Basic accounting relations are developed to analyze the impacts of changes in the age of retirement and in fertility, on economic dependency and on earnings per capita. In general, the magnitude of the first, and both the size and direction of the second of these effects depend on the population and the labor force age distribution; these relations are formally expressed, and a decomposition of the changes in dependency and income due to each one of their factors is carried out in six Latin American countries. The analysis suggests that most of the countries in the region have not reached to date the stage where small reductions in fertility would be clearly detrimental to dependency and per capita income, although all are experiencing negative ageing effects in varying degrees. An application to the study of transfer pension systems indicates that the legal age at retirement would have to increase substantially in some countries in order to compensate for anticipated population ageing, and thus complementary measures are called for if their system's financial balance is not to deteriorate further.
Introduction.

Demographic patterns have been changing rapidly in many Latin American countries, and public concerns are expressed with some frequency in relation to the negative economic implications of population growth on the one hand, and ageing on the other, specially in connection with the current financial difficulties experienced by social security systems. In spite of the public interest in these matters within the region, evaluations of the economic consequences of changing population age profiles are largely absent from recent research and policy analyses, with a few exceptions (e.g., Uthoff, 1990, 1991). Doubtless there are many factors that contribute to this, including some of general policy and institutional nature (United Nations, 1990, chap. 9). Inadequacies of the existing large-scale, long run planning models that are often exceedingly complex and yield results that are difficult to interpret or to validate (Arthur & McNicoll, 1975), have probably prevented their more widespread use for these purposes. In this context, this paper sets to examine, in a highly simplified fashion, some relations between the changes in the population and labor force age structure, economic dependency, and per capita income, that may be of help in guiding policy assessments and discussions by highlighting some basic accounting restrictions which policy formulation and implementation should take into account. Specifically, the relations developed here are used to analyze the effects of changes in retirement age and fertility on dependency and income. Some consequences on pension systems are also examined.

The present discussion is simplified in two major respects. On the one hand, it focuses on the effects of changing age distributions, and does not explicitly consider those of changes of population size in themselves. In spite of the fact that they are related, we do not know of conclusive evidence for the Latin American region regarding the influence of the rate of population growth per se on economic growth, and a whole different set of issues are raised by this broader question. How important are the effects of changing age distributions is not clear either, and this paper examines some ways to assess their importance. The second sense in which this discussion is simplified, is that no behavioural interactions are considered between the demographic and the economic variables analyzed; we use only elementary accounting relations between them. This carries with it obvious limitations, but it allows to concentrate on the basic age-distributional restrictions within which the possible outcomes of alternative changes or actions can vary.

In the first part of the paper some definitions of economic dependency are presented, the effects of changes of its demographic and economic components are specified, and a simple relation between the dependency ratio and income per person is obtained. The derivations carried out subsequently are used to evaluate the impact of changes in fertility and in retirement on the dependency ratio and per capita income. Then, repercussions on the financial balance of pension systems are discussed. Some basic ideas are drawn from the more sophisticated analyses of inter-generational transfers (Arthur & McNicoll, 1978; Lee, 1980, 1990; Preston, 1982), which are generally more concerned with individual lifetime consumption or welfare. The present focus is on period dependency and income for the population as a whole, and ignores the capital dilution effects which are taken up in some
of the cited studies. In the main text, only a few, more crucial equations are presented; all variables and relations are defined and derived in detail in the appendix. Six Latin American countries have been selected to illustrate these relations in the real context of demographic diversity in the region: the set includes cases of countries in the more advanced, intermediate and incipient stages of the demographic transition (Argentina, Chile, Costa Rica, Mexico, Peru and Bolivia).

Readers not interested in compositional population and labor force considerations, may skip to section 2.

1. Economic dependency: definition and basic components.

Defined at the macro level, the concept of economic dependency, as used here, makes reference to the number of persons that must be economically maintained by those who generate income. Given that the main source of income of most people is their labor, and that there are ages where remunerated work is practically nonexistent (young children and the elderly population) a very crude, but very widely used definition of the dependency index is the quotient between the number of persons in the extreme ages and the size of the working age population. This ratio, here called \( R^* \), can be expressed as \( R^* = (1−a)/a \), where \( a \) is the proportion of the population in the working ages. Given the limiting ages, the value of \( R^* \) depends on those purely demographic variables that determine the population age distribution.

However, the 'demographic burden' (as it is sometimes called) does not fall on all the working age population, but mainly on those responsible for the provision of income, i.e., the economically active. This leads to consider a second (also widely used) definition of dependency: the ratio of economically inactive to the active population, which may be written as \( R_a = (1−a)/aB \), where \( B \) represents the total labor force participation rate (i.e., the proportion active among those in working ages). The ILO (1986) estimates and projections show that the economically active population is very heterogeneous. It is possible to distinguish, in particular, three main sub-groups: the middle-aged adults (roughly those between 30 and 55 years of age), who in general have participation rates that are high and stable or increasing over time, and the very young (below 30 years) and older adults (over 55 years) with low and generally decreasing labor force participation.

