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SHORT-RUN TRADE-OFFS BETWEEN OUTPUT AND THE RATE OF INFLATION
IS THERE A PHILLIPS' CURVE IN COLOMBIA?

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SHORT-RUN TRADE-OFFS BETWEEN OUTPUT AND THE RATE OF INFLATION
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An important issue for monetary policy concerns the relation between the monetary growth rate (and rates of price change) and the level of output. According to the well-known Phillips' Curve relation, there is, at least in the short-run, a trade-off which allows higher levels of real output to be "purchased" by accepting higher rates of monetary growth and inflation. Hence, if this trade-off exists, a program to reduce the rate of inflation must tolerate at least a temporary shortfall of output. The purpose of this paper is to quantify this sort of Phillips' Curve relation for the case of Colombia.

The first part of the paper begins with a sketch of some theoretical aspects of relationships which embody an inflation/output trade-off. A general Phillips' Curve relation is then put into estimable form by indicating the forms of some expectation/adjustment mechanisms, and by specifying in detail the appropriate explanatory variables. The second part of the paper is an empirical application of the model which is developed in part 1. Estimates are first presented for the post-World War II experience of the United States. These results indicate a significant short-run trade-off between higher rates of monetary expansion and higher levels of output, though a long-run trade-off appears to be absent. Estimates are then presented for Colombia for the period from 1951 to 1971.

1/ This paper represents part of an investigation undertaken for the Departamento Nacional de Planeación de Colombia. The U.S. equations mentioned in the study are reestimated and developed in much greater detail in Robert Barro "Unanticipated Money Growth and Unemployment in the United States", unpublished.

/These estimates
These estimates indicate an absence of a Phillips' Curve trade-off for Colombia both in the short and the long-run. This surprising result is discussed at the end of the paper. Two possible explanations for the finding are considered: first, the higher variability of the monetary growth in Colombia than in the United States would imply a smaller Phillips' Curve trade-off in the case of Colombia; and second, it is possible that the present model underestimates the extent of the trade-off in Colombia because of the omission of some important explanatory variables. This second possibility constitutes a useful topic for future research.

I. PHILLIPS' CURVE RELATIONSHIPS

The Phillips' Curve describes a relation between the rate of change of prices—or the rate of change of money—and some measure of output or employment (see Fisher, 19; Phillips, 195, and Lipsey, 195, for the original discussions). More recently (e.g. in Friedman, 1968; Phelps, 1967; and Lucas, 1973), output or employment, relative to long-term or normal values, are related to rates of change of prices or money relative to anticipated rates of change of these variables. In this later version of the Phillips' Curve, there is a trade-off between higher output and higher rates of inflation only in the short-run where a change in the inflation rate is unanticipated. Formally, one can write this type of relation as

\[ y = y_n + f(x - x^e), \]

where \( y \) is output, \( y_n \) is normal (or trend) output, \( x \) is the rate of change of either prices or money, and \( x^e \) is the anticipated value of \( x \). The function \( f \) is such that an increase in \( x \) relative to \( x^e \) implies an increase in \( y \) relative to \( y_n \). As long as the average values over time of \( x \) and \( x^e \) are equal, the average value of \( y \) will equal the average value of \( y_n \). Accordingly, in this long-run sense, the amount of inflation—that is, the average value of \( x \)—has no
impact on the average value of output. In the long-run, the average value of y is determined by the average value of normal output, $y_n$. However, in the short-run, if $x$ rises above $x^e$, then it is possible for $y$ to rise above $y_n$, and vice versa for values of $x$ below $x^e$.

A theoretical rationalization for the general relation described in equation (1) has recently been provided by Lucas (1973, op. cit.). Basically, Lucas views output in any sector as responding to price changes in that sector only when these changes are perceived to be relative to those in other sectors. Hence, an increase in aggregate demand—produced, for example, by an increase in the stock of money—will raise output as well as prices only if each sector (incorrectly) perceives its own price change to be, at least in part, a change in relative prices. The larger the fraction of any observed price change which is perceived to be absolute (that is, aggregate), rather than relative, the smaller will be the response of output to a given change in aggregate demand, and the smaller will be the slope of the Phillips' Curve, in the sense of the magnitude of the response of $f$ to $(x - x^e)$ in equation (1). It can also be shown that the more variable (or, rather, unpredictable) the overall rate of inflation in a country, the larger the fraction of any observed price change that will be perceived as absolute rather than relative, and, hence, the smaller the slope of $f$.

