the model, risk has a negative effect on inflation through the following mechanism: a rise in risk is accompanied by an increase in interest on government bonds, to ensure debt solvency, which pushes inflation down.
A second channel for the transmission of fiscal policy is that public spending containment cushions effects on aggregate demand. This eases inflation, which allows the monetary authority to lower the interest rate used to steer inflation towards the desired target, and to reduce government spending on debt servicing (see diagram 4). There are, then, two possible effects, as shown in the flow diagram below.

Diagram 4
Flow diagram 4: channel 2 of fiscal policy transmission

Following analysis of the dynamic equilibrium stability, the determinant may be rewritten as follows:

$$Det = -\sigma \left[ \frac{\partial \pi}{\partial R} (\mu + \beta) + \alpha \frac{\partial b}{\partial R} \right]$$

Thus, assuming a primary surplus and observing that $\frac{\partial b}{\partial R} > 0$, it may be deduced that $\beta > \mu$ is a necessary condition for the stability of the model’s dynamic equilibrium. Since these coefficients indicate the sensitivity of the interest rate to the variation in inflation and in the interest rate spread, as expressed in (5), that condition is consistent with the expectations of the model. This is because controlling price levels is the premier aim in determining monetary policy instruments under an inflation-targeting regime.

In short, under a primary surplus fiscal rule, stable dynamic equilibrium requires that: (i) the base rate of interest be more sensitive to exchange-rate devaluation than to increases in risk; (ii) the fiscal set-up stabilize the confidence of external investors in public debt solvency, as reflected in $\sigma$, and (iii) the monetary authority concern itself more with deviations from target inflation than with the exchange rate.

For a qualitative analysis of the intertemporal trajectory and stability of the dynamic equilibrium, a phase diagram of the dynamic system expressed by equations (2) and (5) provides interesting study.

The system’s convergence towards equilibrium can be studied by examination of the system discriminator, as given by: $D = \{trace(\ A)\}^2 - 4det(\ A)$, where $D$ is the value of the discriminator and $A$ is the matrix analysed.

When $D(\ A) > 0$, the equilibrium will be a stable node. Thus, in the event of a shock that leads to a deviation from equilibrium, the system will return monotonically to equilibrium. When $D(\ A) < 0$, the equilibrium will be a stable focus and the system will revert spirally to equilibrium after a shock.

In the model studied earlier, which represents the coordination of monetary and fiscal policies in the presence of a primary surplus rule, equilibrium is a stable focus and convergence thus occurs in a spiral wave form, where the derivative $\frac{\partial b}{\partial R} > 0$ is very low —and, thus, the primary surplus is small. When the surplus is large and, thus, the derivative is also very large, equilibrium could be a stable node with monotonic convergence.

Diagram 5 shows the scenario in which the system equilibrium is a stable node.
Whether convergence occurs at a focus or node can have important implications in terms of economic policy implementation. This is because a combination of economic policies leading to convergence on equilibrium in a spiral or focal trajectory could weaken the credibility of monetary policy, insofar as the variables affected by economic policy decisions will necessarily register values above or below equilibrium at some points during the trajectory.

The phase diagram below represents a stable focus equilibrium (see diagram 6).

V. Extension of the model: the nominal exchange rate under the effects of default risk

Thus far, the exchange rate has been presented as a function of the interest rate spread. However, in an emerging economy, default risk is an important variable in determining the exchange rate and thus has an indirect effect on inflation. So, one way in which the central bank can lose control of monetary policy\textsuperscript{10} is through the exchange rate which is, in turn, affected by risk.

\textsuperscript{10} Loss of control of monetary policy is understood to mean a situation in which the monetary authority cannot meet its preset inflation target.
In light of these considerations, there follows a study of the comparative statics of the economic model in a situation where the exchange rate is influenced by default risk. Equation (9), which denotes the exchange rate (strictly speaking, the devaluation of the exchange rate), is represented as follows:

\[ E = \rho (i^* - \bar{i}) + \gamma R, \quad \rho < 0; \gamma > 0 \]  

(9.1)

where \( \rho \) measures the exchange rate’s sensitivity to the difference between the domestic and external interest rates and \( \gamma \) measures the exchange rate’s sensitivity to risk. This parameter, in turn, can also be considered a measure of risk aversion on the part of international investors, like \( \sigma \), since at a particular risk level, the larger \( \gamma \), the less willing investors will be to add the respective country’s bonds to their portfolio. This shrinks the supply of foreign exchange and increases exchange-rate depreciation. Following Blanchard (2004), it may also be expected that \( \gamma \) will rise if the debt-to-GDP ratio increases.