It follows that the demographic transition, which carries with it the ageing of the population over the medium to long terms, has at least two distinguishable effects on dependency: an increase in the working-age proportion, which reduces dependency, and on the other hand, the ageing of the adult population, that tends to reduce total participation and therefore to raise the dependency ratio. To the extent that lower fertility is accompanied by greater female labor force participation, the tendency toward a reduced dependency ratio is reinforced.

In figure 1 one can see that \( R_a \) in Chile, Costa Rica, Mexico and Peru, after reaching a peak
around 1970, shows a declining trend afterwards, while in Bolivia it maintains an increasing trend which will probably not be reversed for more than a decade. All countries, except for Bolivia, may see their dependency ratios stabilize or even start to experience a turnaround near the end of the period considered. An algebraic decomposition of $R_g$, allows us to verify (see table 1) that the demographic changes during the last quarter of the present century are in general be favorable, in the sense that they combine to reduce the degree of dependency, counteracting the sometimes unfavorable effects of the changes in labor force participation profiles. The future projections (2000-2025) show that the total 'demographic effect' (the combined effect of increments in the adult proportion and that of ageing of the adult population) will turn out to be favorable in the next few decades if current projections are realized; i.e., dependency will fall in different degrees as a consequence of the projected population trends. All countries, except for Bolivia, are projected to experience negative effects of the ageing of the adult population, but they will not be sufficiently large to compensate for the favorable ones of reduced child dependency.

2. Dependency and per capita earnings.

Among those in the labor force, only those who work effectively generate the income that provides economic support for the whole population. Consequently, it appears useful to consider a third, very rarely used measure of dependency, defined as the quotient of the not-employed over the number of employed in the population, which can be expressed as

$$R_e = \frac{(1-\alpha \beta e)}{\alpha \beta e}$$

(1)

where $e$ represents the employment rate (one minus the unemployment rate) of the labor force. A larger proportion of working-age persons, a greater labor force participation, as well as a lower unemployment rate lead to lower dependency, defined in these terms.

If participation rates were high and stable, and the unemployment rates were low and relatively constant, the three definitions considered would be practically equivalent. But at least in the Latin American reality, labor force participation shows persistent trends, and unemployment fluctuates widely from one quinquennium to another, and even from one year to the next. Thus, in the short-run (one to five years), the changes in $R_e$ are mainly affected by the fluctuations in unemployment, while in the medium and long terms, it is determined more by the trends in labor force participation and the population age structure ($R_\alpha$ and $R_\beta$ depend only on the last two factors).

This way of defining the index of dependency allows us to write a simple relation with the earnings per person ($y$):

$$y = \frac{w}{1+R_e}$$

(2)

where $w$ is the mean wage, or earnings per worker. What this equation says is very simple: on average, each worker (that earns on average a salary $w$), divides his or her income
between him or her self, plus the number of persons -that on average- he or she must maintain ($R_{ij}$). Equation 2 represents a clear way of recognizing the inverse relationship that exists between dependency and earnings per person, and it allows to compare directly any of the above-mentioned effects on $R_{ij}$ with those of variations in real wages. Also, it is possible to show on the basis of this relation than any given percentage change in the dependency ratio will be of a greater relative magnitude that the relative change in per capita income (more specifically, in the proportion $(1+R_{ij})/R_{ij}$). In other words, the same reasoning and methods used to examine the structural changes in economic dependency may be applied to per capita income in all but one respect: the magnitude of relative changes in income per person would be magnified considerably, in the best cases by about one third (when $R_{ij}$ is near 3) and in the worst cases by a factor of 2 (when $R_{ij}$ is near 1).

As in the case of $R_g$, it is possible to carry out a decomposition of the effects on income per person due to changes in each one of its factors; we do this in the following section, in the context of short run simulations.

3. Simulations on dependency and per capita income.

Will a decline in fertility be beneficial (in terms of reducing dependency and raising income per person) for a given country? Under what conditions will the result turn out one way or the other? How do these effects compare with those of changes in other factors, i.e., increases in labor force participation, in active life or reductions in unemployment? Such questions are relevant to the assessment of changes in the structural factors of dependency and average income, and will be answered in the present section for the selected countries.

The examined changes may be thought of as induced by policy, or more generally, as exogenous variations in the given variables. Variables such as the unemployment rate, labor force participation and fertility, are affected in turn by more basic determinants (e.g., aggregate production, inflation, stabilization and adjustment policies in the case of unemployment; education in the case of labor force participation and fertility), and are normally specified as endogenous in causal economic models. For the purposes of the present decomposition analysis their changes may be interpreted as induced by variations in the more basic determinants. In a real policy context, the combination of feasible and most convenient actions will of course vary from one country to the other, depending on their initial labor market situation, their demographic profiles, and the capacity of policy makers to produce, directly or indirectly, the specified changes in the given variables. Initial conditions are to some extent incorporated in the analysis, because the initial structure (as regards to population, fertility, labor force, and unemployment) of each country is incorporated in the specification of each simulation and in the calculation of the effects.

