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GRADUALISM OR SHOCK TREATMENT

Considerations on the Costs of Stabilization

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N O T E

Mathematical Appendix on Stability Conditions of Equilibrium and Note on Determination of the Demand for Credit by a Competitive Firm, are available on request from the authors.

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2010
The following table shows the number of people who have been
in the United States for a period of 10 years or more, by
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GRADUALISM OR SHOCK TREATMENT

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I. INTRODUCTION

One major conclusion arising from the experiences of many countries which in recent years have adopted gradualist and shock treatment strategies is: gradualism is an efficient way to achieve stabilization with smaller costs in terms of reduction in real output.

Until now, most efforts at analyzing stabilization strategies have concentrated upon an explanation of the rate of inflation ^{1/}. Very little has been written in regard to the impact of such policies upon the real side of the economy, that is, upon the rate of growth of output.

This paper attempts to evaluate the effects of monetary and credit policies upon alternative stabilization programs while considering both the rates of inflation and of output growth as endogenous variables in the model. An attempt will be made to show that the rate of monetary expansion is a decisive variable in explaining the long-run behavior of the inflation rate, but that it is relatively

^{★/} The authors are grateful to professor Larry Sjaastad for suggesting the subject matter and to Professors Arnold Harberger and Adolfo C. Diz for valuable comments. They also thank the seminar participants at IPE and at the Department of Economics of the FEA-USP for criticisms of an earlier draft.

^{1/} See, among others, the analysis of Delfim Neto, et. al. (1965), Simonsen (1970), and Pastore (1973) for the case of Brazil; Harberger (1966) and Deaver (1970) for Chile; Diz (1970) for Argentina; and, more recently, Vogel (1973) for several Latin American Countries.

/less efficient

less efficient as an instrument for bringing about short-run changes in the rate. A further object of the study is to show that fluctuations in the rate of change of credit to the private sector substantially affect aggregate supply in the short-run, although their long-run effects on supply may be insignificant.

We will attempt to show as well that expectations of inflation play an important role in the dynamic behavior of the rates of change of prices and that if it were possible to modify exogenously this variable, stabilization could be achieved with smaller social costs.

Finally, the results of the paper indicate that countries that set economic growth as their highest priority objective can be drawn into generating an unstable inflation together with an accompanying inflationary bias. This might explain the post-Keynesian experience of many countries with growing rates of inflation.

The bias would appear because the government would stimulate the economy in order to increase the level of activity. Starting from an equilibrium position the government can succeed in increasing output, but only temporarily, and as the adjustment process gets completed more and more of the increase in demand will be in the form of prices and less and less in the form of output. At this point the government becomes worried about inflation and starts a stabilization policy. The problem this time is that this policy would again affect output first, in this case it will start reducing output, and only later in the adjustment process is that the rate of inflation would diminish. Therefore, stabilization policies are, by the nature of their dynamic process, unpopular. The result is that after obtaining a partial reduction in the rate of inflation or no reduction at all, the government will shift interest from inflation to unemployment and increase aggregate demand again. The U.S. for example has lived this game in the post-war period a few times already.

For reasons of simplicity, the entire analysis will be developed assuming a closed economy, thus leaving aside important information regarding the effects of foreign trade on stabilization strategy.

/Also, we

Also, we will concentrate only upon an explanation of the short-run behavior of the rates of inflation and of output growth, without considering the effects that recessions inherent in certain stabilization mechanisms can have on the long-run rate of output growth. Finally, we will ignore the distorting effects of a continuous inflation (which forces the economy to operate inside its production possibilities frontier) on the level of real output.

II. AGGREGATE DEMAND

The aggregate demand model contains 5 equations. The first is the long-run demand for money and is given:

$$(2.1) \quad \mu^d = \alpha \pi^e + \beta z$$

where μ^d is the logarithm of the desired real stock of money, π^e is the expected rate of inflation, and z is the logarithm of real output. The second equation is that of the adjustment of the desired stock and is given:

$$(2.2) \quad d\mu = a(\mu^d - \mu) + b(\hat{M} - \hat{M}^d) + d\mu^d$$

where d indicates a derivative with respect to time (i.e., $d = d/dt$) and where a circumflex accent indicates that we are taking the percentage rate of change of the given variable. M is the actual nominal stock of money and, M^d is the desired nominal stock. This equation indicates that the actual change in the real stock of money (the left-hand side of the equation) is equal to the desired change (the right-hand side), and that this desired change is the sum of three effects ^{1/}. The first term on the right-hand side is the "stock disequilibrium effect". It is assumed that whenever the desired real stock is greater than the actual stock, individuals will attempt to alter the actual stock in a proportion given by 100a per cent of the

^{1/} See Almonacid (1973), pp. 46-48, and Pastore (1973), pp. 7-8.

total that they desire to accumulate. The economic justification for this component can be found in the fact that there exist two types of costs involved in the decision to accumulate real money balances.

The first is the cost of being out of stock equilibrium (that is, the cost of holding an actual real stock that is less than or greater than the desired stock) and is measured by the loss of returns due to the fact that individuals are not maintaining the optimum real quantity of money. We assume that the larger the difference between the actual and desired stocks (i.e., the greater the stock disequilibrium), the greater, relatively, will this cost be. And the greater this cost, the greater the benefit of a rapid adjustment in the actual stock to make it converge toward the equilibrium stock.

The second cost is that which individuals have to absorb in order to adjust their stocks (i.e., the sacrifice in terms of consumption and the accumulation of other assets). We assume that this cost grows with the velocity with which the real stock of money is adjusted. Clearly, this cost will encourage individuals to reduce the speed of adjustment. The existence of these two costs operating in opposite directions generates an optimum adjustment path that would, by hypothesis be captured by the first term on the right-hand side of (2.2) ^{1/}.

The second term on the right-hand side measures the "shock absorber" effect. If individuals wish to increase their nominal stock at a rate \hat{M}^d , but monetary authorities create money at a rate \hat{M} greater than \hat{M}^d , for example, individuals will keep initially a proportion of that excess in their real stock. This behavior derives from the fact that money is above all a transactions instrument that can be exchanged for any other asset or good; thus, until a decision is made

^{1/} Eisner and Strotz (1963) showed that if the cost of being out of equilibrium and the cost of adjustment were quadratic functions, the coefficient a would be simply the ratio between the marginal cost of being out of equilibrium and the marginal cost of adjustment. The larger the latter cost in relation to the first, the smaller the coefficient a , and the longer the period necessary for adjustment.

regarding the allocation of these additional resources, it is better to maintain them in the form of money. This component of the adjustment of the real stock of money is short-run by its very nature and, therefore, the longer the period considered, the less importance it will have. So when the period of analysis is short enough, we expect that b will be fairly close to 1. In the long-run b ought to be zero. And for intermediate periods, the values of b should vary between 0 and 1.

The last term on the right-hand side represents the growth effect. If μ and μ^d were initially equal and grew at the same rate, we would have $\mu = \mu^d$ continually. If \hat{M}^d and \hat{M} were also equal, we would have $\hat{M}^d = \hat{M}$ continually. In such a case, individuals would accumulate money according to the rate of change in the desired stock, that is, $d\mu^d = d\mu$.

The third equation is simply the definition of the rate of inflation, given by the difference between the rate of monetary expansion and the rate of change in the actual real stock of money:

$$(2.3) \quad \pi = \hat{M} - d\mu$$

The last two aggregate demand equations describe hypotheses about the formation of expectations concerning the rate of inflation and the price level. For the expected rate of inflation we use the model of adaptive expectations proposed by Cagan (1956):

$$(2.4) \quad d\pi^e = c_1 (\pi - \pi^e)$$

/The expected

The expected price level is provided by the model proposed by Almonacid in 1971 ^{1/}:

$$(2.5) \quad dp^e = c_2 (p - p^e) + \pi^e$$

where p is the logarithm of the price level and p^e is the logarithm of the expected level of prices ^{2/}. This model of aggregate demand is similar to one used previously in the analysis of inflation in

- 1/ Equation (2.4) implies that the expected rate of inflation π^e is a moving average with geometrically decreasing weights applied to past rates of inflation. Equation (2.5) establishes a distinction between the expected rate of inflation π^e and the rate of change of the expected price level. The properties of this model are discussed in greater detail in Almonacid (1971). However, we want to emphasize that the distinction between π^e and \hat{p}^e is fundamental for the internal consistency of the model, because in full equilibrium we must have not only the equality $\pi = \pi^e$, but $p = p^e$ as well. If p^e were defined as:

$$p^e(t, \tau) = p_t + \int_t^\tau \pi^e(t, v) dv$$

where t is the moment at which expectations are formed and τ is the moment when the given value of the variable is expected to occur. Given that:

$$p(\tau) = p_t + \int_t^\tau \pi(v) dv$$

in order for p^e to converge to p , it would be necessary that:

$$\lim_{\tau \rightarrow \infty} \int_t^\tau [\pi(v) - \pi^e(t, v)] dv$$

converge to zero, which places an unnecessary restriction on the expectations function of the rate of inflation.

- 2/ The definition of the expected price level (P^e such that $\ln P^e = p^e$), as well as that of the expected rate of inflation, π^e , involves the concept of an expectations horizon defined as the period of time for which expectations are formulated (the relevant value of τ in footnote 1 above). Even if the horizon were nonexistent, we could keep the equations (2.4) and (2.5) by defining π^e and P^e , respectively, as the perceived rate of inflation and the perceived price level. This implies that the holders of the real stock of money do not know exactly either the price level or the (cont.)

other countries and possesses some interesting properties ^{1/}. In order to discuss them, we begin by defining the demand for money in nominal terms which is given by (2.1)'

$$(2.1)' \quad \ln M^d = -\alpha \pi^e + \beta z + p^e$$

From (2.1) and (2.1)', it follows that:

$$d \mu^d = -\alpha d \pi^e + \beta dz$$

$$\hat{M}^d = -\alpha d \pi^e + \beta z + dp^e$$

and using the previous relations we arrive at the reduced form:

$$(2.6) \quad \pi = (1 - b) \hat{M} + b \pi^e + \alpha (1 - b) d \pi^e + a (\mu - \mu^d) - (1 - b) \beta dz + bc_2 (p - p^e)$$

^{1/} (Cont.) rate at which it changes, but that they do have perceptions about this level and this rate. Obviously, the model could be generalized by supposing that expectations (for a given horizon) are formulated by using the perceived rates and levels. This would imply the use of a model with more complex equations than (2.4) and (2.5). Empirically, however, the distinction between perceptions and expectations is impossible to make. This point also is treated in greater detail in Almonacid (1971), chapter 1.

^{1/} See in this regard Pastore (1973a). The basic difference is that in the earlier version of the model, Pastore focussed upon the behavior of π while holding z constant and, thus, he did not include the third term (the growth effect) in the adjustment equation (2.2), nor did he concern himself with price level expectations.