Blanchard (2004) empirically estimated a similar function to the one discussed and found that risk had the expected effect —and of a large magnitude— on the exchange rate. Favero and Giavazzi (2003) found similar results.

Let us consider the Phillips curve presented in equation (7.1):

\[ \pi = \tau (y - \bar{y}) + \theta(E) \]  

(7.1)

By substituting the IS curve presented in (10.1) into the Phillips curve (7.1) and substituting (1) and (4) into (3) —to obtain \( i^* = (r^e + R) + \pi \) — and then inserting the results into the Phillips curve (7.1), along with the exchange-rate function (9.1), we obtain the following for inflation after reorganization:

\[
\pi = \left[ \left( \frac{\tau I_i}{1 - c_y} \right) + \left( \frac{\theta + \frac{\tau X_e}{1 - c_y}}{v} \right) \right] (\rho + \gamma) R + \left[ \left( \frac{\tau I_i}{1 - c_y} \right) + \left( \frac{\theta + \frac{\tau X_e}{1 - c_y}}{v} \right) \rho \right] r^e \\
- \left( \frac{\theta + \frac{\tau X_e}{1 - c_y}}{v} \right) - \bar{i} + \frac{\tau}{v} - g - \frac{\tau}{v} \bar{y}
\]

(7.4)

where \( v = \left[ 1 - \left( \frac{\tau I_i}{1 - c_y} \right) + \left( \theta + \frac{\tau X_e}{1 - c_y} \right) \rho \right] > 1 \)

From which the following partial derivative may be extracted:

\[
\frac{\partial \pi}{\partial R} = \left( \frac{\tau I_i}{1 - c_y} \right) + \left( \frac{\theta}{1 - c_y} \right) \left( \rho + \gamma \right) \]

(7.3.2)

There is some ambiguity in the sign of this partial derivative, because it should be negative in normal conditions —as shown in (7.3.1)— but positive in extreme cases where \( \gamma \)—which represents risk elasticity to the exchange rate— is too high. In other words, the derivative will have the opposite sign to that expressed in (7.3.1), if \( \gamma > \rho \). This is because at high risk aversions, a rise in the interest rate...
may not be enough to contain inflation, since capital flight can push prices up through the mechanism of exchange-rate pass-through.

Similarly, the exchange-rate effect given in (9.1) is plugged into the government’s intertemporal constraint, represented by (6.3). Thus, considering that the government’s budgetary constraint is given by:

$$g - t = (r^e + R + \pi)b$$  \hspace{1cm} (6.3)

We insert (7.4) into (6.3) and solve for the level of public debt, such that

$$b = \frac{g - t}{\left[\left(\tau I_t \right) \left(1 - c_y\right) + \left(\theta + \frac{\tau X_e}{1 - c_y}\right)\right] \left(1 + \frac{r^e}{v}\right) + \left(\theta + \frac{\tau X_e}{1 - c_y}\right)\left(\frac{\tau}{1 - c_y}\right) + \left(\theta + \frac{\tau X_e}{1 - c_y}\right) - \frac{1 - c_y}{v} \left(\rho + \gamma\right) g - \frac{1 - c_y}{v} i}$$  \hspace{1cm} (6.4)

To analyse the effect of a marginal rise in risk on the behaviour of public debt, we extract the following partial derivative:

$$\frac{\partial b}{\partial R} = \frac{(t - g) \left[\left(\tau I_t \right) \left(1 - c_y\right) + \left(\theta + \frac{\tau X_e}{1 - c_y}\right)\right] \left(1 + \frac{r^e}{v}\right) + \left(\theta + \frac{\tau X_e}{1 - c_y}\right)\left(\frac{\tau}{1 - c_y}\right) + \left(\theta + \frac{\tau X_e}{1 - c_y}\right) - \frac{1 - c_y}{v} \left(\rho + \gamma\right) g - \frac{1 - c_y}{v} i}{t^2} > 0$$  \hspace{1cm} (6.3.2)

$> 0$ for $t > g$ or, $< 0$ for $t < g$.

