NATIVE DEPENDENCE AND THE SPATIAL REPRODUCTIVE VALUE

Andrei Rogers
Cecile Hemez

Population Program
Institute of Behavioral Science
University of Colorado
Boulder, CO 80309-0484

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1. INTRODUCTION

The introduction of regions into classical mathematical demography has added several interesting new dimensions to established indices such as stable equivalent population, momentum, and reproductive value. We shall focus on the last of these three measures in this paper and show how its generalization to a multiregional setting provides insights regarding the impacts of spatial population dynamics.

In doing so we shall distinguish between models that are <u>closed</u> and <u>open</u> to international migration and between models that are <u>native independent</u> and <u>native dependent</u> (Ledent and Rogers, 1988). Following Espenshade et al. (1982) and Arthur and Espenshade (1988), our open models will only consider reproductive regimes that are below replacement level. Within that context, this paper develops, for example, the notion of the spatial reproductive value of an average immigrant of a particular age. It then disaggregates this measure to differentiate immigrants who arrive at different entry points in the United States.

Below replacement level fertility has become an important characteristic of economically advanced societies such as the United States, Canada, Japan, Germany, Italy, Austria, the United Kingdom, the Soviet Union, and France. In studying the long-run implications of such demographic behavior, demographers have shown that the size of the future national population in such instances is primarily determined by the magnitudes of the country's net reproduction rate and its immigration flows. They have demonstrated that a fixed level of net immigrants with a specific age composition, entering a country experiencing an unchanging regime of age-specific mortality and below-replacement fertility, ultimately will generate a stationary ("zero growth") population, whose size is strictly determined by the size and age composition of the stream of the immigrants and by the net reproduction rate of the host country---a rate that the immigrants and their descendants are assumed to ultimately adopt. (Espenshade et al., 1982; Mitra, 1983).

2. INTRODUCING NATIVE DEPENDENCE AND IMMIGRATION INTO A BIRTHPLACE-SPECIFIC MULTIREGIONAL POPULATION

2.1 Introducing Native Dependence

In recent years, techniques for constructing multistate life tables and for projecting multistate populations have been fruitfully applied to the study of a number of demographic phenomena, ranging from migration and population redistribution (Rogers and Willekens, 1986) to marital status dynamics (Schoen and Baj, 1984; Espenshade, 1983) to labor force participation (Smith, 1982) and to fertility patterns (Suchindran and Koo, 1980). These applications have recognized that the Markov assumption underlying such multistate models---that age-specific transition intensities depend only on the currently occupied state or status and not on the duration of this occupancy or on the history of previous occupancies---is a restrictive assumption that should be relaxed whenever feasible.

An important effort to include the influences of previous occupancies in multistate analysis has been the incorporation of a place-of-birth dependence. For example, Ledent (1981) demonstrated that introducing place-of-birth-specific (native-dependent) rates of migration into multiregional life table calculations dramatically affected estimated values of life expectancies and survivorship probabilities. Philipov and Rogers (1981) showed that parallel impacts also arose in the associated native-dependent multiregional population projections. More recently, Ledent and Rogers (1988) extended the classical unistate model to incorporate native dependent multistate dynamics.

The introduction of native dependence into the multiradix/multistate model is accomplished by computing an independent uniradix multiregional life table for each place-of-birth-specific cohort. The mortality and migration intensities are those experienced by that cohort, as are the fertility rates that are combined with the life table's person-years lived variables to define the characteristic matrix: $\underset{\sim}{\psi}^{c}(r)$ in equation (1) or $\underset{\sim}{\psi}^{d}(r)$ in equation (2), namely,

$$\left[\sum_{x=\alpha}^{\beta-n} exp[-r(x+\frac{n}{2})]\, \underset{\sim}{F}_x \underset{\sim}{L}_x\right] \{\underset{\sim}{Q}\} = \underset{\sim}{\psi}^c(r)\{Q\} = \{Q\} \tag{1}$$

if derived from the continuous-time formulation of the projection process (Rogers, 1975) and

$$\left[\sum_{x=\alpha-n}^{\beta-n} exp[-r(x+n)]\, \frac{\underset{\sim}{F}_x \underset{\sim}{L}_x + \underset{\sim}{F}_{x+n} \underset{\sim}{L}_{x+n}}{2}\right] \{Q\} = \underset{\sim}{\psi}^d(r)\{Q\} = \{Q\} \tag{2}$$

if derived from its discrete-time formulation (Ledent and Rogers, 1988).