A similar reduced form could be derived from a "structural model" different from the one set out in this paper, such as that elaborated by Sjaastad (1974) and by Friedman (1970) ^{1/}.

Assuming that the money market is in stock equilibrium ($m = m^d$ where m and m^d are the real, actual and desired stocks of money, respectively, i.e., $\ln m = \mu$ and $\ln m^d = \mu^d$) and that the rate of inflation has been constant for a period of time long enough for expectations to be adjusted, that is, $\pi = \pi^e$, and $p = p^e$, equation (2.6) reduces to:

$$(2.7) \quad \pi = \hat{M} - \beta dz$$

which is the traditional quantity equation. Equation (2.7) can also be derived here from stock equilibrium in the money market.

This means that in the long-run, when all the adjustments in the money market are complete, the rate of inflation will be equal to the rate of monetary expansion minus the product of the income

^{1/} See in this regard Friedman (1971). Combining equations 28 and 31 of Friedman, we obtain, after some transformations:

$$\pi = \alpha \Psi \hat{M} + (1 - \alpha \Psi) \pi^e + \alpha \Psi \eta_{L,c} d \pi^e + \alpha \Psi (1nM - 1nM^d) - \alpha \Psi \eta_{L,y} dz + \gamma (z - z^*)$$

where the variables are expressed in terms of our notation and the parameters in Friedman's notation, with z being the logarithm of real permanent income, $\eta_{L,y}$ the income elasticity, and

$\eta_{L,c} = \alpha \pi^e$ the cost elasticity. We note that this is the same reduced form as that of the present model except for two terms: the stock disequilibrium effect of the money market which in our model is expressed in real terms and in Friedman's in nominal terms, and the last term in the equation which in our version represents the disequilibrium between actual and expected prices. This last difference derives essentially from the different way in which the equation which explains prices is related to the real side of the model, as will be analyzed later on.

elasticity of the demand for money (long-run) times the rate of increase of real product. However, in the short-run, if $(1 - b)$ is small, changes in the rate of growth of the money supply will hardly affect the present rate of inflation which will be maintained approximately equal to the expected rate.

Empirical evidence for Brazil shows that the coefficient of the rate of monetary expansion is approximately equal to 0.1 when the model is adjusted to monthly data and to 0.4 for quarterly data. At the same time, the coefficient of the expected rate of inflation is close to 1 for monthly figures and to 0.6 for quarterly data ^{1/}.

The studies of Diz (1966) for Argentina, Harberger (1965) for Chile and, more recently, Vogel (1974) for a series of Latin American countries show that the impact of monetary expansion is not reflected immediately in prices, but is felt only after the passage of a fairly long period of time (a year or more).

Assuming the income level exogenous and constant ($dz = 0$), the model determines the path followed by π and μ when the rate of monetary expansion changes from one constant level to another ^{2/}. This path is typified in Figure 1.

If $\hat{M} = \hat{M}_0$ up to time t_0 , and from then on passes to a new constant level \hat{M}_1 , the path for π , could be as shown in part A of Figure 1 in the case of a stable equilibrium. Initially, the rate of inflation will undergo a small reduction that will be smaller the larger is b . Between t_0 and t_1 , the rate of monetary expansion will be less than π .

^{1/} See Pastore (1973a).

^{2/} The stability conditions are examined in detail in Appendix A. In general, it is known that for plausible values of the coefficients involved, the solution will be stable. Under the assumption that $b = 0$ (absence of the shock absorber effect) and that the term $d\mu^d$ is absent in the adjustment equation, the model here proposed reverts to one used earlier by Mundell (1965). With the assumption that $b = 0$, we will always have $m = m^d$ (instantaneous adjustment of the money market) and the model becomes the one proposed earlier by Cagan (1956).

The actual real stock of money will decrease while the desired money stock will remain initially constant (because neither prices nor expectations react instantaneously), and thereafter will be increasing. The excess of μ^d relative to μ reduces the flow of expenditures on goods and services which diminishes still more the pressure on the rate of inflation which, consequently, will continue to fall.

Later, when the expected rate of inflation falls, the actual rate will be reduced even more, due both to the direct effect of π^e on π and to the increase in the desired stock of money provoked by the reduction in π^e . After a certain point in time (t_1 in the figure), the rate of inflation will be reduced below the rate of monetary expansion. From then on, the actual real stock of money will increase, at t_1 and even at t_2 , thus bringing in the force that in this model will eventually eliminate the difference between μ and μ^d . This difference (the stock disequilibrium) might be increasing if μ^d were growing at that point at a rate faster than μ (μ_{11}^d is a conceivable path for μ^d for the transition period). But at some point in time between t_0 and t_3 , μ^d will start growing at a rate smaller than μ , and the stock disequilibrium will start diminishing. As the stock disequilibrium diminishes, the fall in the rate of inflation will be reduced and after this rate reaches its minimum point (t_2), it will begin to increase again. Eventually, at t_3 in the picture, the stock disequilibrium will disappear and the expected rate of inflation will equal the actual rate again. At that point we would have reached the final equilibrium.

Summarizing: in this model there are two forces altering the rate of inflation: namely: changes in (a) the stock disequilibrium effect and (b) the expected rate of inflation. The stock disequilibrium will be growing, at least, up to t_1 because μ^d is growing and μ is falling. π will be falling both because the stock disequilibrium is growing and because π^e is falling. After t_1 , μ will start rising. At the point when μ starts rising at the same rate as μ^d the stock

/disequilibrium will

disequilibrium will reach a maximum. From there on, as the stock disequilibrium diminishes, there will be a force working to increase the actual rate of inflation. The reductions in π^e will act in the opposite direction. Once the stock disequilibrium effect dominates, the rate of inflation will start rising again which in turn will reduce the forces working to lower π^e . Eventually both π and π^e will converge one to the other and the stock disequilibrium will disappear. At this point (t_3 in the figure) the economy will be in static equilibrium once again.

Yet it is conceivable that π^e goes below \hat{M} so that μ^d will overshoot. Then we might have an oscillatory approach to equilibrium. If we allowed for changes in real income the process would be even more complicated.

Since the new equilibrium will be achieved with smaller rates of monetary expansion and inflation, at the end of the process the cost of holding money will be less. Therefore, the new real stock of money (equal to the desired stock) will be necessarily higher than the original. Consequently, in long-run equilibrium the real stock of money varies inversely with the rate of monetary expansion. In the short-run, however, a reduction in the growth of M leads to a fall in the real liquidity of the economy.

Area B in Figure 1-i will be greater than area A by exactly the magnitude necessary to make the equilibrium real stock of money increase from μ_0^d to μ_1^d . The difference between the two areas is exactly equal to α (the coefficient associated with π^e in the demand for money equation (2.1); the cost elasticity of demand for money is $\alpha\pi^e$) multiplied by the change in the rate of monetary expansion (that is, $B - A = \alpha \Delta \hat{M}$).

The logarithm of actual income velocity is given by the difference between the logarithm of real income and the logarithm of the real quantity of money, $v = z - \mu$. With z constant, a fall in the rate of monetary expansion will initially increase income velocity,

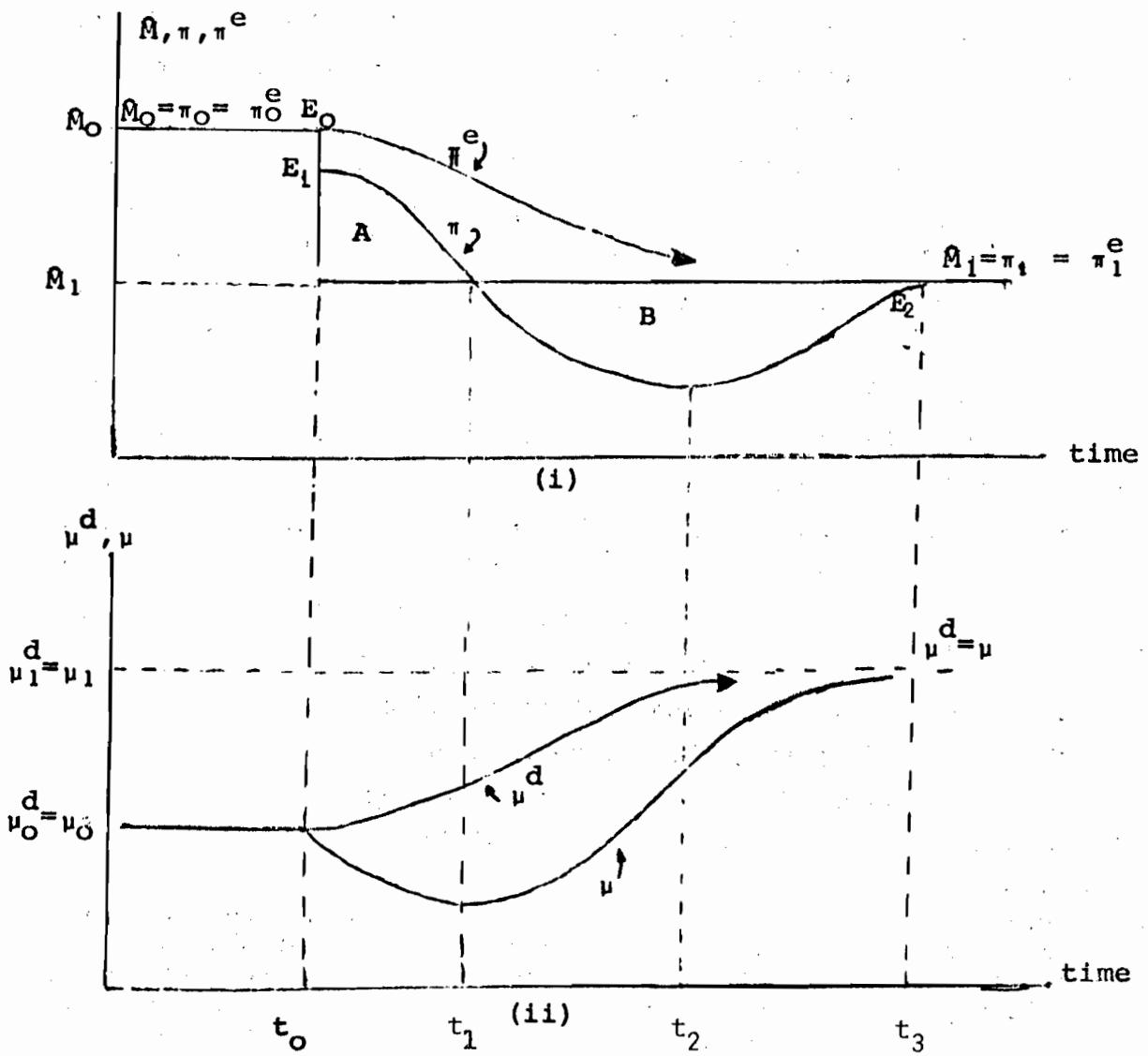


Fig.1. Hypotetical paths of M , π and π^e and for μ and μ^d .

but when

but when the new equilibrium is reached, v will be necessarily smaller. Income velocity in the long-run will be a stable function of real income and the expected rate of inflation. In the short-run, however, if the rate of monetary expansion varies, velocity will tend to fluctuate in the opposite direction ^{1/}.