The following results rely on the derivations detailed in the appendix and take as a basis the

\footnote{Related, although demographically much simpler decomposition techniques have been applied to examine income inequality in Latin American countries (e.g., Altimir & Piñera, 1977; Uthoff & Pollack, 1987).}
demographic, labor force participation, and unemployment conditions observed around 1990. The simulations are *caeteris paribus*: they change one variable at a time, while all others are held constant at their initial values, and are specified with a short-term temporal horizon in mind, (between 0 to 5 years), a time period in which they could possibly be made effective. Table 2 summarizes the mathematical results for the elasticity (i.e., the percentage change in y in response to a one percent increase in a given variable) of per capita earnings with respect to changes in the three mentioned variables, while figure 2 summarizes the results of the three simulations, that are discussed in some detail next.

(a) The effect of an extension of active life. Many of the Latin American countries are starting to manifest financial problems in their social security systems (Mesa-Lago, 1989, 1991), which are often accentuated by the increase of life expectancy and by the reduction in the mean age at retirement. The demographic factors are, more often than not, of secondary importance compared to the effects of the severe recent economic crises, but they constitute a non-negligible element of the problem, particularly with regard to its medium to long term trends. In some countries the legal age at retirement has been raised, and in some others legislation aimed at that objective (or more strictly enforcing existing regulations) is being considered. The repercussions on the financial balance of pension systems are presented in the next section; the potential for changing per capita earnings is examined here.

The magnitude of the positive effect of extending the length of active life on dependency and average income, depends on the way in which a given number of additional active years are distributed over the life cycle. Three alternative types are illustrated in figure 3. In the simplest case (case 1), where participation rates increase in a constant proportion at all ages, per capita income rises in the same proportion as active life. If the increase in active years is tilted toward older ages (case 2) the impact on per capita earnings is somewhat lower. For example, when the force of retirement is reduced by a fixed amount at all ages (and the force of entry into the labor force increases in the same constant quantity),

\[ \dot{R}_t = \frac{-1 + R_t}{R_t} A_L \hat{S} \]

and

\[ \dot{y} = A_L \hat{S} \]

where the dot above any letter represents the percentage change in that variable, S is the (expected) number of working years, and where \( A_L \) and \( \hat{A}_L \) denote the average ages of the labor force in the population and in the reference stationary population, respectively. Since in a growing population \( A_L \) is lower than \( \hat{A}_L \), the ratio \( A_L / \hat{A}_L \) is less than one, and the effect is therefore smaller than in the first case. The reason is that a given increase in labor force participation is applied with greater intensity to an age segment that has relatively little weight in the population. Finally, if participation rates increase proportionately more at
younger ages, and less at older ages (case 3), then the effect could be substantially higher, the reason being that the increase in participation occurs more heavily on the age groups that have the greatest proportional weight in the working age population.

For the present simulation, active life is assumed to increase in two years, an amount equivalent to less than 10 per cent of the values observed around 1990. Case 2, expressed in equations (3) and (4), represents more closely the scenario of increasing the average age at retirement; this is the case used for the reporting of effects in figure 2. The results show relatively uniform effects, of a larger overall magnitude than in the unemployment simulation: between 7 and 8 per cent in Argentina, Chile, Bolivia and Peru, and around 6.5 per cent in Mexico and Costa Rica. It is interesting to note, however, that a similar effect in Argentina and Bolivia is obtained for different reasons and under very different demographic conditions: in Argentina the large effect is due to its closer proximity to stationarity (producing a large $A_t/A_t$ elasticity) while in Bolivia, in spite of its younger age structure, the mean length of active life was low to start with, and therefore the percentage change in active life represented by the same two years is relatively large (a similar contrast can be made between Chile on the one hand, and Peru on the other).

Evidently, the above relations could serve as well to figure out the extent of decline in per capita income as a result of reductions in $S$, should governmental intervention to the contrary fail, and trends toward earlier retirement continue.

(b) The effect of a reduction in fertility. A reduction of fertility produces a greater proportion of the population in working ages, and with a time lag, a lower proportion of the young among the adults. In the Latin American countries, the net effect tends to be a reduction of dependency (as illustrated in figure 1) and therefore to increase average income, but depends in general on the repercussions on all the components in equation 1.

Precise formulae for the effect of a change in fertility on dependency and per capita income in any given population could be obtained in principle on the basis of B. Arthur's (1984) 'linkage method'. However, the approximations used here derived for reference stable populations can be more readily compared with the previous expressions, and are good enough for the purposes of this analysis. They can be written as:

$$\dot{R} = \frac{(1+R_e) (\Delta_l \Delta)^m}{R_e A_m}$$

For an interesting discussion of the effects of mortality decline (not taken up here) on the population age structure and lifetime consumption, see Lee, 1990.
\[
\dot{y} = \left[ -\frac{(A_\alpha - A)}{A_m} + \frac{(A_\alpha + A_\ast)}{A_m} \right] \dot{m}
\]

which can be simplified to:

\[
\dot{y} = -\frac{(A_\ast - A)}{A_m} \dot{m}
\]

where \(\dot{m}\) = the birth rate (i.e., the population-weighted mean fertility level), \(A\) = the average age of the population, \(A_\ast\) = the average age of the working age population (aged \(a\) and older), \(A_m\) = the average age of childbearing, with all values referred to the reference stable population. The first term of equation (6) is an approximation of the short-term effect of fertility change, which is always negative, since the mean age of the working age population (i.e., those aged \(a\) and older) is by necessity larger than that of the total population. The more substantive reason is that in the short term, lower fertility brings about a reduction in child dependency and therefore a proportionate increase in the working age population, leaving unaltered the age composition of the labor force. This component may thus be termed the 'child dependency reduction' effect.