In order to proceed empirically, equation (1) must be extended in several respects. First, the determinants of $y$ must be specified; second, the measure of $x$ and the determinants of $x^e$—related perhaps to the historical time path of $x$—must be specified, and, finally, some additional lagged adjustment of $y$ to the right side of equation (1) may be appropriate.

For the United States, a satisfactory empirical counterpart of the general form of equation (1) has been attained with the following detailed specifications:

/First, normal
First, normal output is related to population—intended to be a trend element which captures the effects of long-term real growth—and the real amount of government spending, which determines the fraction of output which is available for private uses and which may also affect the overall productive capacity of the economy. In the typical case the effect of government spending on \( y_n \) would be positive. The specific form which has been used is log-linear:

\[
\log (y_n)_t = a_1 + a_2 \log (POP)_t + a_3 \log (GOV)_t + u_{1t}, \tag{2}
\]

where \( t \) is a time subscript, \( POP \) refers to population, \( GOV \) to real government spending (on goods and services, but excluding transfer payments), and \( u_1 \) is a stochastic term with the usual properties.

Second, the variable \( x \) has been measured as the annual rate of change of the money stock (currency plus demand deposits). This variable will be referred to subsequently as \( DM_t \). The use of the rate of change of money, rather than of prices, is desirable on two counts. First, measured prices—though not necessarily true prices—are often influenced by various types of price controls, while money measures are not. Second, prices must be endogenous to a fully-specified model, whereas the money supply may be, at least to a considerable extent, exogenous. Hence, the use of money rather than prices may avoid some serious statistical problems of identification.

Third, the anticipated rate of change of money, \( DM_t^e \), has been related to several lagged values of \( DM \), and to a measure of the national government budget relative to gross national product. The inclusion of the latter measure derives from the idea that increased monetary growth is a method of financing government spending, and the incentive for this finance increases the higher the relative size of the government budget. With respect to lagged values of \( DM \), it turned out empirically for annual post-World War II experience in the United States

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1/ The positive effect of government spending is on measured real gross national product, which includes the government spending itself valued at cost. There is no implication here that government spending is socially productive at the margin.
that only a single lag, $DM_{t-1}$, had significant predictive value with respect to $DM_t$. However, two lagged variables, $DM_{t-1}$ and $DM_{t-2}$, have been retained in the regressions which are reported in this paper. In particular, it is supposed that $DM_t$ is generated by a function of the form,

$$DM_t = b_1 + b_2 DM_{t-1} + b_3 DM_{t-2} + b_4 \text{log}(FED)_t + u_2 t,$$

(3)

where $FED_t$ is the level of the national government budget relative to (the previous year's) gross national product, and $u_2$ is another disturbance term. The anticipated value, $DM_t^e$, is interpreted as a prediction of $DM_t$ based on information available at date $t-1$. A time series for $DM_t^e$ is constructed using the following procedure: First, estimates of the b-coefficient in equation (3) are obtained by standard regression techniques with $DM_t$ as the dependent variable. Second, the estimated values of $DM_t$—based on these estimated b-coefficients—are used to provide a time series of estimated values of $DM_t^e$. It is, of course, likely that additional pieces of information are available to predict the value of $DM_t$. An obvious candidate would be the exchange rate, which may be perceived rapidly. However, the present analysis does not include this variable—see the discussion at the end of the paper.