This behavior is captured by the model, for when b is large, the money supply, has, in the short-run, a very reduced impact upon π . The model also shows that in the long-run, on the other hand, it is not possible to carry out any stabilization program without a control of \hat{M} .

Equation (2.6) allow us to draw the aggregate demand curve in the $\pi - \hat{y}$ plane (where y is real income, so $\hat{y} = dz$ is the relative rate of change in income). Assuming initially full equilibrium the long-run aggregate demand curve with $\hat{M} = \hat{M}_0$ is situated in the position DL_0 . After full adjustment with $\hat{M} = \hat{M}_1 < \hat{M}_0$, the aggregate demand curve shifts to the position DL_1 . Initially after \hat{M} is reduced to \hat{M}_1 , however, the shift of the aggregate demand curve will be less than indicated by DL_1 and given that in the short-run $\partial \pi / \partial \hat{y}$ is less than in the long-run, the short-run aggregate demand curve is more elastic, as illustrated by DC_1 . In time the aggregate demand curve will keep shifting until reaching the position DL_1 .

^{1/} Even if the income elasticity of demand for money were in the long-run greater than one, it would still be possible for it to be less than one in the short-run. In this case, income velocity of money will vary positively with income in the short-run and negatively in the long-run. In this way, the model permits (for given values of the parameters involved) a behavior for the income velocity of money identical to that recorded by Friedman in 1959 for the United States. However, the existence of a positive correlation between z and v cyclically and a negative correlation in the long-run would not be interpreted as the result of the hypothesis that the demand for money is a stable function of permanent income (instead of current income). Such a finding would instead be related to the existence of lags in the adjustment of the real money stock.

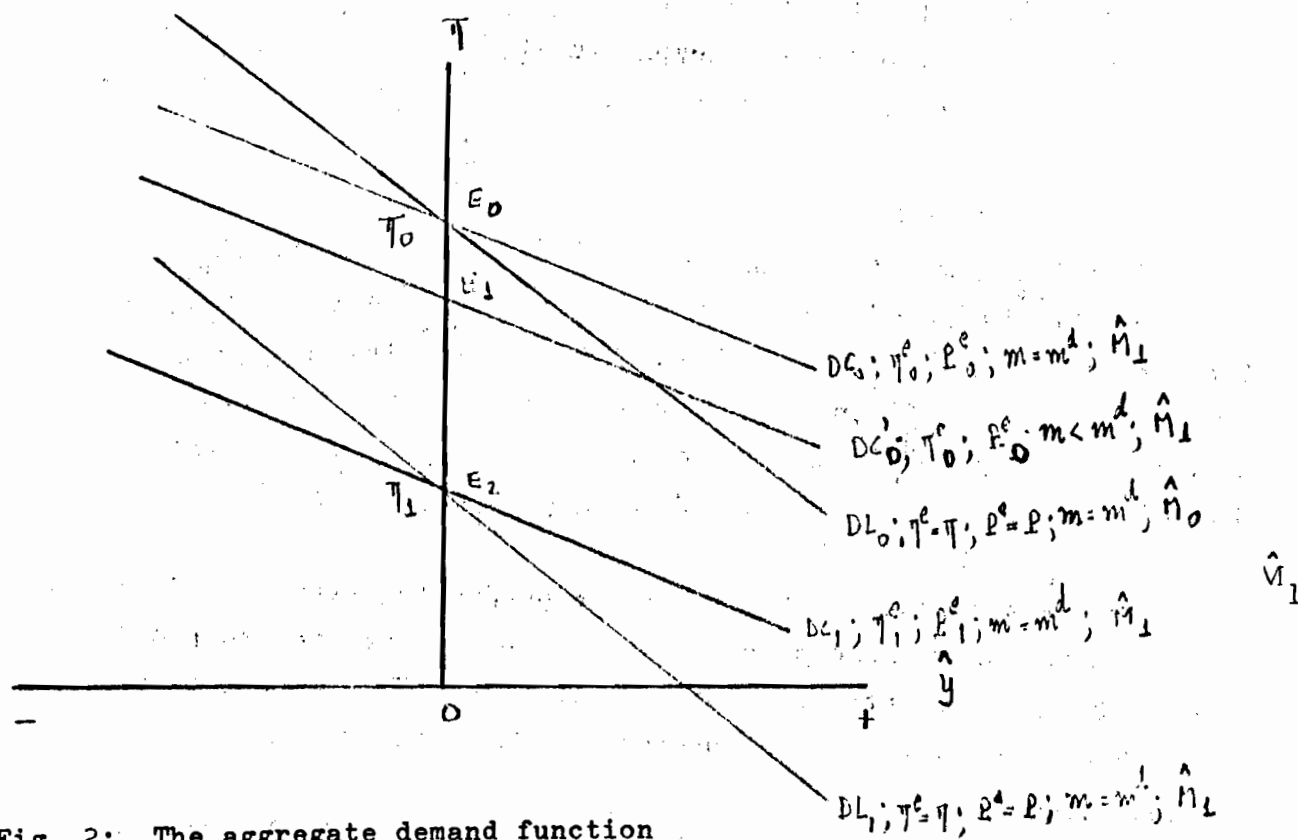


Fig. 2: The aggregate demand function

III. AGGREGATE SUPPLY

III. AGGREGATE SUPPLY ^{1/}

One of the most serious criticisms of the majority of models designed to explain inflation relates to the omission in such models of an aggregate supply function. This means that real income must be an exogenous variable, which makes it extremely difficult to discuss the costs of alternative stabilization strategies.

In order to introduce the supply function, we begin with the assumption that the production function can be expressed as follows:

$$(3.1) \quad y = F(N^S, K^S)$$

where y is the flow of real output, and N^S and K^S are the flows of productive services of labor and capital, respectively. Assume that the function in (3.1) is everywhere differentiable with continuous first and second derivatives. Assume further that the marginal productivities of labor and capital, F_{N^S} and F_{K^S} , are positive and diminishing, that is, $F_{N^S N^S}$ and $F_{K^S K^S} < 0$, and finally, that $F_{N^S K^S} = F_{K^S N^S} > 0$.

Our first problem is to define the productive services of capital. Let us assume that these productive services are related to plant (PL), machinery (MQ), raw materials (MP), inventories (I), depreciation (D), real credit (L/P), etc., by a production function such as ^{2/}:

^{1/} This section draws heavily upon Almonacid (1971), Chapter I, Part 2.

^{2/} It is evident that the productive services of raw materials confound themselves with the consumption of raw materials in the productive process. In this case, the output defined in (3.1) is expressed in terms of gross output and not of value added. This presents no difficulty for the results obtained in this section since it is possible to show that if we define y as gross product and thereafter define a new variable y' as output in terms of value added, the new aggregate supply function corresponding to y' will have exactly the same properties as the one related to y . In this regard, see Almonacid (1971).

$$/(3.2) \quad K^S =$$

$$(3.2) \quad K^S = G(PL, MQ, MP, I, D, L/P \dots)$$

By combining those capital goods which are stocks (plant, machines, etc.) into a single stock of capital concept (K), by assuming that real credit is given for firms and equal to $C = L/P$, and by grouping together all other inputs (raw materials, depreciation, etc.) into a new variable X, we may express the relation (3.2) in the following form:

$$(3.3) \quad K^S = G(K, X, C)$$

For each component of X we will have a price denoted P_{xj} , where j varies from 1 to n, with n being the number of inputs that together comprise X. We may also define an average price level of X, given by P_x . Minimizing the costs of producing capital services ^{1/}, we can, using (3.3), derive a marginal cost curve of producing capital services given by ^{2/}:

$$(3.4) \quad CM(K^S) = h(K^S, K, C, P_x)$$

This relation is homogeneous of degree one in P_x and, thus, can be expressed as:

^{1/} If we were to define X in terms of value at constant prices, P_x would take on the dimension of a price index.

^{2/} The relation in (3.4) was obtained under the assumption that the real stock of credit is given for firms. In Note B of the Appendix, we analyze the determination of the demand for credit by a competitive firm. It is shown that this demand is a function of P_x , K^S , K, and i, where i is the rate of interest on loans. If we sum the demands of all firms and assume the existence of a supply of credit (i.e., the demand for loan assets on the part of banks), the model would determine endogenously values for i and L.

$$/(3.4): CM(K^S) =$$

$$(3.4)' \quad CM(K^S) = P_x H(K^S, K, C)$$

where $H_{KS} > 0$ and $H_K < 0$. Finally, marginal costs decline with the greater availability of operating capital so that $H_C < 0$. Furthermore, it is reasonable to assume that proportional changes in K^S , K and C will leave the marginal cost of capital services invariable (i.e. the H function is homogeneous of zero degree in K^S , K and C). Then the following relationship must hold:

$$(3.4)'' \quad K^S H_{KS} + KH_K + CH_C = 0$$

We can imagine two alternative assumptions with regard to wage determination in the labor market. Under the first assumption, we let nominal wages be institutionally determined; under the second, we assume that they are determined simultaneously by the interaction of the demand for and supply of labor. In any one of the two cases, the profit function will be given by:

$$(3.5) \quad Q = PF(N^S, K^S) - WN^S - \int_0^{K^S} P_x H(v, K, C) dv - FC$$

where FC represents other fixed costs and P is the relevant price level for products sold by the firm.

The necessary conditions for profit maximization are given by

$$\frac{\partial Q}{\partial K^S} \text{ and } \frac{\partial Q}{\partial N^S} = 0 \text{ from which we obtain:}$$

$$(3.6) \quad PF_{KS} = P_x H(K^S, K, C) \text{ and}$$

$$(3.7) \quad PF_{NS} = W$$

These equations indicate that the value of the marginal product of each input is equal to its respective marginal cost. Equation (3.6) includes two different price indexes. A price index for real output appears on the left-hand side, while the right-hand side contains an input price index. To simplify matters, and without justifying in

this paper the assumptions necessary to reach such a result ^{1/}, we will assume that economic agents invest more to obtain information about the prices of those products that affect them most directly and significantly; thus, they foresee with greater precision movements in the prices of such goods. We consequently assume that entrepreneurs have correct expectations with respect to wages and output prices, but that they may commit errors in predicting the prices of other inputs. At the same time, workers have correct expectations with respect to wages, but may be mistaken with regard to movements in the general price level. Denoting the expected input price level by P_x^e , (3.6) could be re-written as:

$$(3.6)' \quad PF_{KS} = P_x^e (K^S, K, C) = D$$

where D is the nominal rental rate on capital services, which is analogous to the wage rate for the workers. In order to study the properties of the aggregate supply function, we compute the total differential of the system (3.6)-(3.7) which, expressed in matrix form, is:

$$(3.8) \quad \begin{bmatrix} PF_{KSKS} - P_x^e H_{KS} & PF_{KSNS} \\ PF_{NSKS} & PF_{NSNS} \end{bmatrix} \begin{bmatrix} dK^S \\ dN^S \end{bmatrix} = - \begin{bmatrix} D \\ 0 \end{bmatrix} (\hat{P} - \hat{P}_x^e) -$$

$$- \begin{bmatrix} 0 \\ W \end{bmatrix} (\hat{P} - \hat{W}) + \begin{bmatrix} P_x^e K H_K \\ 0 \end{bmatrix} \hat{K} + \begin{bmatrix} P_x^e C H_C \\ 0 \end{bmatrix} (\hat{L} - \hat{P})$$

where as before a circumflex accent above a variable indicates that we are taking the percentage rate of change of that variable.