The second term reflects the effect of ageing of the adults that starts to be manifested only after 20 years or so, which can be positive or negative. Since lowered fertility twists the proportional age distribution in the direction of the people that have the lowest activity rates, per capita earnings are normally reduced, but the converse outcome could occur in young populations where workers retire relatively late. Thus the total long run effect can be positive or negative, depending on the balance of the two components: in high fertility countries, where the total population is young compared to those in the labor force, a reduction in fertility will increase per capita earnings, and vice-versa. Estimates of the relevant average ages for the calculation of both the short and the long term effects appear in table 3.

Equation (6') has some resemblance to the well known 'intergenerational transfer effect' on lifetime consumption or welfare as it appears, for example, in Arthur & McNicoll (1978, 244) or Lee (1980, 1134), which implies that whether individual lifetime consumption or welfare improves or not as a result of a reduction in fertility depends on whether income flows are on average 'downward' or 'upward'; i.e., whether consumption occurs on average at younger ages than production. Beyond the apparent similarity however, what should stand

---

4 A result similar to that of equation (6') for changes in the stable growth rate is found in Preston, 1982, 257, which can be worked out directly, without the intermediate step of equation 6. The present derivation, obtained for changes in fertility, uses a relation given in Arthur, 1984, 115.
out is the key difference between them: the lifetime effect depends on the difference between the mean age of production and that of consumption, whereas the period income effect takes the difference with respect to the mean age of the population. Both numbers could coincide, for example, in the special case where the consumption age schedule is flat (consumption is the same at all ages), but in general they may differ. To the extent that they do, no necessary relationship holds between the changes in per capita income and in lifetime welfare; in principle, there may be cases where per capita income is raised while welfare deteriorates as a result of a reduction in fertility, and vice-versa. For example, estimates for the United States, United Kingdom and Japan by Lee (1988) and Ermish (1988) suggest that the average age of consumption exceeds that of earning by about four years, so that lifetime consumption would be reduced by a decline in fertility. On the other hand, the mean age at earning exceeds that of the population in these as in most countries, and thus period per capita income would go up as a result of lower fertility.

For this simulation, we take the changes of fertility between 1985-90 and 1990-95, based on CELADE's 'low' fertility projections of each country. These values were adopted with the idea of specifying a reduction of an amount close to the limit to which fertility could reasonably be expected to fall in each country during the next five years. The reported results correspond to the short term (i.e., child dependency reduction) effect only; the long term effect can be easily calculated from the data in table 3. As expected, some high fertility countries, i.e., Mexico and Peru, show the greatest potential from fertility reduction, with effects above 5.5 per cent. Costa Rica has intermediate fertility, but a relatively large expected decline according to the low projection which result in an effect comparable to Peru's. Argentina and Chile display changes below 3 per cent. Bolivia is a case of high fertility combined with a small expected fertility decline, thus its moderately low fertility effect (3.4 per cent).

Under the current demographic and labor force participation conditions, all countries (see table 3 for the relevant mean ages) have working age populations that are older than their respective labor force, and thus show unfavorable adult ageing effects (second term of equation 8). These latter effects, however, are relatively small and are not sufficient to compensate the favorable short term child dependency reduction effects. Both in the short and longer term, there are important differences in the degree to which a given fertility decline would improve per capita income: the changes tend to be substantial in high fertility (young age structure) countries and very reduced in countries with lower fertility (older populations).

---

How is it possible that individuals can be made worse off while measured per capita is increasing? The reason is that the period measure of earnings per person says nothing about how this income is allocated among members of different age groups; when consumption is more intense at older ages, lower fertility reduces the relative number of family members and taxpayers that can finance transfers to the elderly and therefore the implicit rate of return to these transfers is reduced, even when the compositional shifts can determine a rise in period per capita income.
Note further that the present 'ageing' estimates are conservative since they implicitly assume that productivity and consumption do not vary with age; to the extent that older people consume more (e.g., due to more onerous health expenditures), these would be more important, and perhaps dominant. More precise measurements of these factors could be made in countries where appropriate data exist, i.e., age profiles of productivity and consumption, including public transfers (see Lee, 1988, 1990). Also, 'ageing' is essentially relative in this context: a population can be labeled 'old' or 'young' only in relation to the timing and intensity of labor force participation by age. For example, Peru has a slightly older population age distribution than Bolivia, but has also an older age profile of economic activity, so the ageing effects turn out to be very similar in both countries; the comparison between contemporary Cuba and Argentina offers an example of two countries with very similar population age structure but different labor force age schedules, where ageing effects consequently differ.

(c) The effect of a reduction in unemployment. From a policy standpoint, the measured effects of variations in unemployment is the least realistic of all the simulations presented here since unemployment is normally thought of as driven by changes in production and income, not the reverse; it is included here for completeness and comparison with the changes in the two other major structural components. In this particular case, fertility, salaries and the labor force participation schedule are unchanged, so that the simulation could be thought of as a government employment creation policy in an excess labor supply context, where the higher employment levels are achieved without changing the equilibrium wage. This will be more valid in countries such as some in the less developed world where unemployment is high and/or the informal sector is large.