Fourth, the possibility of lagged adjustment of $y_t$ to the right side of equation (1) (which can be rationalized by the existence of adjustment costs for changing output) is introduced by using the specific log-linear form,

$$\log (y)_t = c_1 + c_2 \log (y_n)_t + c_3 (DM_t - DM_t^e) + c_4 \log (y)_{t-1} + u_3 t,$$

(4)

where $(DM_t - DM_t^e)$ now substitutes for $x - x^e$ in equation (1), the $y_{t-1}$ term accounts for a lagged response of $y_t$ to the right side of equation (1), and $u_3$ is another disturbance term. The lag coefficient, $c_4$, must be in the interval between zero and one, where a smaller value of $c_4$ in this interval signifies a more rapid adjustment of $/y_t$ to
\( y_t \) to the right side of equation (1). If adjustment were complete during one year, then the coefficient on \( y_{t-1} \) in equation (4) would be zero. One possible difficulty with the form of equation (4) is that it prescribes a constant and symmetric effect of \( (DM_t - DM^e_t) \) on \( \log (y)_t \). It may be preferable to have this effect depend on the gap between \( y_t \) and \( y_{nt} \)—which would determine the extent of current "excess capacity". However, this possibility has not yet been explored.

Finally, substitution for \( y_{nt} \) from equation (2) into equation (4) yields the form which is used for estimation purposes,

\[
\log(y)_t = \alpha_1 + \alpha_2 \log(POP)_t + \alpha_3 \log(GOV)_t + \alpha_4 (DM_t - DM^e_t) + \alpha_5 \log(y)_{t-1} + u_t, \tag{5}
\]

where \( DM^e_t \) is determined as the estimated value from equation (3), the \( \alpha \)-coefficients can be derived from the a- and c-coefficients of equations (2) and (4), and \( u_t \) is a disturbance term which involves the disturbance terms from equations (2), (3) and (4). 

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Since \( DM^e_t \) is not observed directly but is, instead, obtained as an estimated value from equation (3), the \( (DM_t - DM^e_t) \) variable is effectively observed with a random error. This situation does produce some bias in the estimates of the \( \alpha \)-coefficients in equation (5). The bias will be less serious the smaller the standard error of the parameter combination, \( \hat{\beta}_1 + \hat{\beta}_2 DM_{t-1} + \hat{\beta}_3 DM_{t-2} + \hat{\beta}_4 \log(PED)_t \), where a carat over a coefficient indicates an estimated value. An alternative, unbiased procedure could be designed by first substituting for \( DM^e_t \) from equation (3) into equation (5). Unbiased estimates of all the coefficients could then be obtained (abstracting from the problem of the inclusion of a lagged dependent variable) by the application of standard regression methods to the single equation which includes \( DM_{t-1}, DM_{t-2}, \) and \( \log (PED)_t \) along with the other variables (other than \( DM^e_t \)) which appears in equation (5). However, because this procedure seems to involve greater multicolinearity problems and because it seems to be much less robust to mis-specifications of the forms of equations (3) and (5), I have actually used the two-stage estimation procedure which involves separate estimation of equation (5).
II. EMPIRICAL RESULTS

A. United States

For comparison with the Colombian results, I will first present estimates of equations (3) and (5) for the United States during the post-World War II period. Using annual data from 1947 to 1972, the estimated equations, with standard errors of the coefficients in parentheses, are

$$DH_t = 0.15 + 0.38 DH_{t-1} + 0.16 DH_{t-2} + 0.079 \log (FED)_t \tag{6}$$

$$(.05) (.22) (.16) (.030)$$

$$R^2 = 0.52, S.E.E. = .016,$$

$$\log (y)_t = 2.08 + 0.93 \log (POP)_t + 0.130 \log (G6V)_t + 0.81 (DH_t - DH^c_t) \tag{7}$$

$$(.92) (.37) (.045) (.32) (.7)$$

$$+ 0.55 \log (y)_{t-1}$$

$$(.15)$$

$$R^2 = 0.995, S.E.E. = .022.$$ 

Equation (6), which is an estimated form of equation (3), indicates positive serial correlation of DH, although only the first lag coefficient ($DH_{t-1}$) is statistically significant. The size of the federal budget relative to the gross national product also has a significantly positive effect on DH. The standard-error-of-estimate (S.E.E.) for this equation indicates an error of about 1.6 per cent per year in the monetary growth rate.