Now, by making use of the equilibrium conditions (3.6)', employing Cramer's rule to solve the system, and denoting by Δ the determinant of the matrix of coefficients associated with the endogenous variables (the principal determinant), we obtain:

^{1/} See Almonacid (1971).

$$\Delta = (PF_{K^S K^S} - P_x^e H_{K^S}) PF_{N^S N^S} - P^2 (F_{K^S N^S})^2$$

and the profit maximization condition implies that $\Delta > 0$. The expression for the increments in the productive services of capital is:

$$(3.9) \quad dK^S = \frac{1}{\Delta} \{ -PF_{N^S N^S} D(\hat{P} - \hat{P}_x^e) + PF_{K^S N^S} W(\hat{P} - \hat{W}) \\ + PF_{N^S N^S} P_x^e K H_K \hat{R} + PF_{N^S N^S} P_x^e C H_C (\hat{L} - \hat{P}) \}$$

In the same way the expression for the increments in the productive services of labor is:

$$(3.10) \quad dN^S = \frac{1}{\Delta} \{ PF_{N^S K^S} D(\hat{P} - \hat{P}_x^e) - PF_{K^S K^S} P_x^e H_{K^S} W(\hat{P} - \hat{W}) \\ - PF_{N^S N^S} P_x^e K H_K \hat{R} - PF_{N^S N^S} P_x^e C H_C (\hat{L} - \hat{P}) \}$$

In order to obtain the expression for the increase in real output, we may differentiate (3.1) totally, obtaining:

$$\hat{y} = \frac{1}{y} (F_{K^S} dK^S + F_{N^S} dN^S)$$

and by substituting dN^S and dK^S for the equilibrium values in the use of inputs, we then obtain:

$$(3.11) \quad \hat{y} = a_1 (\hat{P} - \hat{P}_x^e) + a_2 (\hat{P} - \hat{W}) + a_3 \hat{K} + a_4 (\hat{L} - \hat{P})$$

where: $a_1 = a_0 D(-DF_{N^S N^S} + W F_{N^S K^S})$

$$a_2 = a_0 W [DF_{K^S N^S} - F_{N^S} (PF_{K^S K^S} - P_x^e H_{K^S})]$$

$$a_3 = a_0 P_x^e K H_K (DF_{N^S N^S} - W F_{N^S K^S})$$

$$a_4 = a_0 P_x^e C H_C (DF_{N^S N^S} - W F_{N^S K^S})$$

with a_1, a_2, a_3 , and $a_4 > 0$, where $a_0 = \frac{1}{y \Delta}$

Therefore, aggregate

$$(3.15) \quad dN^S = -e_o \{ W (PF_{KS} K^S - P^e H_{KS}) - D PF_{NS} K^S \} (\hat{P} - \hat{P}^e) \\ - e_o (PF_{NS} K^S P^e K H_K) \hat{K} + e_o P^e N G_N (PF_{KS} K^S - P^e H_{KS}) \hat{N} \\ - e_o (PF_{NS} K^S \cdot P^e C H_C) \hat{C}$$

for the productive services of labor.

In order to obtain the expression for the increase in total output, we may differentiate totally (3.1) one more time to find:

$$\hat{Y} = \frac{1}{Y} (F_{KS} dK^S + F_{NS} dN^S)$$

and substituting dN^S and dK^S for the equilibrium values in input markets, we get:

$$(3.16) \quad \hat{Y} = d_1 (\hat{P} - \hat{P}^e) + d_2 \hat{K} + d_3 \hat{N} + d_4 \hat{C}, \text{ finally, using (2.5), we get:}$$

$$(3.16)' \quad \hat{Y} = d_1 (\pi - \pi^e) + d_2 \hat{K} + d_3 \hat{N} + d_4 \hat{C} - c_2 d_1 (p - p^e)$$

where

$$d_1 = d_o [-D^2 (PF_{NS} N^S - P^e G_{NS}) + 2W PF_{KS} N^S - W^2 (PF_{KS} K^S - P^e H_{KS})]$$

$$d_2 = d_o [D (PF_{NS} N^S - P^e G_{NS}) - W PF_{NS} K^S] P^e K H_K$$

$$d_3 = -d_o [D PF_{KS} N^S - W (PF_{KS} K^S - P^e H_{KS})] P^e N G_N$$

$$d_4 = d_o [D (PF_{NS} N^S - P^e G_{NS}) - W PF_{NS} K^S] P^e C H_C$$

with d_1, d_2, d_3 , and $d_4 > 0$, where $d_o = \frac{1}{Y.P.\Delta}$

Therefore, aggregate

Therefore, aggregate supply is a function homogenous of degree zero in P and P^e and may be expressed in the following form:

$$y^S = S(P/P^e, K, N, C)$$

The rate of growth of output that result if real credit grew at the same rate as the capital stock and if $\hat{P} = \hat{P}^e$ is given by:

$$(3.17) \quad \hat{y} = (d_2 + d_4) \hat{K} + d_3 \hat{N} = n \quad 1/$$

which yields the natural rate of growth, now determined by the growth rate of the capital stock, of the labor force and of real credit.

Making use of (3.17) equation (3.16)' can be rewritten as:

$$(3.18) \quad \hat{y} = n + d_1 (\pi - \pi^e) - c_2 d_1 (p - p^e) + d_4 (\hat{C} - \hat{K})$$

It would be easy to add in (3.1) a variable reflecting technological progress which would appear explicitly in expressions (3.11), (3.14) and (3.15), without altering essentially the form of the aggregate supply function (3.16), although such a procedure would add a new term to the nature rate of growth.

In the limiting case in which $\pi = \pi^e = \hat{W}$, $\hat{C} = \hat{K}$ and $P = P^e$, the aggregate supply curve in the $\pi - \hat{y}$ plane is vertical at the level of the natural rate of growth. This relation will be defined as the long-run aggregate supply curve. Along this curve, the rates of utilization of labor and capital are constant.

Points to the right of the long-run aggregate supply curve define situations in which employment and capital services grow at rates greater than those corresponding to long-run equilibrium. Points to the left indicate situations in which the same items grow at rates less than those of long-run equilibrium.

1/ Once again it can be shown that if (3.4)" holds, if the production function is homogenous of the first degree, and if $\hat{K} = \hat{N}$ then $d_2 + d_3 + d_4 = 1$, and all variables would grow at the same relative rate. See footnote 1 on p. 20 above.

If real credit were growing relative to the capital stock (assuming $P = P^e$ and $\tilde{W} = \hat{W} = \tilde{\pi}^e$) the aggregate supply curve would be to the right of the long-run curve; if real credit were diminishing relatively, the curve would lie to the left of n . The aggregate supply curve assumes similar positions with respect to the long-run supply curve when wages are fixed administratively if $\hat{W} < \tilde{W}$ or $\hat{W} > \tilde{W}$, respectively.

Let us assume now that $\tilde{\pi} = \hat{L} = \hat{W}$ and that the expected price level is adjusted ($P = P^e$), but that the expected rate of inflation is rigid at the level $\tilde{\pi}^e = \tilde{\pi}_0^e$. In this case, since $d_1 > 0$, we know from (3.18) that the aggregate supply curve will be positively sloped in the $\tilde{\pi} - \hat{y}$ plane. This short-run aggregate supply curve for an expected rate of inflation $\tilde{\pi}_0^e$ cuts the long-run aggregate supply curve at the point where $\hat{W}_0 = \tilde{W}_0^e$. If $\tilde{\pi}^e = \tilde{\pi}_0^e$ (a constant) and given that $d_1 > 0$, a lower rate of inflation implies a lower rate of growth, and, likewise, a higher rate of growth implies a higher rate of inflation, i.e., higher than $\tilde{\pi}_0^e$.

For different values of $\tilde{\pi}^e$, the short-run aggregate supply curve shifts. Thus, if $P = P^e$, $\hat{C} = \hat{K}$, $\hat{W} = \tilde{W}$, and $\tilde{\pi}^e$ decreases from $\tilde{\pi}_0^e$ to $\tilde{\pi}_1^e$, short-run aggregate supply shifts to the right.