The vector *{Q}* is a vector of regional stable equivalent births, $\underset{\sim}{L}(x)$ is the matrix of life table person-years lived, and $\underset{\sim}{F}(x)$ is the diagonal matrix of age-specific birth rates. Solving for the intrinsic rate, r, by, say, the method of functional iteration (Keyfitz, 1968) one also obtains the vector *{Q}* (Ledent and Rogers, 1988). The intrinsic rate of growth is the value of r that gives the matrix $\underset{\sim}{\psi}^c(r)$ in equation (1) or $\underset{\sim}{\psi}^d(r)$ in equation (2) a characteristic root of unity; the corresponding characteristic vector is denoted by *{Q}*.

To introduce native-dependence, the matrix product $\underset{\sim}{F}_x \underset{\sim}{L}_x$ in equation (1) or (2) is replaced by

$$\left[\sum_{p=1}^{m} {}^{(p)}\underset{\sim}{F}_x\, {}^{(p)}\underset{\sim}{L}_x\, {}^{(p)}\underset{\sim}{M}\right] \tag{3}$$

where ${}^{(p)}\underset{\sim}{F}_x$ = a diagonal matrix of regional birth rates experienced by the cohort born in region p;

${}^{(p)}\underset{\sim}{L}_x$ = a matrix of person-years lived by the cohort born in region p; and

${}^{(p)}\underset{\sim}{M}$ = a matrix with all of its elements equal to zero except for a one in the $p^{\underline{th}}$ row and $p^{\underline{th}}$ column position.

The matrix ${}^{(p)}\underset{\sim}{M}$ ensures that only the $p^{\underline{th}}$ column of ${}^{(p)}\underset{\sim}{L}_x$ enters into the calculations for the $p^{\underline{th}}$ radix. In the likely event that place-of-birth-specific fertility rates are unavailable, assume that

$$ {}^{(p)}\underset{\sim}{F}_x = \underset{\sim}{F}_x \qquad \textit{for all } p = 1,\, 2,\, \ldots,\, m $$

and simplify equation (3) to:

$$ \underset{\sim}{F}_x \sum_{p=1}^{m} {}^{(p)}\underset{\sim}{L}_x \, {}^{(p)}\underset{\sim}{M} = \underset{\sim}{F}_x \underset{\sim}{\overline{L}}_x \tag{4} $$

where $\underset{\sim}{\overline{L}}_x$ is a matrix whose $p^{\underline{th}}$ column is the $p^{\underline{th}}$ column of ${}^{(p)}\underset{\sim}{L}_x$. The iterative algorithm used to solve for r remains unchanged.

2.2 Introducing Immigration

Demographers interested in learning how the standard "closed" unistate stable population model can be "opened" to include international migration can turn to two streams of literature for enlightenment: the continuous-time formulation discussed by Espenshade et al. (1982), Mitra (1983), Mitra and Cerone (1986), and Cerone (1987), or the discrete-time formulation described by Pollard (1966, 1973) and Keyfitz (1968). This literature reveals that for a national population, an ultimate consequence of a fixed stream of immigration is a stationary population if fertility is below replacement level, a linearly increasing population if fertility is exactly at replacement level, and an exponentially increasing population if fertility is above replacement level.

In this paper's examination of the open model, attention will be focused on results for a below-replacement level fertility regime and a fixed net immigration vector. (One can, of course, deal with the case of a <u>gross</u> immigration flow by treating emigration as a form of death.)

The continuous-time model set out in Espenshade at al. (1982) is defined by two fundamental relationships, one relating to total annual births and the other to total population. The derivation of the multiregional form of their two basic relationships is straight-forward. We begin with the matrix expression for the vector of total annual regional births $\{B(t)\}$, find the constant vector of regional annual births to foreign-borns $\{B_1\}$, and then solve for the vector of regional stationary equivalent populations $\{N\}$.

i.) <u>Total Annual Births by Region (State)</u> = Annual Births to Native-borns + Annual Births to Foreign-borns:

$$\begin{aligned} \{B(t)\} &= \int_{\alpha}^{\beta} \underset{\sim}{m}(a)\,\{N(a,t)\}\,da \\ &= \int_{\alpha}^{\beta} \underset{\sim}{m}(a)\,\underset{\sim}{\ell}(a)\,\{B(t-a)\}\,da + \int_{\alpha}^{\beta} \underset{\sim}{m}(a)\,\{H_I(a)\}\,da \end{aligned} \tag{5}$$

where the age-specific foreign-born population by region of residence at age a is

$$\{H_I(a)\} = \int_o^a \underset{\sim}{\ell}(a)\underset{\sim}{\ell}(x)^{-1}\{i(x)\}dx \tag{6}$$

and where $\{i(x)\}$ is a fixed vector of regional annual migration streams at age x, $\{N(a,t)\}$ is a vector of regional populations at age a at time t, $\underset{\sim}{m}(a)$ is a matrix of annual regional fertility rates at age a, α and β are the lower and upper limits of the childbearing ages, respectively, and $\underset{\sim}{\ell}(a)$ is a matrix of regional life table probabilities of survival from birth to age a, disaggregated by region of birth and region of residence (Rogers, 1975). Following Espenshade et al. (1982), we shall focus on the evolution of the female population, which like births may be represented as the sum of two subpopulations, native-born and foreign-born.