The changes in $R$ and $y$ due to a percent change in unemployment ($\mu$) are obtained by differentiating equations (1) and (2), and may be expressed as

$$
\dot{R} = \frac{(1+R)\mu}{R(1-\mu)}
\tag{7}
$$

$$
\dot{y} = -\frac{\mu}{(1-\mu)}
\tag{8}
$$

In the simulation, a reduction of 50% in the unemployment rate with respect to the 1990 value in each country is assumed. The results suggest that Bolivia and Peru, which had large unemployment rates around 1990, can expect significant effects (between 4 and 6.5 per cent) on mean earnings. Somewhat lower would be the effects in Argentina, Chile and Costa Rica (around 3.4 per cent), while in Mexico it is very reduced (below 2 per cent). These estimated effects are less stable in time than those for $\alpha$ and $\beta$, due to the greater short-run

---

6 different age patterns for the change in $\mu$ could be specified; a global change was chosen here since this simulation is intended to serve only as a rough basis for comparison with the rest.
volatility of unemployment rates.

4. Repercussions on pension systems.

Some of the relations, and the general approach used here, may be extended in many directions. A natural application, and a relevant one to the countries under study, is to transfer pension systems with partial or no funding, which are the norm in the Latin American region. Typically, the analyses of demographic factors in pension systems assume an equilibrium of the system's income (or 'contributions') with respect to expenditures ('pensions'). This is rarely the case even in transfer schemes that are actuarially balanced; most partially-funded systems are designed to obtain the actuarial balance over a given period of time precisely by contemplating a period of financial surplus, followed by one of deficit. Systems with no funding also experience imbalances in practice due to economic fluctuations. We admit the possibility of temporary (negative or positive) financial imbalances by specifying the deficit (\( D \)) of a pension system in any given period of time, expressed as a fraction of contributions, as:

\[
D = \phi R - 1 \tag{9}
\]

where \( \phi \) is the ratio of the average pension over the mean salary contribution by the economically active, and \( R \) is the ratio of the (eligible) retired population to the labor force in the system (the balanced budget case is a particular one where \( \phi R = 1 \)). It is clear from equation 9 that as long as \( \phi \) remains constant in time, any change in \( R \) will be transmitted on a one-to-one basis, in percentage terms, to the system's deficit.

The question now is how do changes in the population age structure and in retirement age would affect \( R \), and thus the financial balance of the system. Requirements for eligibility to receive pension benefits normally include both a minimum number of active contributory years and a minimum legal age at retirement. The number of compulsory contributory years is a binding restriction in most Latin American countries only for a small fraction of workers, since it typically varies from 10 to 15 years (IDB, 1991, 234-281), which is far less than what most people work over their lifetimes. Nonetheless, if the length of active life increases, either spontaneously (e.g., as a result of greater female labor force participation) or in response to legal requirements, the ratio of the retired to the active population will fall and the financial deficit will be reduced in an amount given by

\[
\dot{D} = \left[ \frac{1 + R_p^i}{R_p^f} \right] \dot{S} \tag{10}
\]

if activity rates increase in the same proportion at all ages (case 1), where \( R_p^i \) is the ratio of the inactive to the economically active among those in pensionable ages. The change is given by
\[
\dot{D} = -\left[ \left( \frac{A_{tp}}{A_t} \right) + \frac{(A_t / A_{tl}) R_p}{R_p} \right] \dot{S}
\]  

(10')

if the increase in participation rates is tilted toward the older ages (case 2), where \( A_{tp} \) is the mean age of those in pensionable ages that are still economically active. By use of the same assumptions as in the preceding section, a simulation of an increase in two years of active life was carried out, giving results consistent with the previous simulation on per capita income in the sense that the effects are relatively uniform, ranging from a potential of a 9 per cent reduction of the deficit in Argentina, to 14 per cent in Bolivia.

Changes in the minimum (legal) age at retirement are, at least in principle, under more direct governmental control. The rationale for increasing the legal age at retirement (or at any rate, for the attempts at it) is precisely that it is needed to improve the financial balance of the system; the more so when the gradual, but persistent population ageing trends are contemplated within a given planning period. Instead of inquiring separately about the impacts of changes in the population age distribution and in the legal age at retirement, we pose a single question that integrates both aspects: How much would the legal age at retirement have to increase to compensate for the projected population ageing? Purely numerical computer calculations can be made, but an analytical expression obtained for the reference stable populations, is also useful for gaining some insight into the factors that affect the results. Letting \( r \) be the (stable) intrinsic rate of natural increase and \( z \) the legal age at retirement, the required change in the age at retirement can be written as

\[
\Delta z = -\Phi(r,z) (A_p - A_t) \Delta r
\]

(11)

where \( \Phi(r,z) \) is the ratio of the economically active in pensionable ages over the inactive at age \( z \) in the reference stable population, and \( A_p \) is the average age of retirees\(^7\). A 30-year population projection (1990-2020) suggests that in order to compensate only for population ageing during this period, the legal age at retirement would have to increase between 0 and 3 years in Bolivia and Argentina, and between 5 and 10 in Costa Rica, Mexico and Peru\(^8\). The required changes are significant in most countries, except for Bolivia (which will experience a transitory adult population rejuvenation), and are clearly impractical in the last three; in these countries, other measures will have to be used in combination with retirement age requirements if the deficit is not to worsen further.