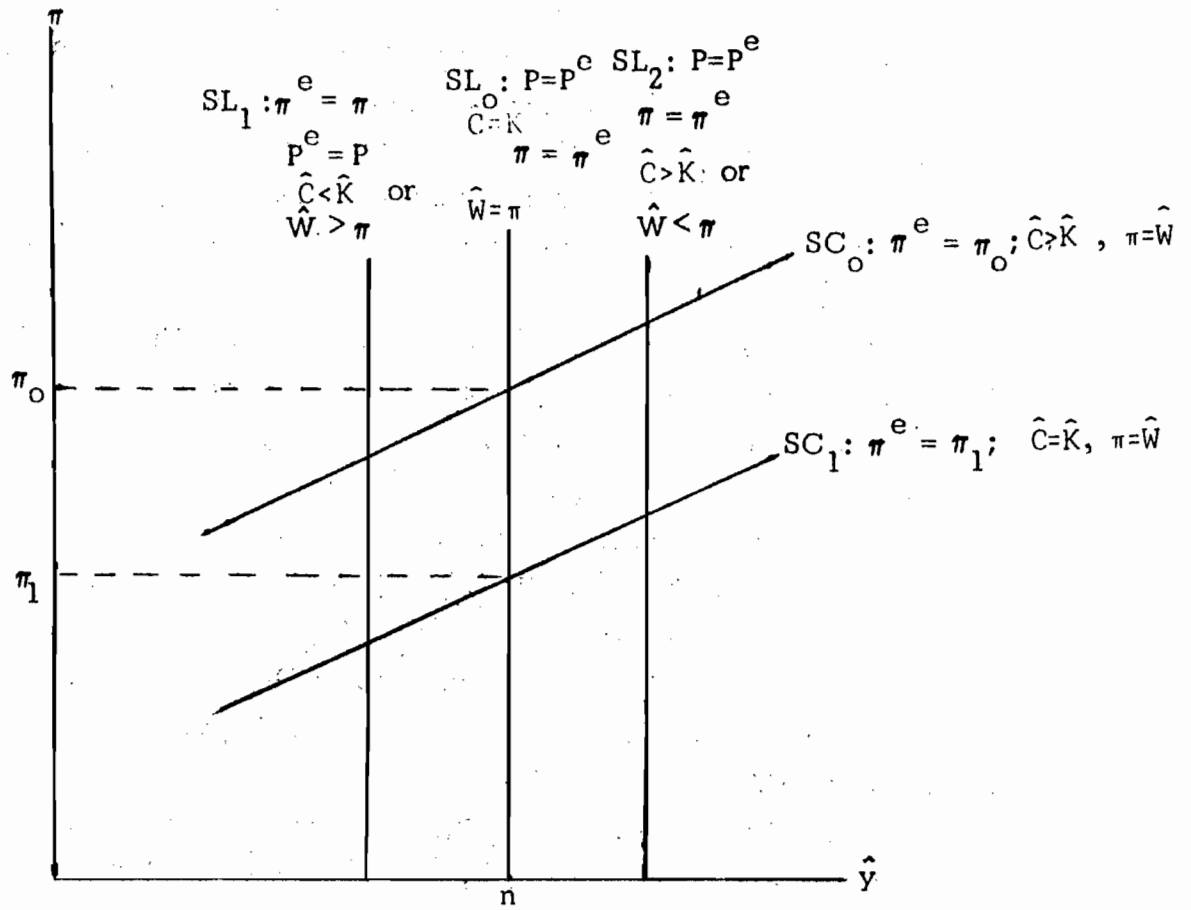


Fig. 3. The aggregate supply function.

IV. THE SUPPLIES

IV. THE SUPPLIES OF MONEY AND CREDIT (ψ)

The previous analysis was based on the assumptions that the rate of change of the nominal money stock plays an important role in the determination of the long-run rate of inflation while the rate of change of the real stock of credit explains part of the behavior of aggregate supply in the short-run. Accordingly, policies designed to restrict the rate of growth of the money supply which at the same time imply a reduction in the real stock of credit will have a negative impact on aggregate supply. In such cases, therefore, stabilization will only be achieved at the cost of a larger reduction in the rate of growth of real output.

The object of this section is to investigate the extent to which it is possible to change the supplies of money and credit independently so as to minimize the recessionary effects of a monetary contraction ^{1/}.

We may begin the analysis by defining the nominal stock of money as:

$$(4.1) \quad M = kB$$

where k is the money multiplier, B the monetary base, and the stock of money, M is defined as the sum of currency in the hands of the public and total bank deposits, that is, $M = M_p + D$.

In the Brazilian case, the monetary base is given by the sum of M_p plus the total free and required reserves of commercial banks,

(ψ) This section is based on the institutional set up existent in Brazil where the monetary authorities also act as a commercial bank. There would be no essential change, however, if the monetary authorities were not to include such a bank.

^{1/} The ideas developed in this section draw heavily upon Pastore (1973a) and (1973b).

R, plus deposits of the public in the Banco do Brasil, D^{BB} ^{1/}, that is:

$$(4.2) \quad B = M_p + R + D^{BR}$$

We are going to assume the following three behavioral relationships:

1. The public wishes to hold a proportion h of the total money supply in the form of cash; 2. the ratio between reserves (free and required) and total deposits in the banking system is constant and equal to r ; and 3. the average propensity to deposit in the Banco do Brasil is equal to g . Formally, these three assumptions can be written as:

$$(4.3) \quad M_p = h M$$

$$(4.4) \quad R = r D^{BC}$$

$$(4.5) \quad D^{BB} = g D$$

where

$$(4.6) \quad D = D^{BB} + D^{BC}$$

in which D^{BC} stands for the deposits of the public in commercial banks. The expression for the multiplier will then be:

$$(4.7) \quad k = \frac{1}{1 - (1-h)(1-r)(1-g)} = \frac{1}{(1-\delta)}$$

It follows that k varies inversely with h , r and g ^{2/}.

^{1/} The Banco do Brasil is simultaneously a commercial bank and the financial agent of the monetary authorities. On the one hand, the Banco do Brasil has more freedom than other banks since: (a) it is not subject to minimum reserve requirements; and (b) it need not fix its level of free reserves since it enjoys unlimited access to the vault of the Central Bank. On the other, it has less freedom because it cannot establish its lending policy in the same way other banks do. Thus, its loans (L^{BB}) are not determined in relation to its deposits (D^{BB}) and excess reserves, but by guidelines established by the monetary authorities.

^{2/} If no commercial bank were part of the monetary authorities, the monetary case, B , would be defined in the usual way, that is, $B = M_p + R$. g would equal zero and the leakages in the multiplier would be just h and r , δ would reduce to $\delta = (1-h)(1-r)$ and the denominator of the multiplier would be simply $1-\delta = h+r(1-h)$, as its traditionally expressed.

The monetary base when analyzed from the point of view of sources rather than uses is given by:

$$(4.8) \quad B = L_g + L^{BB} + F_x$$

where L_g is liquid credit to the government, L^{BB} is the balance of loans of the Banco do Brasil to the private sector, and F_x is that part of the monetary base which was generated in the purchase of foreign exchange, and which may have a negative sign.

Once the supply of money is fixed with a given value for δ , the credit that monetary authorities can supply to the private sector is obtained by isolating L^{BB} in (4.8). Then, by using the money multiplier to transform the monetary base into the total supply of money, we have:

$$(4.9) \quad L^{BB} = (1 - \delta) M - (L_g + F_x)$$

The relationship in (4.8) is the budget constraint for the monetary authorities. Once the desired level of the monetary base is fixed, it is possible to increase the nominal loans of the Banco do Brasil to the private sector only by reducing relatively the liquid balance of $L_g + F_x$. By modifying the channels through which money is injected into the economy, that is, by altering the composition of the monetary base, it is possible to change the ratio between L^{BB} and M .

If we ignore the capital of commercial banks, and to simplify matters, assume that these banks lend all of their free resources (the difference between deposits and reserves), then the amount of loans of commercial banks will be given by $\frac{1}{\delta}$:

^{1/} The model may be easily extended to cover the case in which banks use their resources not only to make loans, but also to purchase any other type of financial asset. Such a generalization would certainly enrich the model, but it goes beyond the scope of the present paper.

$$/(4.10) \quad L^{BC} = (k - 1)$$

$$(4.10) \quad L^{BC} = (k - 1) B = \frac{k-1}{k} M = \delta M$$

and summing term the relationships (4.9) and (4.10), we get:

$$(4.11) \quad L = M - (L_g + F_x)$$

By **regrouping** terms in (4.11) in order to isolate M on the right-hand side, the left-hand side becomes simply the assets of the consolidated balance of the banking system with the monetary authority. This expression shows that the supply of money must be identical to the sum of credit granted to the government and to the private sector plus the accumulated past results in the balance of payments which are converted into cruzeiros according to the exchange rate ruling at time of transaction.

The loans-money ratio will be given by:

$$(4.12) \quad \frac{L}{M} = 1 - (1 - \delta) (1 - \frac{L^{BB}}{L})$$

where $\frac{L^{BB}}{L}$ is the proportion in the monetary base of loans to private sector provided by the Banco do Brasil. It is clear that $\frac{\partial (L/M)}{\partial \delta} > 0$ and $\frac{\partial (L/M)}{\partial \frac{L^{BB}}{L}} > 0$. That is, the loans-money ratio increases with

either an increase in the monetary base multiplier or in the share of credit provided by the Banco do Brasil to the private sector. By altering the reserve ratio or the composition of the monetary base and by creating incentives for the private sector to modify the M_p/M ratio or the D^{BB}/D ratio, the monetary authorities can increase or decrease the L/M ratio.

By re-arranging terms in (4.12), it is possible to obtain a relationship to explain the behavior of the real amount of credit to the private sector, C, given by ^{1/}:

^{1/} Another simplification used in this section is that only commercial banks lend resources to the private sector. This is not a realistic assumption, as is apparent from the experience of recent years during which there has been a great development of the non-bank financial system. Certainly, an interesting extension of the present paper would be to include the rest of the financial system in the model, but this is not an avenue we are able to pursue at this stage of our analysis.

$$/C = \frac{L}{P}$$

$$C = \frac{L}{P} = \frac{M}{P} [1 - (1 - \delta) (1 - \alpha_L^{BB})]$$

We may now introduce into the analysis some results derived in Section 2. We know that there is evidence that the demand for money is a stable function of income and the expected rate of inflation. In a situation in which real income is constant at a level corresponding to full equilibrium, the actual and expected rates of inflation will be equal to the rate of monetary expansion. Since the demand for money varies inversely with the expected rate of inflation, we know that if the rate of monetary expansion is reduced from one constant level to another, then once the economy has reached the new full equilibrium position the real stock of money will be larger. Consequently, in the long-run, a reduction in the rate of monetary expansion implies a relative increase in the real stock of loans to the private sector.

We also saw that the larger is b , the less efficient is monetary policy in the short-run since the rate of inflation is then initially less flexible with respect to variations in the rate of monetary expansion.

Figure 1-ii depicts the typical path of the real stock of money when the rate of monetary expansion is reduced from one constant level to another. It will be noted that the first effect of a reduction in \hat{n} is to reduce M/P .

Even if the rates of growth of L_g , F_x and L^{BB} were reduced in the same proportion, thus provoking no change in the composition of the monetary base, and the coefficients that enter into the monetary multiplier remained constant, relations (2.6) and (4.13) show that with $1 > (1 - b) > 0$, a reduction in the rate of monetary expansion will necessarily generate a fall in the real stock of credit.

/If to

If to obtain a reduction in \hat{M} was possible only through an even greater reduction in the rate of growth of private sector credit provided by the Banco do Brasil (for reasons of rigidities in the fiscal deficit of the government, for example, or because it is not desirable to lose liquid foreign reserves), the fall in the real stock of credit in the initial phases of the stabilization program will be even more marked than the fall in the real money stock.

The reduction in M/P resulting from a decrease in \hat{M} contracts aggregate demand, an essential development for a fall in the inflation rate, but the fall in C contracts aggregate supply. The principal effect of these two movements (in the initial stages of stabilization) will be upon the real side of the economy, being reflected more strongly in a fall in the rate of output growth than in a fall in the rate of inflation.

This also shows that an institutional organization of the monetary authorities that maintains the Banco do Brasil not as a common commercial bank but as an integral part of the monetary authorities, permits greater flexibility in the conduct of monetary policy. Such an institutional framework, by permitting L to be more freely manipulated independently of M , increases the possibility of carrying out a stabilization strategy successfully without at the same time increasing costs in terms of losses in real output.