ii.) <u>Total Population by Region (State)</u> = Native-borns + Foreign-borns:

$$\{N(t)\} = \int_o^\infty \underset{\sim}{\ell}(a)\{B(t-a)\}da + \int_o^\infty \{H_I(a)\}da \tag{7}$$

The long-run behavior of $\{B(t)\}$ and $\{N(t)\}$ under a <u>nationally</u> below replacement level fertility regime can be identified by a multistate matrix generalization of the argument in Espenshade et al. (1982), with the condition for achieving an asymptotic limit now being that the dominant characteristic root of the net reproduction matrix $\underset{\sim}{R_0}$ be less than unity. In that event the asymptotic limit of $\{B(t)\}$ is

$$\{B\} = \left(\underset{\sim}{I}-\underset{\sim}{R}_0\right)^{-1}\{B_1\} \tag{8}$$

where

$$\{B_1\} = \int_\alpha^\beta \underset{\sim}{m}(a)\,\{H_I(a)\}\,da \tag{9}$$

Finally, substituting a constant vector of births $\{B\}$ into equation (7) yields:

$$\{N\} = \underset{\sim}{e}(0)\{B\} + \{H_I\} \tag{10}$$

where $\underset{\sim}{e}(0)$ is the life expectancy at birth matrix $\int_o^\infty \underset{\sim}{\ell}(a)\,da$, and $\{H_I\}$ is the vector of regional foreign-born populations $\int_o^\infty \{H_I(a)\}\,da$. Finally, replacing $\{B\}$ by its value in equation (8) gives

$$\{N\} = \underset{\sim}{e}(0)\left(\underset{\sim}{I}-\underset{\sim}{R}_0\right)^{-1}\{B_1\} + \{H_I\} \tag{11}$$

which is the multistate stationary equivalent population in this model. Note that the inverse may be expressed as the sum of a geometric series, i.e.,

$$\left(\underset{\sim}{I}-\underset{\sim}{R}_0\right)^{-1} = \underset{\sim}{I}+\underset{\sim}{R}_0+\underset{\sim}{R}_0^2+\underset{\sim}{R}_0^3+\ldots = \sum_{k=0}^{\infty} \underset{\sim}{R}_0^{\,k}.$$

3. THE SPATIAL REPRODUCTIVE VALUE IN A CLOSED POPULATION

3.1 The Nonspatial Reproductive Value

The concept of reproductive value, as developed by Fisher (1929), revolves around the notion of regarding the offspring of a child as the repayment of a debt. Specifically if the birth of a baby is viewed as a loan of a life and if the future offspring of this child are viewed as the subsequent repayment of this loan, suitably discounted at the annual rate *r* and compounded momently, then the present value of the repayment may be taken to be

$$\int_0^\infty \exp(-ra)\, m(a)\, \ell(a)\, da \tag{12}$$

Equating the loan with the discounted repayment, one gets

$$1 = \int_0^\infty \exp(-ra)\, m(a)\, \ell(a)\, da, \tag{13}$$

which is recognizable as the characteristic equation used to solve for r, the intrinsic rate of growth. Thus, as Keyfitz points out,

> "the equation can now be seen in a new light: the equating of loan and discounted repayment is what determines ***r***, ***r*** being interpretable either as the rate of interest of an average loan or as Lotka's intrinsic rate of natural increase" (Keyfitz, 1975, page 588).

In the same essay, Keyfitz considers how much of the debt is outstanding by the time the child has reached the age *x*. He defines this quantity to be *v(x)*, the reproductive value at age *x*, where

$$v(x) = \int_x^\infty \exp[-r(a-x)]\, m(a)\, \frac{\ell(a)}{\ell(x)}\, da, \tag{14}$$

and v(0) is scaled to equal unity.

Goodman (1969) develops a somewhat different unistate (and hierarchical multistate) formulation starting with a single person in the first age group and adopting a discrete age-time framework.