---

\(^7\) This result bears a close resemblance to that related to promotion in stable organizations due to Keyfitz (1985, pp. 107 & 108). By reinterpreting the ratio of high-rank to lower-rank workers as our \( R \), and the age at promotion as that of retirement, it becomes clear that the two problems have basically the same form.

\(^8\) the calculations assume that the remaining parameters (labor force participation, social security tax, mean level of wages in relation to pensions, coverage of the system) maintain their current levels. Chile is excluded from these calculations since its current private, fully funded capitalization system does not fit into this particular specification.
5. Summary and Conclusion.

The effects of extending active life and of fertility reductions on dependency and earnings per capita depend on the population and the labor force age distributions. By expressing these basic relations formally, a decomposition of the changes in dependency and income due to each one of their component elements was carried out, and the changes compared across six Latin American countries. The simulations suggest that changes in active life and fertility can have significant impacts on dependency and per capita income, although the favorable effects are not obtained without costs: funds must be made available to increment employment, higher incomes are obtained at the cost of more lifetime work effort when retirement is postponed, and lowered fertility, if continued in the longer run, may eventually produce some larger negative effects. On the other hand, changes in the extension of active life and fertility have a more stable character than those in employment and salaries, which at least in Latin America, have displayed great volatility in the past.

In a broader perspective, it is worthwhile to remember that the present results shown refer to first-order effects, that may be partially offset in the case of unemployment and age at retirement, but most likely be reinforced in the case of fertility\(^9\), and that the relations presented evidently do not suffice to justify particular policies; other considerations may be equally or more relevant as a basis for demographic or other social policies (Preston, 1988; United Nations, 1990, chap. IX). Nevertheless, the venue taken up in this paper, which is to highlight some basic accounting restrictions under which certain changes or policies take place, can be useful when utilized with appropriate care.

It is noted that a change in fertility need not affect total lifetime consumption in the same way as per capita income, so the use of the latter as a welfare index is to be taken with caution even when no other caveats apply. The analysis suggests that most of the countries of the region have not yet reached the stage where small reductions in fertility would be clearly detrimental to dependency and per capita income, although negative adult ageing affects are detected in all of them in varying degrees. The unfavorable aspects of ageing trends are more evident regarding their repercussions on the financial balance of pension systems, where the present results indicate that the required compensatory changes in the

\(^9\) The foregoing decomposition exercises, as noted earlier, hold everything else constant; i.e., give an idea of first-order effects only. How would the conclusions change if interactions were allowed to take place? Lacking actual measurements, theoretical considerations are of some help. Regarding an increase in active life, there may be three additional effects: a rise in unemployment rates and some reduction in wages in response to the increased labor supply, and a reduction in fertility, if some of the increased participation is of women. The first two will act in a direction opposite to the third, so that the first-order effect would most likely dominate, although it could be diminished to some extent. An exogenous reduction in unemployment may induce a short-term increase in births, which is likely to be small in comparison to its medium-term trend, but would nonetheless tend to ameliorate the positive first-order effect. A reduction in fertility, on the other hand, would tend to increase the labor force participation of women, which would directly contribute to augment per capita income. Indirectly and with a time lag, higher incomes and more intense female labor force participation may further reduce fertility, so that re-enforcing effects should be expected beyond the short term.
legal age at retirement in some countries will be very difficult to attain in practice, and therefore complementary measures are needed if the financial balance is not to deteriorate further.
References

Altimir, O. and Piñera, S. (1977) "Análisis de descomposición de las desigualdades de los ingresos en países de América Latina". Santiago, CEPAL.


Figure 1. Dependency ratio R(b)*
6 Latin American countries, 1950-2025

*R defined as the ratio of the inactive over the economically active. Source: CELADE (1990), ILO (1986).
Variables:

S = number of economically active years
m = mean age-specific fertility
u = unemployment rate

The age profiles as of 1990 are taken as a basis for calculations. The simulations are: a 50% reduction in the unemployment rate, an increase of 2 years in the mean age at retirement, and a reduction in fertility according to CELADE "low" projection in each country (see text for further details).
In case 1, activity rates increase in the same proportion at all ages; in case 2 participation rates increase more at older ages, and in case 3 the increase in more tilted toward the younger ages.
Table 1. Changes in the dependency ratio $R_g$ due to changes in the age structure and in labor force participation rates