This derivative, which indicates the risk elasticity of public debt, will be positive in the case of a primary surplus and negative in the case of a deficit, as described in section III.2.

However, it should be noted that once risk is included as an explanatory variable for the exchange rate, unforeseen effects may be anticipated in the previous configuration of the model. That is, if $\gamma > \rho$, the derivative may again have a different sign than expected. Following the example given in IV.1, deductions can be made about possible changes in the long-term stability of the equilibrium for the model with a primary surplus fiscal rule, this time including the hypothesis of risk as an explanatory variable for the exchange rate.

Although the model matrix remains identical to that presented in (14) under section IV.1, now—with a nominal exchange rate influenced by risk and, thus, possible alterations in the direction of the partial derivatives—results other than those expected may occur.

The stability of the model will depend, as noted earlier, on the signs of the trace and determinant of the matrix. These also remain the same as those presented before, with the exception of possible changes in sign generated by the impact of risk on the exchange rate. Thus:

$$Trace = \sigma \left(-\frac{\partial \pi}{\partial R}\right) + \mu = ?$$

$$Det = -\sigma \left[\frac{\partial \pi}{\partial R}(\mu + \beta) + \alpha \frac{\partial b}{\partial R}\right]$$
The difference between this configuration and the previous one is that now stability is influenced by the effect of risk on the exchange rate. Even with a primary surplus, which supports an asymptotically stable equilibrium in the first model, too high a risk aversion (as measured by \( \gamma \)) can lead to instability in the event of an exogenous shock generated by the rise in risk itself. In other words, even assuming all the necessary conditions for stability—as in section IV.1—, the Jacobian determinant may be negative if \( \gamma > \rho \), because then the partial derivatives that measure the impact of risk on inflation and public debt may present unexpected signs. As seen earlier, at very high levels of risk aversion, a risk shock can lead to an exchange-rate devaluation that pushes up inflation and induces instability in the model.

**VI. Analysis of findings and concluding remarks**

This article has analysed the coordination of monetary and fiscal policies in an emerging economy with an inflation-targeting regime, in a context in which default risk shocks can lead to macroeconomic disequilibria. It has sought to understand how the economy adapts to exogenous shocks to maintain an asymptotically stable equilibrium.

On the basis of a model proposing a monetary policy rule that takes into account not only the deviation of inflation from its target (\( \pi - \pi^* \)), but also the deviation of public debt from its desired level (\( b - b^* \)) and the spread between domestic and external interest rates (\( i^* - i \)), we identified the model's equilibrium stability conditions, which are summarized below. The form used was a model of simultaneous first-order differential equations, analysing their intertemporal equilibrium and their stability. It was concluded that:

(a) For a model in which the exchange rate is defined by interest rate parity and a primary surplus regime, stability requires that: (i) monetary policy afford more importance to the deviation of inflation from its target than to the interest rate spread; (ii) shifts in the interest rate spread have a greater influence on monetary policy than the need to adapt short-term interest rates to risk shocks. It may be deduced from these conditions that controlling price levels should be the chief concern of monetary policy in an inflation-targeting regime and should thus be afforded more importance than the need to adapt to external interest rate shocks. At the same time, fiscal oversight is needed to make the economy less vulnerable to default risk shocks.

(b) Under a scheme of policy coordination in which the nominal exchange rate is defined by the interest rate spread and by the risk factor, a very high level of risk aversion, as measured by \( \gamma \), could have a large enough impact on the nominal exchange rate to destabilize the model. This is because a rise in interest could be interpreted as a greater probability of default and could consequently lead to exchange-rate devaluation and, potentially, loss of monetary policy control over inflation. For this reason, in economic policy terms, a policy of fiscal austerity is recommended. The other conclusions mentioned remain valid under this configuration. These results are similar to those obtained by Blanchard (2004) and Favero and Giavazzi (2003).

Generally speaking, the results suggest that inflation control should be the monetary authority's main objective in an inflation-targeting regime. At the same time, fiscal policy should be passive, generating surpluses to stabilize the public debt and ensure its solvency over time, which will, in turn, stabilize the default risk and avoid the risk of fiscal dominance.
Bibliography