3.2 The Native Independent Spatial Reproductive Value

Keyfitz's arguments have their spatial (multiregional) counterparts (Rogers and Willekens, 1978). To develop these it is convenient to reexpress equation (14) for arbitrary values of $v(0)$, that is,

$$v(x) = v(0)\int_x^\infty \exp[-r(a-x)]\,m(a)\,\frac{\ell(a)}{\ell(x)}\,da = v(0)\,n(x)\,,$$

where

$$v(0) = v(0)\int_o^\infty \exp(-ra)\,m(a)\,\ell(a)\,da = v(0)\,\psi(r) = 1$$

and $n(x)$ denotes the total discounted number of baby girls expected to be born to a woman now aged x. This form of the equation immediately suggests the multiregional analog: find the (row) vector $\{v(x)\}'$, such that

$$\{v(x)\}' = \{v(0)\}'\int_x^\infty \exp[-r(a-x)]\,\underset{\sim}{m}(a)\,\ell(\underset{\sim}{a})\,[\underset{\sim}{\ell}(x)]^{-1}da \tag{15}$$

$$= \{v(0)\}'\underset{\sim}{n}(x)\,, \tag{16}$$

where

$$\{v(0)\}' = \{v(0)\}'\int_0^\infty \exp(-ra)\underset{\sim}{m}(a)\ell(\underset{\sim}{a})\,da = \{v(0)\}'\underset{\sim}{\psi}(r), \qquad (17)$$

that is, where $\{v(0)\}'$ is the left characteristic row vector associated with the unit dominant root of the characteristic matrix $\underset{\sim}{\psi}(r)$.

The matrix $\underset{\sim}{n}(x)$ represents the expected total number of female offspring per woman at age x, discounted back to age x. The element $n_{ij}(x)$ gives the discounted number of female children to be born in region j to a woman now x years of age and a resident of region i. The vector $\{v(x)\}'$ represents the reproductive values of x-year-old women, differentiated by region of residence. Observe that the elements of $\{v(x)\}'$ depend on the scaling given to $\{v(0)\}'$, the left characteristic vector associated with the unit dominant characteristic root of the characteristic matrix $\underset{\sim}{\psi}(r)$. Thus in the multiregional model, the reproductive value of a baby girl depends on where she is born.

Equations (15), (16), and (17) may be given the following demographic interpretation. If lives are loaned to regions according to the (column) vector (Q) then the amount of "debt" outstanding x years later is given by the (row) vector $\{v(x)\}'$, the regional expected values of subsequent offspring discounted back to age x. The elements of this vector therefore may be viewed as spatial (regional) reproductive values at age x.

A slightly modified perspective of the spatial reproductive value is adopted in this paper. Specifically we shall distinguish between the terms <u>number</u> and <u>value</u> when referring to births in the various regions of a closed multiregional system. The two expressions have identical meanings in the uniregional model, but variations in regional fertility and mortality schedules give them different meanings in any multiregional model in which internal migration is represented.

Recall the definition of the multiregional characteristic matrix $\underset{\sim}{\psi}(r)$ in equations (1) and (2). An element ${}_i\psi_j(r)$ denotes the discounted total number of daughters born in region j to a mother born in region i. The discounted number of female births per woman born in a particular region ${}_i\psi(r) = \Sigma_i\psi_j(r)$, may be less than unity. Although this suggests that she does not repay the full amount of her "debt" to society, we shall show that this may be true only of number but not of value. Thus if the investment in one life in a region is viewed as a debt of an individual to society, then in a stable equilibrium each individual must repay that debt to society at an annual interest rate r. The repayment does not have to take place in the region of birth, however. Part of it can occur in other regions, where births may be worth more (or less) than in the region of birth. Thus we may conclude that individuals pay back their debt to society in values $v(0)$, whereas regions pay back their debt in numbers Q. The former distribution is defined by equation (17); the latter derives from equations (1) or (2).

Spatial reproductive values at age x, $v_i(x)$, may be appropriately consolidated to yield total spatial reproductive values, v_i, by means of the relationship

$$\begin{aligned} \{v\}' &= \int_o^\infty \{v(x)\}' \underset{\sim}{k}(x)\,dx \\ &= \{v(0)\}' \int_o^\infty \underset{\sim}{n}(x)\,\underset{\sim}{k}(x)\,dx \\ &= \{v(0)\}' \underset{\sim}{n} \end{aligned}$$

where $\underset{\sim}{k}(x)$ is a diagonal matrix with $k_{ii}(x)$ representing the number of women at age x in region i, and $\underset{\sim}{n}$ is a matrix of total discounted number of female offspring associated with that population. The total reproductive value of the multiregional population then is $v = \{v\}'\{1\}$.