<table>
<thead>
<tr>
<th>Country</th>
<th>changes in $R_g$ due to:</th>
<th>Country</th>
<th>changes in $R_g$ due to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$c(x)$</td>
<td>$\beta(x)$</td>
</tr>
<tr>
<td>Argentina</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950-1975</td>
<td>-0.048</td>
<td>0.122</td>
<td>0.021</td>
</tr>
<tr>
<td>1975-2000</td>
<td>-0.075</td>
<td>0.077</td>
<td>0.048</td>
</tr>
<tr>
<td>2000-2025</td>
<td>-0.138</td>
<td>0.040</td>
<td>-0.027</td>
</tr>
<tr>
<td>Chile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950-1975</td>
<td>0.004</td>
<td>0.043</td>
<td>0.075</td>
</tr>
<tr>
<td>1975-2000</td>
<td>-0.360</td>
<td>-0.018</td>
<td>0.113</td>
</tr>
<tr>
<td>2000-2025</td>
<td>-0.217</td>
<td>0.183</td>
<td>-0.049</td>
</tr>
<tr>
<td>Costa Rica</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950-1975</td>
<td>-0.065</td>
<td>-0.028</td>
<td>0.062</td>
</tr>
<tr>
<td>1975-2000</td>
<td>-0.556</td>
<td>0.015</td>
<td>0.084</td>
</tr>
<tr>
<td>2000-2025</td>
<td>-0.332</td>
<td>0.164</td>
<td>-0.052</td>
</tr>
</tbody>
</table>

Note: $\alpha = \text{proportion of the population in working ages (15 and over)}$; $c(x) = \text{working age population age distribution}$; $\beta(x) = \text{labor force participation rates by age (x)}$; $R_g = \text{dependency ratio, defined as the quotient of the inactive over the economically active. The decomposition is based on the total differentiation of } R_g$: $dR_g = -(1+R_g)\left[\frac{d\alpha}{\alpha} + \frac{1}{\beta} \sum_{15}^{\infty} \Delta c(x)\beta(x) + \sum_{15}^{\infty} \Delta \beta(x)c(x)\right]\]
Table 2. Elasticity of per capita earnings with respect to changes in active life, fertility and unemployment

**Length of Active Life**

(a) proportional at all ages: \(1\)

(b) tilted toward old: \(\frac{A_L}{\bar{A}_L} < 1\)

(c) tilted toward young: \(\frac{(Q-A_L)}{(Q-\bar{A}_L)} > 1\)

**Fertility**

(a) child dependency reduction: \(-\frac{(A_{a+} - A)}{A_m} < 0\)

(b) aging of adult population: \(\frac{(A_{a+} - A_L)}{A_m} \geq 0\)

(c) total effect: \(-\frac{(A_L - A)}{A_m} \geq 0\)

**Unemployment**

\(-\frac{\mu}{(1-\mu)} < 0\)

**Notation:**

\(\mu = \) unemployment rate  
\(A = \) average age of the population  
\(A_L = \) average age of the labor force  
\(\bar{A}_L = \) average age of the labor force in the reference stationary population  
\(A_{a+} = \) average age of the working-age population (aged \(a\) and older)  
\(A_m = \) average age at childbearing  
\(\Omega = \) oldest age of labor force participation

The values involved in the fertility elasticities correspond to the reference stable population. For the derivations, see the appendix.
Table 3. Estimated average ages around 1990.

The mean ages are of: the population (A); the working age population (Aₐₙ); childbearing (Aₘₚ); the labor force (Aₗ); the labor force in the reference stationary population (Aₗ₁)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>Aₐₙ</th>
<th>Aₘₚ</th>
<th>Aₗ</th>
<th>Aₗ₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>31.5</td>
<td>41.2</td>
<td>27.1</td>
<td>36.2</td>
<td>38.0</td>
</tr>
<tr>
<td>Chile</td>
<td>27.9</td>
<td>38.7</td>
<td>26.4</td>
<td>35.8</td>
<td>38.8</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>24.1</td>
<td>36.4</td>
<td>25.8</td>
<td>33.8</td>
<td>39.5</td>
</tr>
<tr>
<td>Mexico</td>
<td>23.4</td>
<td>35.7</td>
<td>26.9</td>
<td>33.9</td>
<td>39.7</td>
</tr>
<tr>
<td>Peru</td>
<td>24.0</td>
<td>36.0</td>
<td>27.5</td>
<td>35.6</td>
<td>40.9</td>
</tr>
<tr>
<td>Bolivia</td>
<td>23.2</td>
<td>35.2</td>
<td>28.2</td>
<td>34.2</td>
<td>39.5</td>
</tr>
</tbody>
</table>

Appendix

Notation and basic definitions.

Let:
- \( P \) = the size of the population
- \( M \) = the working-age population
- \( L \) = the size of the labor force
- \( E \) = number of employed persons
- \( w \) = average earnings per worker
- \( Y \) = total earnings
- \( a = M/P \) = proportion of the population in working-age
- \( B = L/N \) = total labor force participation rate
- \( c = E/L \) = employment rate, with \( p = 1 - c \) = unemployment rate
- \( y = Y/P \) = income (or more strictly, earnings) per capita.

Dependency ratio \( R \) and per capita income \( y \).

\( R = (P-E)/E = (P/P-E)/E \). Since \( E/P = (E/L)[L/N][N/P] = \epsilon \delta \),

\( R = (1-\alpha)/\alpha \) \hspace{1cm} (1)

Per capita income is:

\( y = Y/P = wE/(P-E+E) = w(E/E)/(E-E/E) \)

\( y = w/(1+R) \) \hspace{1cm} (2)

which can also be written as \( w\delta \).

Effects of a change in the number of active years \( S \).

In any given population with population density function \( N \), and labor force participation function \( p \), the total labor force participation rate is

\( B = \int_{a}^{\infty} p \, dx \). \hspace{1cm} where \( a \) is the youngest working age.