V. EXPECTATIONS, THE ANNOUNCEMENT EFFECT, PRICE GUIDELINES, AND INFLATIONARY CORRECTION

In Section 2, we made the assumption that the expected rate of inflation was obtained by a process of adaptive expectations identical to the one proposed by Cagan in 1956. Equation (2.4) may be expressed in the form:

$$(5.1) \quad \pi^d(t) = \frac{c}{c+d} \pi(t)$$

This equation implies that the expected rate of inflation can be expressed as a moving average of past rates of inflation using geometrically decreasing weights and written in the following form:

$$(5.2) \quad \pi^e(t) = \int_t^\infty f(\xi) \pi(t - \xi) d\xi$$

where $f(\xi) = c e^{-c\xi}$ is the function that describes the weighting structure.

Several empirical studies show that the profile of $f(\xi)$ reveals an extremely long weight structure. Although the weights are declining in geometric fashion, there are nonetheless relatively high values attached to rates of inflation fairly distant in the past. Accordingly, expectations would have a high degree of inertia and be difficult to reduce, especially when the economy is moving out of a relatively prolonged phase of rising inflation.

We do not want to leave the impression that we feel the model in (5.1) is the best to describe the way in which expectations are generated; other models have been used in empirical studies with relative success ^{1/}. However, all the models used share a common

^{1/} A case in point is the Pascal-type distributed lag functions of the form $\pi^e(t) = \left(\frac{mc}{d+mc}\right)^m \pi(t)$ proposed by Solow (1960) or more general models, such as the one proposed by Jorgenson with the form $\pi^e(t) = \frac{F(d)}{G(d)} \pi(t)$, where $F(d)$ and $G(d)$ are polynomials in the operator d . Clearly (5.1) is a special case of these two forms.

difficulty: the only information used to structure expectations of future rates of inflation is the behaviour of past rates of inflation.

Is it reasonable to think that individuals use no other information in forming their expectations? Would not a series of announcements by the government regarding target rates of inflation for future periods affect expectations? Why is it that government announcements at times are effective in modifying the expectations of individuals while at other times they are not?

Let us suppose that the government can announce continually the rate of inflation, π^* , it hopes to obtain and that individuals grant to the announced targets a certain degree of credibility. In this case, we could postulate a formation of expectations' model to incorporate the announced rate such as the following:

$$(5.3) \quad \pi^e(t) = (1 - \theta) \frac{c}{c+d} \pi(t) + \theta \pi^*(t)$$

where the expected rate of inflation is a weighted average of its historical component (given essentially by the relation (5.1)), and the announcement component with its corresponding weight given by a coefficient of credibility θ , a number which varies between zero and one ^{1/}.

^{1/} θ could be a coefficient that measures the subjective probability economic agents assign to the achievement of the target rate of inflation. The values that this coefficient will assume depend upon the management of those instruments of economic policy that economic agents believe to be most important in explaining the behavior of inflation rates (such as minimum wage readjustments, modifications in the rate of exchange, etc.). If economic agents orient their behavior not according to their observation of these variables, but according to observations of the consistency of past announced goals (in the sense of the extent to which those goals were achieved in the past) θ could be likened to a psychological correlation coefficient between π and π^* , of the form:

$$\theta = \text{cov}(\pi, \pi^*) \cdot \{ \text{var}(\pi) \cdot \text{var}(\pi^*) \}^{-1/2}$$

where:

$$\text{cov}(\pi, \pi^*) = \int_t^\infty f(\xi) \{ \pi(t-\xi) - E\pi(t-\xi) \} \{ \pi^*(t-\xi) - E\pi^*(t-\xi) \} d\xi$$

(cont.)

As Θ increases to unity, individuals give less importance to the historical component; when Θ is equal to zero, they consider only the historical component in the formation of expectations ^{1/}.

The model developed in previous sections demonstrated both that actual rates of inflation influence expectations and that, in turn, expectations affect the rate of inflation itself in the short-run. If we assume that an announcement can exogenously reduce expectations, then the rate of inflation decreases. This reduction in π will affect the historical component of the expected rate and will thus reduce expectations even further, reinforcing the tendency toward a decrease in inflation.

If the announcement effect really were an important element in the stabilization program, a justification would be provided for government efforts to affect individual behavior through the declaration of target rates of inflation.

In the initial phases of a stabilization program, for example, or when a new government is installed, individuals might be reluctant to believe the announced target due to the lack of past information on the ability of the government to implement its economic policy.

^{1/} (Cont.)

which would be restricted exclusively to cases of zero or positive covariance and in which:

$$\text{var } (\hat{\pi}) = \int_t^{\infty} f(\xi) \{ \hat{\pi}(t-\xi) - \Theta \hat{\pi}(t-\xi) \}^2 d\xi$$

$$\text{var } (\hat{\pi}^*) = \int_t^{\infty} f(\xi) \{ \hat{\pi}^*(t-\xi) - \Theta \hat{\pi}^*(t-\xi) \}^2 d\xi$$

where $f(\xi)$ is the weight structure arising from (5.1). If the government announces targets that are consistently achieved, Θ will tend toward 1, whereas if the targets are never achieved, Θ will tend toward zero.

^{1/} For all values of Θ the model possesses the known asymptotic property that when the rate of inflation remains constant for a period of time long enough for the entire adjustment to take place, the expected rate will be equal to the actual rate. When Θ is equal to zero, we are in the Cagan case. When $\Theta = 1$, the government will be continually announcing targets that are perfectly correct, and we will also have: $\hat{\pi} = \hat{\pi}^* = \pi^e$.

/Even if

Even if the program has already produced some results, if the available information derives from a relatively short period of time, credibility may be low ^{1/}.

This implies that a policy of announcements will have more limited effects in the initial stages and will be able to be used with greater success only after some positive results have been observed. However, the effectiveness of the announcement policy is dependent upon the creation of a system to maintain individuals informed of the government's targets.

The second limitation on such a policy arises from the restriction that normally an announcement must be made on a rational basis, i.e., it must realistically take into account the possibility of following the type of monetary and fiscal policies needed to attain the rate of inflation desired by the government. If the announcement were inconsistent with these policies (for example, if in order to attain the announced rate a much greater reduction in the rate of monetary expansion were needed than the government can actually manage), discrepancies would occur between announced and actual rates, leading to a decline in the degree of credibility and limiting the future usefulness of this policy instrument.

One of the methods which can be used to divulge effectively the inflation targets of the government is a system of price guidelines. What we have in mind is not a rigid system of price controls, which generally becomes a wage-price freeze with important distortionary effects upon the economy, but rather one similar to the system which has operated in Brazil in recent years. In such a framework, a dialogue is established between entrepreneurs and the government's price guideline agency the purpose of which is to revise price readjustment plans proposed by firms in order to maintain prices consistent with inflation targets.

^{1/} In terms of statistics, if the sample size used for the calculation of the correlation coefficient ρ is small, it is likely to be significant only with a low level of confidence.

To the extent that targets are reached, credibility will increase and the price guidelines will have the effect of reducing expectations without adverse effects on market efficiency. The announcement of infeasible targets, on the contrary, will reduce credibility, thus eliminating the effectiveness of the policy instrument and forcing the government to adopt a more rigid system of price controls. This creates rather serious distortions and encourages the appearance of black markets which, in turn, result in the adoption of less efficient forms of production and a subsequent drop in the total volumen of production. Many times these consequences occur despite the establishment of strong government sanctions against price control violators.

We believe that the effectiveness of the government's announcements depends, on the one hand, upon the success of the system of price guidelines and, on the other, upon the coordination between the announcements and monetary and fiscal policies. The violation of either of these two conditions will cause expectations to be dependent solely upon the historical behavior of prices and will reduce the degrees of freedom in stabilization policy.

The most important effect of an announcement, when adequately accepted by individuals, lies in the possibility of reducing the discrepancy between actual and expected changes in the general price level. This implies that one important source of short-run fluctuations in the level of output would be reduced or even eliminated.

The same result could be obtained without recourse to a policy of announcement (which is obviously subject to the limitations referred to above) by using the mechanism of inflationary correction or indexing. If all contracts in the economy were perfectly adjusted by a price index (assuming, obviously, that the variations in the price index reflect the true rate of inflation), the result would be equivalent to the case in which contracts are formulated in real terms.

/This would

This would not only reduce or eliminate the differences in expectations among demanders and suppliers of productive services, for instance, but would make it unnecessary to devote resources to the very formation of expectations. Assuming a constant level of real credit, the aggregate supply curve would be vertical at the level of the natural rate of growth of real output and price stabilization could be obtained without significant fluctuations in y .

Although the use of escalator clauses would correct or even eliminate this source of distortion on the aggregate supply side, the cost of holding money will continue to depend upon the expected rate of inflation and this friction will continue to manifest itself in the demand for money.

In an ideal situation with the introduction of a perfect inflation adjustment clause, the model analyzed in the previous sections would explain only the dynamics of inflation; the rate of growth of output would behave as if it were fixed exogenously, thus the analysis would revert to the classical case of perfect flexibility of the price level.

VI. A STABILIZATION PROGRAM WITH PURE MONETARY SHOCKS

We can now analyze in detail the behavior of the rate of inflation and the rate of growth of real output in the course of a stabilization program with pure monetary shock. Let us assume that the aggregate demand and supply curves, respectively, are given by:

$$(2.6) \quad \pi = (1-b) \hat{M} + b\pi^e + \alpha(1-b) d\pi^e + a(\mu - \mu^d) - (1-b)\beta dz + bc_2(p-p^e) \quad \text{and}$$

$$(3.18) \quad \hat{y} = n + d_1(\pi - \pi^e) - d_1 c_2(p - p^e) + d_4(\hat{C} - \hat{R})$$

The third equation of the model is the budget constraint of the monetary authorities and banking system together is simply equation (4.11) expressed in the form of rates of change, i.e.:

$$(6.1) \quad \hat{M} = \gamma_1 \hat{L} + \gamma_2 \hat{L}_g + (1 - \gamma_1 - \gamma_2) \hat{F}_x$$

Let us assume, further, that the government does not desire or is not able to use any announcement of inflation targets and that there is no type of inflation correction clause in the economy.