3.3 The Native Dependent Spatial Reproductive Value

In Section 2.1 of this paper we learned that the introduction of native dependence into the multiregional model could be accomplished by computing an independent uniradix multiregional life table for each birthplace-specific cohort. Denote the p^{th} cohort's life table survival probability matrix ${}^{(p)}\underset{\sim}{\ell}(x)$. Retain only the p^{th} column of that matrix and with it define the p^{th} cohort's survival probabilities. Collect all such columns to form a composite matrix $\bar{\underset{\sim}{\ell}}(x)$ to represent the survival regime of all cohorts and enter it in place of $\underset{\sim}{\ell}(x)$ in the equations defining the spatial reproductive value. (We assume, once again, that birthplace-specific data on fertility are unavailable, so the matrix $\underset{\sim}{m}(a)$ remains unchanged.)

Entering $\bar{\ell}(x)$ into equations (15) and (17), along with the corresponding new value of the stable rate of growth (now denoted by $\bar{r}$), one obtains the corresponding native dependent values for the spatial reproductive value.

4. THE SPATIAL REPRODUCTIVE VALUE IN AN OPEN POPULATION

4.1 The Nonspatial Reproductive Value

In a more recent paper, two of the authors of the original Espenshade et al. (1982) article re-examine the topic of immigration and the stable population model, focusing on the influence of the immigrant stream's age composition (Arthur and Espenshade, 1988). By changing the order of integration in the equation for the stationary female population equivalent, N, they are able to show that

$$N = \frac{e(0)}{1-R_0} \int_o^\infty i(x)\, v(x)\, dx + \int_o^\infty i(x)\, e(x) \tag{18}$$

where R_0, as before, is assumed to be less than unity, $e(x)$ is the remaining life expectancy at age x, and "$v(x)$ [is] the average number of daughters remaining to be born to a female immigrant admitted at age x" (Arthur and Espenshade, 1988, p. 318). As we have seen, $v(x)$ is the reproductive value at exact age x.

By expressing the ultimate total stationary population N in terms of life expectancies and reproductive values, Arthur and Espenshade make explicit the influences that age at admission of immigrants has on that population.

"...migrants contribute to the size of the ultimate stationary population in two ways: first, through their presence in the population and, second, through their offspring, who set in motion a chain of descendants from one generation to the next. And it is now clear that the age distribution of immigrants is a crucial determinant of ultimate population size. Because *e(x)* and *v(x)* slope downward over much of the relevant age range, increasing immigrants' ages at admission will typically reduce the ultimate stationary population size" (Arthur and Espenshade, 1988, p. 319).

4.2 The Native Independent Spatial Reproductive Value

The multistate generalization of equation (18) is straightforward (Rogers, 1990). Changing the order of integration in

$$\{N\} = \underset{\sim}{e}(0)\,(\underset{\sim}{I}-\underset{\sim}{R_0})^{-1}\int_o^\infty\int_o^\infty \underset{\sim}{m}(a)\,\underset{\sim}{\ell}(a)\,\underset{\sim}{\ell}(x)^{-1}\{i(x)\}\,dx\,da + \int_o^\infty\int_o^\infty \underset{\sim}{\ell}(a)\,\underset{\sim}{\ell}(x)^{-1}\{i(x)\}\,dx\,da \tag{19}$$

we find that

$$\{N\} = \underset{\sim}{e}(0)\,(\underset{\sim}{I}-\underset{\sim}{R_0})^{-1}\int_o^\infty \underset{\sim}{z}(x)\,\{i(x)\}\,dx + \int_o^\infty \underset{\sim}{e}(x)\,\{i(x)\}\,dx \tag{20}$$

where $\underset{\sim}{z}(x)$ denotes the matrix of expected total number of female offspring per immigrant woman at age *x*, discounted back to age *x*, with $z_{ij}(x)$ representing the expected discounted number of female children to be born in state *j* to an immigrant woman now *x* years of age and entering state i (Rogers and Willekens, 1978). The matrix $\underset{\sim}{z}(x)$ is the open model's counterpart to the closed model's matrix $\underset{\sim}{n}(x)$ defined in equation (16). To transform $\underset{\sim}{z}(x)$ into the multistate row vector of reproductive values $\{v(x)\}'$, one needs to premultiply it by the corresponding vector $\{v(0)\}'$, the left characteristic row vector associated with the dominant root of the net reproduction rate matrix, $\underset{\sim}{R}_0$:

$$\{v(0)\}' = \{v(0)\}'\underset{\sim}{R}_0 \tag{21}$$

Reproductive values and expected number of offspring are equivalent concepts in unistate demography; they are not in multistate demography. In the unistate case, $v(0)=1$, $v(x)=z(x)$; but in the multistate case only one state-specific reproductive value (the "numeraire") is set equal to unity. Hence one needs to adopt the relative weighting (Rogers and Willekens, 1978):

$$\{v(x)\}' = \{v(0)\}'\underset{\sim}{z}(x) \tag{22}$$

Following Arthur and Espenshade (1988), let us now simplify the analysis and assume that all net immigrants enter at a single exact age, x_0. Then the integrals disappear, and

$$\{N\} = \underset{\sim}{e}(0)(\underset{\sim}{I}-\underset{\sim}{R}_0)^{-1}\underset{\sim}{z}(x_0)\{i(x_0)\} + \underset{\sim}{e}(x)\{i(x_0)\} \tag{23}$$