In equation (7), which is an estimated form of equation (5), the two trend variables, population and government purchases, have significantly positive effects. The lag coefficient is also positive and significant. The estimated coefficient on $\log (y)_{t-1}$ of 0.55 indicates that $\log (y)_t$ adjusts to the current year's influences with a coefficient of 0.45. The S.E.E. indicates an equation error of about 2.2 per cent of output.

Most importantly,
Most importantly, equation (7) indicates a significantly positive effect of the monetary growth rate \( (\text{DM}_t - \text{DM}_t^e) \), on real output. The estimated coefficient of 0.81 implies that, with \( \text{DM}_t^e \) held fixed, an increase by 1 per cent per year in \( \text{DM}_t \) would raise \( y_t \) by about 0.81 per cent of output—that is, by about 10 billion dollars per year in the United States in 1974. The increase in output produced by the increase in the rate of monetary expansion (which would also be accompanied by an increase in the rate of price change) expresses the familiar Phillips' Curve trade-off. However, the trade-off is only temporary in this model since, eventually, an increase in DM would induce an equivalent increase in \( \text{DM}_t^e \) which would then eliminate the expansionary effect of \( \text{DM} \) on \( y \). Hence, in the long-run, a higher value of \( \text{DM} \) leads to more inflation, but to no increase in output.

It is also of interest to consider a regression equation for the United States which includes the absolute monetary growth rate, \( \text{DM}_t \), as well as the difference from the anticipated rate \( (\text{DM}_t - \text{DM}_t^e) \). This estimated equation is

\[
\log (y)_t = 2.63 + 1.09 \log \text{(POP)}_t + 0.747 \log \text{(GOV)}_t + 1.41 (\text{DM}_t - \text{DM}_t^e) \\
\text{(95)} \quad \text{(37)} \quad \text{(044)} \quad \text{(48)} \\
- 0.72 \text{(DM)}_t + 0.51 \log(y)_{t-1} \\
\text{(44)} \quad \text{(16)}
\]

\[ R^2 = 0.993, \text{S.E.E.} = .021. \]

The \( \text{DM}_t \) variable has the "wrong" sign—that is, a negative effect—in this equation and the estimated coefficient is at the margin of being significantly different from zero. In any case the results from equation (8) support the theoretical hypothesis that it is only \( \text{DM}_t \) relative to \( \text{DM}_t^e \), and not the absolute size of \( \text{DM}_t \), that has a positive influence on real output. Hence, there is empirical support from the post-World War II United States experience for a short-run Phillips' Curve trade-off, but no evidence for a long-run trade-off.

/B. Colombia
D. Colombia

Empirical results for Colombia on annual data from 1951 to 1971 have been obtained in the same forms as equations (6)-(8). The results are

\[ DH_t = 0.29 - 0.45\, DH_{t-1} - 0.24\, DM_{t-2} + 0.016\, \log(FED)_t \]
\[ R^2 = 0.20, \, S.E.E. = .046, \]

\[ \log(y)_t = 0.11 + 0.49\, \log(POP)_t + 0.118\, \log(GOV)_t + 0.10\, (DH_t - DH^e_t) \]
\[ R^2 = 0.991, \, S.E.E. = .032, \]

\[ \log(y)_t = 0.09 + 0.49\, \log(POP)_t + 0.177\, \log(GOV)_t + 0.03\, (DH_t - DH^e_t) + 0.07\, (DH)_t + 0.57\, \log(y)_{t-1} \]
\[ R^2 = 0.991, \, S.E.E. = .033. \]

The results for the DM equation, as shown in equation (9), are very different from those for the United States, which are shown in equation (6). The Colombian results show a pattern of significant negative serial correlation for DM, particularly on the first lag term, \( DM_{t-1} \); while the United States results exhibited a positive lag relation. Further, the federal budget variable does not have a significant effect on DM in the Colombian case. The standard-error-of-estimate indicates an equation error of about 4.6 per cent per year in the monetary growth rate for Colombia—roughly three times the error for the United States.
The Colombian output regression in equation (10) shows significant positive effects on $y$ for the two trend variables, although the population coefficient is about half the size of that for the United States (indicating a slower trend rate of growth of output over the sample period for Colombia). However, the estimated coefficient on government spending for Colombia is very close to that for the United States. The lagged output coefficient for Colombia is significantly positive and is also very close to the coefficient which was estimated for the United States. The standard-error-of-estimate for Colombia is about 3.2 per cent of output, which is about 50 per cent higher than the corresponding error for the United States.