Let \( S \) = the (expected) number of economically active years. For a cohort (or a stationary population) with survivorship function \( l \), defined for exact age \( x \),

\( S = \int_{a}^{\infty} p \, dx \).

Consider an increase in the number of active years equal to \( dS \), that corresponds to an increment in labor force participation for each age \( x \), of an amount \( dp \). There are infinitely many possible life-cycle changes in \( p \) that could match a given \( dS \); a few general types are considered next. In general, the derivations rely on the following equalities:

\( \frac{dr}{ds} = \frac{dr}{dk} \cdot \frac{dk}{ds} \)

\( \frac{dy}{ds} = \frac{dy}{dk} \cdot \frac{dk}{ds} \)

where \( B \) and \( S \) are functions of a parameter \( k \). From equations (1) and (2), it can be deduced that \( \frac{dr}{ds} = -(1+R)/B \)

and \( \frac{dy}{ds} = y/8 \). Only \( \frac{dk}{ds} \) and \( \frac{dk}{ds} \) remain to be determined.

Case 1. If \( p^* = (1+k) \), \( S^* = \int_{a}^{\infty} l(1+k)p \, dx \), so

\( ds = S \)

and similarly \( dk = B \). Substituting in the above expressions for \( dr/ds \) and \( dy/ds \), and denoting discrete
differentials by \( \delta \),

\( \delta R = -(1+R)/\delta \)

\( \delta Y = \delta \frac{S}{S} \)

Case 2. If \( p^* = p \delta \), \( S^* = \int_{a}^{\infty} l p \delta \, dx \)

\( ds = \int_{a}^{\infty} x l p \delta \, dx = A \delta \)

evaluated at \( k=0 \), where

\( A = \int_{a}^{\infty} x l p \, dx \)

is the mean age of the labor force in the reference stationary population.

This result is analogous to the effect of an increase in the force of mortality on life expectancy at birth (Keyfitz, 1985, p. 62). Similarly,

\( \delta d = A \delta \), again evaluated at \( k=0 \), and where \( A \) is the mean age of the labor force in that population.
\[ \Delta R. = (1+R.) \frac{\Delta S}{A} \]  \hspace{1cm} (3) \\

\[ \Delta Y = \frac{\Delta S}{y} \frac{A}{A S} \]  \hspace{1cm} (4) \\

Case 3. If \( p^* = p^e \), where \( \Omega \) is the oldest age of labor force participation, 
\[
S^* = \int_{0}^{\Omega} p^e^{(\xi-\Omega)/\alpha} d\xi
\]
\[
dS^* = \int_{0}^{\Omega} (\xi-\Omega) p^e^{(\xi-\Omega)/\alpha} d\xi = (\Omega-\Lambda) S^*, \text{ evaluated at } k=0.
\]
\[
dS = (\Omega-\Lambda), \text{ also evaluated at } k=0. \text{ Thus,}
\]
\[
\Delta R. = -(1+R.)(\Omega-\Lambda) \frac{\Delta S}{\Omega-\Lambda} \\
\Delta Y = \frac{(\Omega-\Lambda)}{\Omega-\Lambda} \frac{\Delta S}{\Omega-\Lambda} \\
\]

Effects of a change in fertility (stable population). 

In a stable population, the proportion of the population in working-ages is:
\[
\alpha = \int_{0}^{\infty} 1.e^{-\alpha}d\alpha / \int_{0}^{\alpha} 1.e^{-\alpha}d\alpha
\]
where \( r \) is the (stable) rate of natural increase. A change in \( r \), of an amount \( dr \), induced by a change in the age-specific fertility rates \( m_\alpha \), of amount \( dm \) (result derived in Arthur, 1984, p. 115), is \( dr = (1/m_\alpha)dm \), where \( m \) is the birth rate and \( A_\alpha \) the mean age of childbearing in the stable population. Taking the derivative of \( \alpha \) with respect to \( r \) and using the above expression for \( dr \),
\[
\Delta \alpha = -(A_\alpha - A) \frac{\Delta m}{m}
\]
where \( A_\alpha \) and \( A \) denote the mean age of the population aged \( a \) and older, and of the total population respectively. By a similar process, the change in \( B \) induced by a change in \( m \) is obtained as:
\[
\Delta B = (A_\alpha - A) \frac{\Delta m}{m}
\]
Using the fact that \( dy/y = [(1/\alpha)\delta\alpha/\delta m + (1/\alpha)\delta B/\delta m] dm \),
\[
\Delta Y = \left[ -(A_\alpha - A) + (A_\alpha - A) \right] \frac{\Delta m}{m}
\]
which simplifies to
\[
\Delta Y = - \frac{A_\alpha - A}{m} \frac{\Delta m}{m}
\]
(6')

Effects of a change in the unemployment ratio (\( \mu \)).

The changes in \( R. \) and \( y \) are obtained by differentiating equations (1) and (2), and using the fact that \( \Delta \mu = -\Delta \mu \):
\[
\Delta R. = g(1+R.) \frac{\Delta \mu}{1-\mu} \hspace{1cm} (7)
\]
\[
\Delta y = \frac{-\mu. \Delta \mu}{(1-\mu)} \hspace{1cm} (8)
\]