Transforming the above to a per immigrant formulation, by dividing each N_j by the corresponding $i_j(x_0)$, gives

$$\{N_I\} = \underset{\sim}{e}(0)(\underset{\sim}{I}-\underset{\sim}{R}_0)^{-1}\underset{\sim}{z}(x_0)\{1\} + \underset{\sim}{e}(x_0)\{1\} \quad (24)$$

which makes clear that for a given age x_o, $\{N_I\}$ varies as a function of $\underset{\sim}{z}(x_0)$ and $\underset{\sim}{e}(x_0)$. As in the unistate case, the term $\underset{\sim}{e}(x_0)$ represents the remaining average lifetime of a new immigrant who enters at exact age x_0, but the relationship now varies by region of entry and residence as well as by age.

4.3 The Native Dependent Spatial Reproductive Value

The procedure for computing the open model's native dependent spatial reproductive value is the same as for the closed model's. One replaces the survival probability matrix $\underset{\sim}{\ell}(x)$ by its composite counterpart $\bar{\underset{\sim}{\ell}}(x)$ in the defining equations (19), (20), and (21). Since r=0 in the open model, no composite counterpart for the stable growth rate needs to be calculated. First compute $\bar{\underset{\sim}{z}}(x)$, using the definitions embedded in equations (19) and (20). Then, calculate $\{v(0)\}'$ using equation (21); the corresponding values for all ages x follow from equation (22).

5. CONCLUSION

A national multistate population exposed to fixed fertility, mortality, and internal migration, and a constant annual number and age distribution of immigrants, will ultimately become a zero growth stationary population, if national fertility is below replacement level. If fertility is at replacement level, the population will increase linearly, and if fertility is above that level then the growth will be exponential. This can be demonstrated both with the discrete-time and the continuous-time models.

The ages at which immigrants are admitted can be shown to make a significant difference in the ultimate population size and spatial distribution, <u>and so can the region of entry</u>. Multistate versions of the life expectancy and the reproductive value may be used to assess these impacts.

Finally, although the numerical illustrations in the Appendix to this paper deal with regional populations linked by internal migration streams, many other multistate models come to mind, in which regions are replaced by statuses, such as marital states, employment states, and states of health. Nothing in the mathematical apparatus presented precludes such applications of the general model.

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APPENDIX: NUMERICAL EXAMPLES

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC
REPRODUCTIVE VALUES: V(x)
United States, 1980

I. THE CLOSED MODEL

	Nonspatial	Spatial: Native Independent		Spatial: Native Dependent	
	v(0)=1, r=-.004382	{v(0)}'=[1 1.114767], r=-.003955		{v(0)}'=[1 1.206076], r=-.003734	
	Region of Residence	Region of Residence		Region of Residence	
Age	USA	North	Southwest	North	Southwest
0	.997172	.993666	1.115866	.991360	1.209208
5	.984387	.974287	1.109284	.970008	1.203963
10	.963134	.948312	1.090949	.945444	1.183206
15	.878830	.868504	.990844	.866585	1.073770
20	.657025	.660341	.728440	.658928	.788809
25	.363588	.370435	.398433	.369416	.431522
30	.140048	.140600	.155658	.140194	.168604
35	.035750	.034534	.041070	.034459	.044467
40	.005489	.005099	.006513	.005094	.007048
45	.000270	.000230	.000343	.000230	.000371

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC
REPRODUCTIVE VALUES: V(x)
United States, 1980

2. THE OPEN MODEL

	Nonspatial $v(0)=1,\ r=0$	Spatial: Native Independent $\{v(0)\}'=[1\quad 1.119484],\ r=0$		Spatial: Native Dependent $\{v(0)\}'=[1\quad 1.213848],\ r=0$	
	Region of Residence	Region of Residence		Region of Residence	
Age	USA	North	Southwest	North	Southwest
0	.899676	.895842	1.025555	.911484	1.111341
5	.907816	.899543	1.038182	.912565	1.126503
10	.907813	.895985	1.040023	.906833	1.128874
15	.844127	.837347	.959679	.845318	1.041714
20	.639623	.645250	.713727	.649339	.774508
25	.357180	.364551	.393748	.365758	.427214
30	.138467	.139036	.154759	.139237	.167905
35	.035510	.034293	.041002	.034313	.044479
40	.005478	.005087	.006528	.005087	.007080
45	.000270	.000230	.000344	.000230	.000373