Let us begin the exercise from an initial situation of equilibrium with

$\pi = \pi^e$ and $\hat{L} = \hat{K} + \pi$ and let us assume that the desired and actual stocks of money are equal, i.e., $m = m^d$; assume also that expectations initially do not change, that is, $d\pi^e = 0$. These assumptions allow us to draw in figure 5

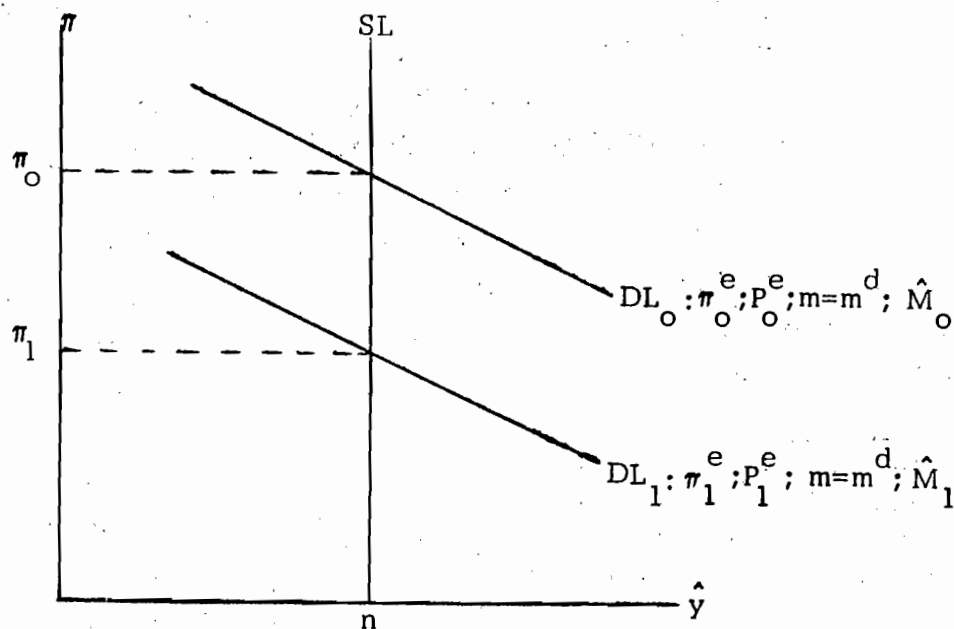


fig. 5. Long-run aggregate demands and supply

/the long-run

the long-run supply and demand curves, DL_0 and SL , which intersect at a rate of inflation $\bar{\pi}_0$. Assume that the government has in mind a quantity theory such as that represented by equation (2.7) and that it believes the rates of monetary expansion and of inflation can be reduced without affecting the rate of growth of real output. This might occur in an ideal situation in which expectations of inflation can be adjusted to the new situation (if, for example, the new target could be announced and accepted with complete credibility). In such circumstances, the government reduces the rate of monetary expansion from \hat{M}_0 to \hat{M}_1 with the expectation that the rate of inflation will stabilize at $\bar{\pi}_1$ after a fairly short period of time (as the aggregate demand curve shifts along the supply curve SL) without any effect on real output.

As a consequence of the analysis in Sections 2 and 4, we know that if the composition of the monetary base does not change, the reduction in \hat{M} will imply a fall in the rate of growth of credit. But, in order to add a new ingredient to the strategy, let us assume that the reduction in the rate of monetary expansion is brought about via a considerable cut in the rate of growth of private sector so that \hat{L} falls more than \hat{M} ^{1/}.

The path in the direction of the equilibrium point may be analyzed with the aid of Figure 6. Initially, the aggregate demand curve shifts not to DL_1 , but to the position represented by DC_1 . This smaller shift (with a flatter slope in the $\bar{\pi} - \hat{y}$ plane) occurs because the reduction in \hat{M} affects aggregate demand fundamentally through the stock disequilibrium effect. Supposing the shock absorber effect is important (implying that b is large), aggregate demand will

^{1/} This is a situation that may occur as a result of lack of coordination between fiscal and monetary policies. It happens for example when the government follows an expansionary fiscal policy pressuring the monetary authorities while at the same time tries to maintain the supply of money under control by reducing relatively credit to the private sector.

fall only after a discrepancy has been created between m and m^d (the coefficient of M in (2.6) is small). The flatter slope of the short-run aggregate demand curve is due to the fact that the slope of this curve with respect to \hat{y} is now $(1-b)\beta$ instead of β . Again, if b is large the demand curve will be only slightly inclined.

If the expected rate of inflation were fixed at $\hat{u}^e = \hat{u}_0^e$, and if $\hat{L} = \hat{M} + \hat{K}$, the short-run aggregate supply curve would be in the position denoted by SC_0 . However, since $\hat{L} < \hat{M} + \hat{K}$, the supply curve shifts to the left, reaching the position SC_1 . At the end of the process, when full equilibrium has been attained, the inflation rate will be reduced to \hat{u}_1 , provided that there is no change in the composition of M between L and the other assets. But in order to reach this point, the rate of inflation and the rate of growth of real output must pass initially through E_1 , which implies an initial reduction in \hat{y} and an indeterminate effect upon \hat{u} . This effect may amount to a small reduction (Figure 6), no change at all, or even an increase in the rate of inflation. This is because the larger the reduction in the rate of expansion of credit, the greater will be the shift of the aggregate supply curve to the left and, for a given demand curve DC_1 , the smaller the reduction in the rate of inflation; the reduction in supply might lead to an increase in the rate of inflation and, thus, to an even more marked reduction in the growth of real output.

In Figure 6, we represent a possible path toward equilibrium. The study of the stability conditions (see Note B, Appendix), shows however, that convergence may be oscillatory and that, in general, the necessary period for convergence to the new equilibrium will be long. The analysis of stability conditions reveals further that both rate of inflation and the rate of growth of output must overshoot their equilibrium values before full equilibrium is once again attained. In the Appendix, this is shown formally so that here we may concentrate upon the economic justification for this fact.

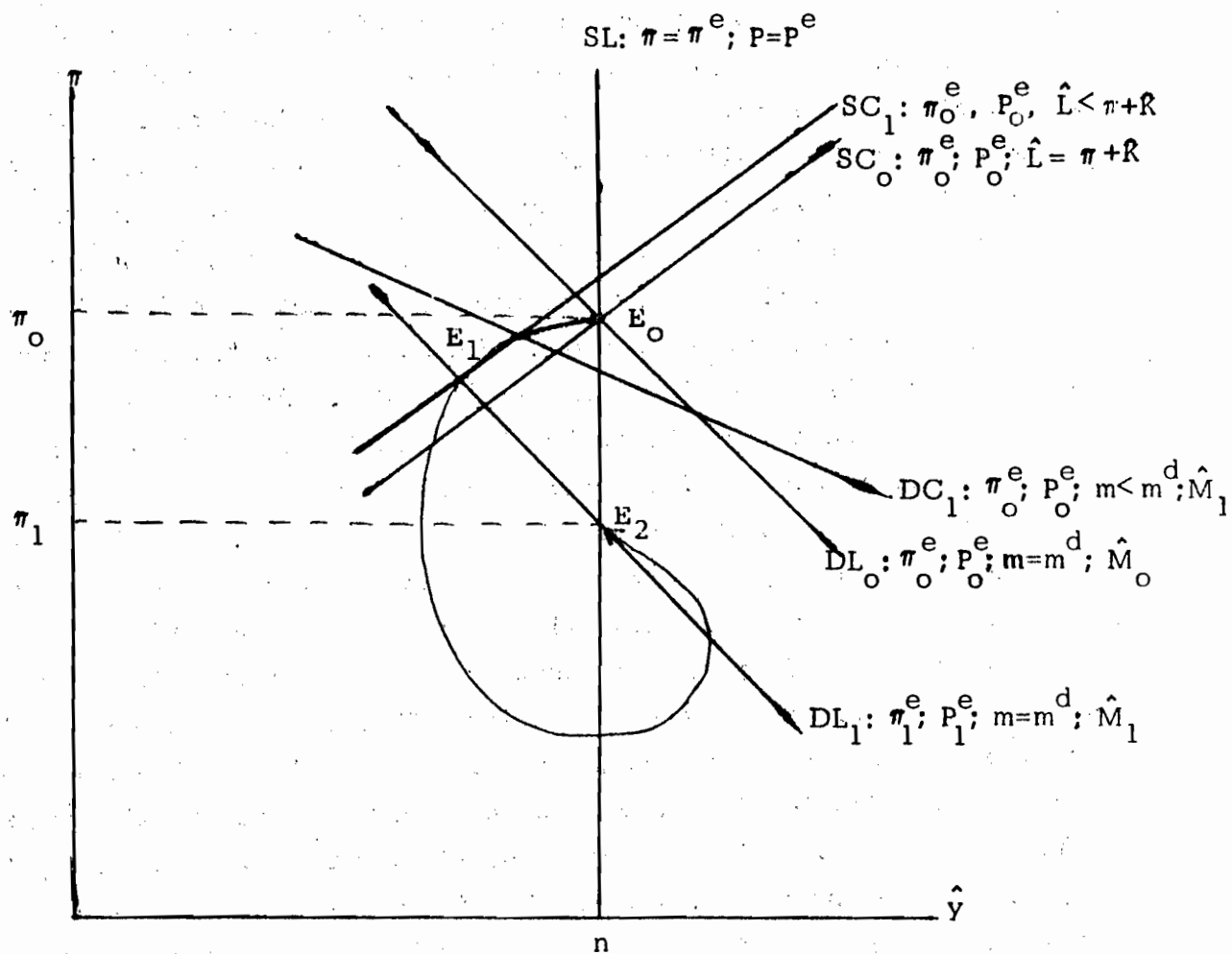


fig. 6. The dynamics of output and prices in the case of a pure monetary shock.

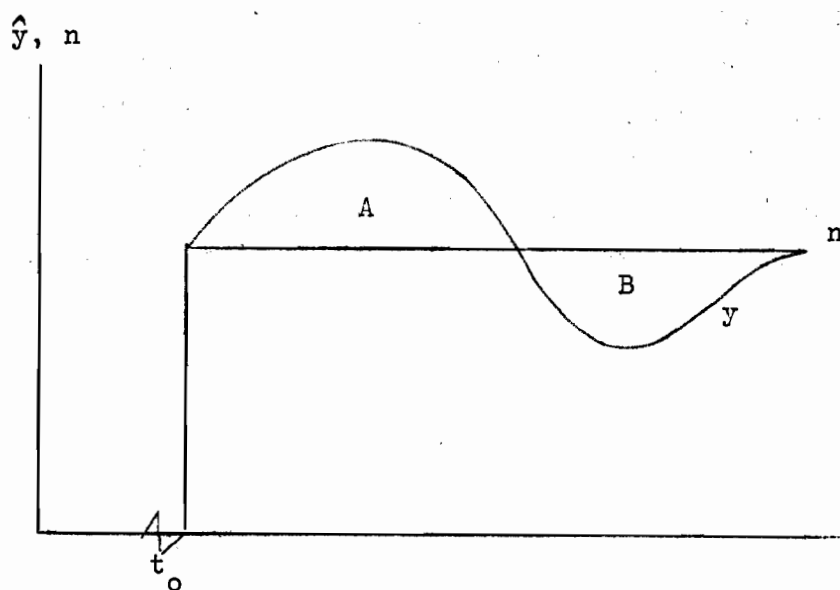
The argument

The argument to show that the rate of inflation will overshoot the equilibrium level is similar to the one developed in Section 2. When the rate of monetary expansion is reduced and the rate of inflation remains relatively stable, the actual real stock of money is reduced. Throughout the adjustment process, the behaviour of the real desired stock of money is different from that assumed in Section 2, since here, the rate of output growth is different from zero and variable. The value of m^d will be affected by a declining expected rate of inflation (which increases the demand for money) and by an income level that is growing at a rate less than the natural one (which will make m^d grow at a rate smaller than before) and, possibly, even decreasing (which would reduce the demand for money). In any case, we know that at the end of the process the level of real output will probably be greater (allowing for growth) and the rate of inflation (with actual rate equal to expected rate) smaller, implying that the demand for money necessarily will be greater. Since the actual real stock was reduced in the initial stages of the adjustment process, the rate of inflation necessarily will have to fall below the rate of monetary expansion (equal to the equilibrium rate) in order to restore equilibrium (of flows and stocks) in the money market. The rate of growth of real output will also undergo an initial reduction. And as long as \hat{y} is less than n , the rates of growth of the productive services of labor and capital will be less than those of the long-run. Since in the new equilibrium the rates of growth of the productive services of labor and capital will return to their long-run values, in equilibrium \hat{y} will once again be equal to n .