The most important difference between the United States and the Colombian results concerns the coefficient of the monetary growth rate variable. In equation (10) the estimated coefficient of $(\Delta M_t - \Delta M^e_t)$ is only 0.10 and is insignificantly different from zero. The tentative conclusion from the estimated output equation is that the Phillips' Curve--relating output to the monetary growth rate--in Colombia has a negligible slope. That is, there is no apparent trade-off in Colombia—even in the short run—between higher rates of monetary expansion (and inflation) and higher real output. If this tentative conclusion is correct, the implication for monetary policy is that it should be concerned solely with obtaining a desired rate of inflation, and not at all with short-run losses to output associated with stabilizing the rate of inflation. The evidence for Colombia at this point is that stabilizing the rate of inflation would not lead to any short-run losses of output.

The results in equation (11), which also include the absolute monetary growth rate, $\Delta M_t$, do not alter the above conclusion. In equation (11) the coefficients of $\Delta M_t$ and of $(\Delta M_t - \Delta M^e_t)$ are both insignificantly different from zero, and the sum of the two coefficients is 0.10, the same value as the coefficient of $(\Delta M_t - \Delta M^e_t)$ in equation (10).
The result that there is no significant Phillips' Curve trade-off in Colombia is surprising, and this conclusion cannot be regarded as definitive on the basis of the tentative results which have been obtained thus far. It may be useful to mention here some possible explanations for the result, and also to outline some future research which may either confirm or alter the result.

First, because the monetary growth rate is much less predictable (from the standpoint of a year earlier) in Colombia ($S.E.E. = 0.46$ in equation (9)) than for the United States ($S.E.E. = 0.16$ in equation (6)), the Lucas-type Phillips' Curve theory which was outlined in part I of this paper would predict a smaller Phillips' Curve coefficient for Colombia than for the United States. Essentially, since the unpredictability in the overall monetary growth rate is larger for Colombia than for the United States, one would be more inclined to attribute an observed price fluctuation in Colombia to aggregate, rather than relative, forces. Hence, the response of output to a given shift in aggregate demand would also be smaller in the Colombian case. However, while this line of argument would account for a smaller coefficient on $(D_M_t - D_{M_t}^e)$ in the Colombian case, it does not explain the insignificant coefficient which was obtained in equation (10).

A second possible source of explanation involves systematic effects on $D_M_t$ and on $y_t$ that have been omitted from the estimating equations. If the omitted variables from these equations are correlated in a certain fashion, then the estimated coefficient of $(D_M_t - D_{M_t}^e)$ could be biased in a manner which would account for the present results. For example, suppose that the yields of agricultural products has a positive effect on $y_t$ in equation (10) and a negative effect on $D_M_t$ in equation (9). The latter effect could result if the central bank reacts to a shortfall in agricultural production by increasing its loans to this sector—and, hence, by increasing the monetary base $^1$. These relationships would create a negative correlation.

$^1$ However, reduced agricultural exports would have an opposite effect on the monetary base in this circumstance, so the net effect on $D_M$ would be ambiguous.
between $\Delta h_t$ and $y_t$ which would produce a downward bias in the estimated coefficient of $(\Delta h_t - \Delta h_t^e)$ in equation (10). Analogous types of effects might arise through the influences of export and import prices, devaluations, etc. In particular, if a change in the exchange rate is perceived rapidly as implying a movement of the overall price index, then the equation which is used for relating output to the rate of monetary expansion, equation (9) for Colombia, could be directly affected. If these types of effects are important for Colombia, the correct procedure would be to introduce the appropriate additional explanatory variables into equations (9) and (10). Conceivably, the estimated coefficient of $(\Delta h_t - \Delta h_t^e)$ would then increase. However, this change in the coefficient is only a conjecture at this point, and the resolution of this conjecture must await future research on the Colombian case.