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC
REPRODUCTIVE VALUES: V(x)
THE CLOSED MODEL

1940

Spatial: Native Independent
$\{v(0)\}' = [1 \quad 1.357758]$, $r = -.004145$

Age	North	Southwest
	Region of Residence	
0	.993510	1.358686
5	.974510	1.349313
10	.949154	1.325917
15	.870775	1.204002
20	.662817	.885469
25	.371158	.484845
30	.140645	.189560
35	.034531	.050022
40	.005098	.007933
45	.000230	.000418

Spatial: Native Dependent
$\{v(0)\}' = [1 \quad 1.526788]$, $r = -.004041$

Age	North	Southwest
	Region of Residence	
0	.992217	1.529131
5	.972265	1.519619
10	.947950	1.492588
15	.870280	1.354970
20	.662309	.996609
25	.370732	.545844
30	.140479	.213405
35	.034501	.056294
40	.005096	.008923
45	.000230	.000469

1960

Spatial: Native Independent
$\{v(0)\}' = [1 \quad 1.099056]$, $r = -.004263$

Age	North	Southwest
	Region of Residence	
0	.993241	1.101747
5	.973548	1.096592
10	.948092	1.076618
15	.868121	.977401
20	.659840	.719116
25	.370092	.393304
30	.140467	.153603
35	.034517	.040508
40	.005098	.006422
45	.000230	.000338

Spatial: Native Dependent
$\{v(0)\}' = [1 \quad 1.172291]$, $r = -.004153$

Age	North	Southwest
	Region of Residence	
0	.990765	1.178622
5	.969016	1.175883
10	.944861	1.152153
15	.865083	1.045093
20	.657715	.768467
25	.369096	.420321
30	.140189	.164110
35	.034468	.043256
40	.005095	.006853
45	.000230	.000360

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC REPRODUCTIVE VALUES: V(x)
THE OPEN MODEL

1940

Spatial: Native Independent
$\{v(0)\}' = [1 \quad 1.375062], r=0$

Age	North	Southwest
	Region of Residence	
0	.897515	1.250800
5	.900046	1.267382
10	.895912	1.270744
15	.837851	1.173943
20	.646487	.874112
25	.365089	.482780
30	.139118	.189915
35	.034299	.050339
40	.005087	.008018
45	.000230	.000423

Spatial: Native Dependent
$\{v(0)\}' = [1 \quad 1.551827], r=0$

Age	North	Southwest
	Region of Residence	
0	.899246	1.415434
5	.900278	1.434745
10	.896285	1.437508
15	.838186	1.327304
20	.646350	.988141
25	.364845	.545697
30	.139022	.214599
35	.034283	.056857
40	.005086	.009051
45	.000230	.000477

1960

Spatial: Native Independent
$\{v(0)\}' = [1 \quad 1.103402], r=0$

Age	North	Southwest
	Region of Residence	
0	.889661	1.006612
5	.894414	1.020651
10	.892014	1.022110
15	.834384	.943539
20	.643384	.702814
25	.363788	.388081
30	.138846	.152553
35	.034269	.040413
40	.005086	.006434
45	.000230	.000339

Spatial: Native Dependent
$\{v(0)\}' = [1 \quad 1.179572], r=0$

Age	North	Southwest
	Region of Residence	
0	.891497	1.079335
5	.893575	1.097291
10	.890749	1.097292
15	.832018	1.012461
20	.641216	.753744
25	.363020	.415809
30	.138694	.163319
35	.034243	.043241
40	.005084	.006881
45	.000230	.000363

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC
REPRODUCTIVE VALUES: V(x)
United States, 1980

. THE CLOSED MODEL

	Nonspatial v(0) = 1, r = -.004366	Spatial: Native Independent {v(0)}' = [1 1.114767], r = -.003955		Spatial: Native Dependent {v(0)}' = [1 1.206076], r = -.003734	
	Region of Residence	Region of Residence		Region of Residence	
Age	USA	North	Southwest	North	Southwest
0	.997207	.993666	1.115866	.991360	1.209208
5	.984503	.974287	1.109284	.970008	1.203963
10	.963325	.948312	1.090949	.945444	1.183206
15	.878980	.868504	.990844	.866585	1.073770
20	.657005	.660341	.728440	.658928	.788809
25	.363524	.370435	.398433	.369416	.431522
30	.140038	.140600	.155658	.140194	.168604
35	.035748	.034534	.041070	.034459	.044467
40	.005490	.005099	.006513	.005094	.007048
45	.000271	.000230	.000343	.000230	.000371

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC
REPRODUCTIVE VALUES: V(x)
United States, 1980

2. THE OPEN MODEL

Age	Nonspatial $v(0)=1,\ r=0$ Region of Residence USA	Spatial: Native Independent $\{v(0)\}'=[1 \quad 1.119484],\ r=0$ Region of Residence North	 Southwest	Spatial: Native Dependent $\{v(0)\}'=[1 \quad 1.213848],\ r=0$ Region of Residence North	 Southwest
0	.899676	.895842	1.025555	.911484	1.111341
5	.907816	.899543	1.038182	.912565	1.126503
10	.907813	.895985	1.040023	.906833	1.128874
15	.844127	.837347	.959679	.845318	1.041714
20	.639623	.645250	.713727	.649339	.774508
25	.357180	.364551	.393748	.365758	.427214
30	.138467	.139036	.154759	.139237	.167905
35	.035510	.034293	.041002	.034313	.044479
40	.005478	.005087	.006528	.005087	.007080
45	.000270	.000230	.000344	.000230	.000373

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC REPRODUCTIVE VALUES: V(x)
THE CLOSED MODEL

1940

	Spatial: Native Independent $\{v(0)\}'=[1 \quad 1.357758]$, $r=-.004145$		Spatial: Native Dependent $\{v(0)\}'=[1 \quad 1.526788]$, $r=-.004041$	
	Region of Residence		Region of Residence	
Age	North	Southwest	North	Southwest
0	.993510	1.358686	.992217	1.529131
5	.974510	1.349313	.972265	1.519619
10	.949154	1.325917	.947950	1.492588
15	.870775	1.204002	.870280	1.354970
20	.662817	.885469	.662309	.996609
25	.371158	.484845	.370732	.545844
30	.140645	.189560	.140479	.213405
35	.034531	.050022	.034501	.056294
40	.005098	.007933	.005096	.008923
45	.000230	.000418	.000230	.000469

1960

	Spatial: Native Independent $\{v(0)\}'=[1 \quad 1.1099056$, $r=-.004263$		Spatial: Native Dependent $\{v(0)\}'=[1 \quad 1.172291]$, $r=-.004153$	
	Region of Residence		Region of Residence	
Age	North	Southwest	North	Southwest
0	.993241	1.101747	.990765	1.178622
5	.973548	1.096592	.969016	1.175883
10	.948092	1.076618	.944861	1.152153
15	.868121	.977401	.865083	1.045093
20	.659840	.719116	.657715	.768467
25	.370092	.393304	.369096	.420321
30	.140467	.153603	.140189	.164110
35	.034517	.040508	.034468	.043256
40	.005098	.006422	.005095	.006853
45	.000230	.000338	.000230	.000360

NUMERICAL ILLUSTRATIONS OF AGE-GROUP-SPECIFIC REPRODUCTIVE VALUES: V(x)
THE OPEN MODEL

1940

	Spatial: Native Independent $\{v(0)\}'=[1 \quad 1.375062]$, $r=0$		Spatial: Native Dependent $\{v(0)\}'=[1 \quad 1.551827]$, $r=0$	
	Region of Residence		Region of Residence	
Age	North	Southwest	North	Southwest
0	.897515	1.250800	.899246	1.415434
5	.900046	1.267382	.900278	1.434745
10	.895912	1.270744	.896285	1.437508
15	.837851	1.173943	.838186	1.327304
20	.646487	.874112	.646350	.988141
25	.365089	.482780	.364845	.545697
30	.139118	.189915	.139022	.214599
35	.034299	.050339	.034283	.056857
40	.050870	.008018	.005086	.009051
45	.000230	.000423	.000230	.000477

1960

	Spatial: Native Independent $\{v(0)\}'=[1 \quad 1.103402]$, $r=0$		Spatial: Native Dependent $\{v(0)\}'=[1 \quad 1.179572]$, $r=0$	
	Region of Residence		Region of Residence	
Age	North	Southwest	North	Southwest
0	.889661	1.006612	.891497	1.079335
5	.894414	1.020651	.893575	1.097291
10	.892014	1.022110	.890749	1.097292
15	.834384	.943539	.832018	1.012461
20	.643384	.702814	.641216	.753744
25	.363788	.388081	.363020	.415809
30	.138846	.152553	.138694	.163319
35	.034269	.040413	.034243	.043241
40	.005086	.006434	.005084	.006881
45	.000230	.000339	.000230	.000363