/This observation,

This observation, however, is not enough to determine whether there has occurred some loss in the level of output during the process ^{1/}. In order to answer this question it is necessary to observe the behavior of the model in terms of the levels of real output and of factors of production, not solely in terms of their rates of change. If the rates of increase of the capital stock (K) and the labor force (N) were exogenous to the model, they could be considered as constants. As long as y is decreasing, the ratios N^S/N and K^S/K will also be decreasing. But at the end of the process when the price levels of products and factors have restored equilibrium in factor markets (that is, when the expected and actual prices levels are equal), these ratios (N^S/N and K^S/K) will return to their natural level and output will be on its long-run trend. Output will be greater since we now have a larger stock of capital and a larger work force.

- 1/ The areas between the values reached by \hat{y} and n (\hat{y} and n are expressed as functions of t) determine the difference in the level of the logarithm of actual output (z) and the one that would prevail if the economy grew continuously at its natural rate. Then actual output will be smaller than the natural growth output if the area between \hat{y} and n while \hat{y} falls short of n (A) is larger than the area while it exceeds it (B) (Fig. 7).



Fi. 7. Actual and natural rate of growth and the level of output.

/This shows

This shows that with these assumptions there will be no loss of real output in the long-run and that the area between \hat{y} and n while \hat{y} is less than n will necessarily be equal to the area between \hat{y} and n while \hat{y} is greater than n .

In terms of the present value of output, however, there will occur a loss given by:

$$(6.4) \quad V = \int_{t_0}^{\infty} [\hat{y}(t) - n] e^{-\rho t} dt$$

where ρ is the rate of time preference and t_0 is the moment at which the stabilization program is begun. V will be higher the greater the value of ρ and the greater the initial reduction in output.

The welfare loss will occur not only because $\rho > 0$, but also because of the allocative distortions that the process might generate. Since the transition toward the new equilibrium comes about with an initial reduction in output, it is possible that the rate of capital accumulation will not remain constant, but instead decline. This is so because one of the most evident characteristics of stabilization programs similar in framework to the one analyzed here is that the rate of investment is reduced ^{1/}.

The same occurs with the rate of technological innovation and with investment in human capital. This means that during the transition the natural rate of growth must fall. In such a case, the area of losses must be greater than the area of gains. Equation (6.4) offers us in this way a lower limit for the costs of stabilization in terms of present value of lost output; the loss actually registered will possibly be greater.

^{1/} An obvious limitation of the present model in the form in which it is structured is the absence of a demand for capital and for investment goods. Its inclusion would permit a study of the behavior of K through time and would make possible a more precise evaluation of the costs of stabilization.

VII. THE GRADUALIST APPROACH

In the coefficient b in equation (2.6) is in reality close to one in the short-run, the actual rate of inflation will be extremely sensitive to changes in the expected rate of inflation. In this case, successful price stabilization without the recessionary effects that follow a pure monetary shock will depend upon the possibility of affecting the expected rate of inflation.

Let us consider again equation (2.6):

$$\pi = (1-b) \hat{M} + b\pi^e + \lambda(1-b) d\pi^e + a(\mu - \mu^e) - (1-b)\beta dz + bc_2 (p-p^e)$$

If the stabilization strategy manages to reduce π^e immediately, then π would undergo an immediate reduction. Monetary policy would have to be manipulated in order to reduce the rate of monetary expansion to make it compatible with the lower rate of inflation.

Observe that if b were large, the reduction in \hat{M} would have a small impact on π in the short-run. The two rates would become compatible only so far as stock equilibrium is guaranteed in the money market, that is, in so far as the equality $m = m^d$ is maintained. Since the demand for money is a stable function of the cost of holding money, π^e , the reduction of expectations will provoke immediately an increase in the desired stock of money, m^d .

Given that in all empirical works $\lambda < 1$, if the rate of monetary expansion were reduced to a level correspondingly higher than the rate of inflation π , so as to maintain the actual real stock growing at approximately the same rate as m^d , the eventual disequilibrium in the money market would be slight. If monetary authorities achieve this fine control of economic policy, we will have the rate of inflation decreasing at the same speed as the expected rate of inflation while the actual and desired stocks of money increase in a parallel manner.

The consequence is that the reduction in π will be accomplished approximately at the rate predicted by equation (2.7) of the simplest quantity theory.

/We now

We now turn our attention to aggregate supply (3.18):

$$\hat{y} = n + d_1 (\pi - \pi^e) - d_1 c_2 (p - p^e) + d_4 (\hat{L} - \pi - \hat{K})$$

If an approximate equality is achieved between π and π^e , and if $\hat{L} = \pi + \hat{K}$ the aggregate supply curve will be maintained in a vertical position at the height of the natural rate of growth and stabilization will be brought about with successive reductions in the rate of inflation, thus guaranteeing a slowing of inflation without visible costs in terms of reductions in the pace of economic activity.

The principal objection to gradualism, in general, is that it is very slow in showing results. In reality, the only factor limiting the speed with which the rate of inflation is reduced (without negative impact upon output) is the ability of the government to affect expectations. By means of a succession of well planned announcements and with a price guideline system that leads entrepreneurs and workers alike to the realization that the targets of the government are feasible so that they must act accordingly, it is possible to go about gradually reducing inflationary expectations.

However, expectations cannot be completely malleable since to a certain extent the past behaviour of the inflation rate is an important piece of information in the formation of expectations. For this reason, in order to maintain a high degree of credibility in the announced rates, targets must be set within feasible limits which implies that, in fact, it is not possible to carry out more than a policy of rather slow gradualism.

Nevertheless, we simply make the important observation that the strategy should always be centered on the announcement of the government's targets, with efforts being made to obtain final results not too distant from the objectives set by those targets. In this case inflation will diminish slowly but steadily and the path followed by π and \hat{y} will be like that represented in Figure 8:

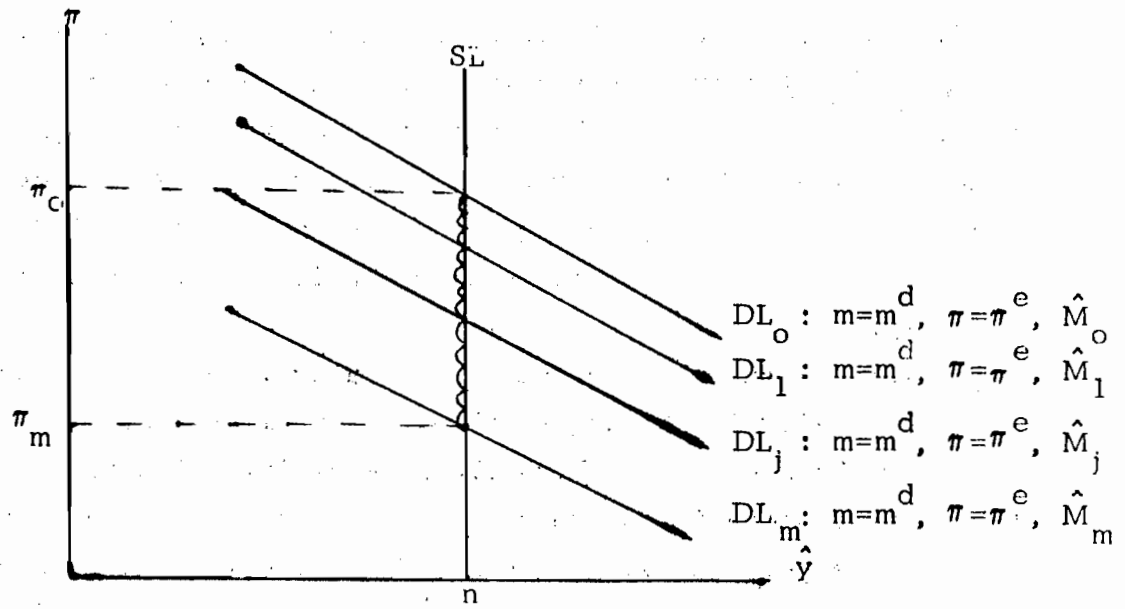


fig. 8. The dynamics of output and prices in the gradualist approach.

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We begin with a situation of full equilibrium in all markets with a rate of inflation given by π_0 . Next, the government provokes a reduction in expectations (through announcement for example) simultaneously lowering the rate of monetary expansion to \hat{M}_1 , thus causing the aggregate demand curve to shift to a slightly lower position. Since real credit was not reduced and since throughout this first phase π will be approximately equal to π^e , the economy will be moving along the long-run aggregate supply curve and the rate of inflation will diminish as the aggregate demand curve simply slides down along SL.

Through the application of further anti-inflation doses like this first one, the rate of inflation will continue decreasing, and real output will be maintained at the level of the natural rate of growth.

During the period of the stabilization strategy we would have an increasing real stock of money and since by assumption all other variables that affect L/M are constant, the real supply of credit will also be increasing. Clearly, successful price stabilization with a greater chance of maintaining the natural rate of growth can be achieved through the use of a policy of relative expansion of real credit which would induce successive shifts to the right of the aggregate supply curve. Such a policy is possible because, as was analyzed in section 4, the government possesses additional degrees of freedom such as the ability to change required reserves policy or the channels through which money is injected into the economy. In such ways, it is possible to obtain a relative increase in the real stock of credit. These increases in supply will help achieve the reduction in the rate of inflation more rapidly and at a rate of output growth equal to or even slightly greater than the natural rate.

The use of a gradualist approach can lead to certain problems which should be avoided. The first critical problem is that an attempt to induce changes in expectations can be confused with a

/rigid control

rigid control over prices and wages; it is possible to show that this strategy will not always give the best results.

If the government decides to freeze prices and wages, the first change in the model with respect to the short-run aggregate supply curve is now represented by equation (3.11) and (3.12)

$$(3.11)' \quad \hat{y} = n + a_1 (\tilde{\pi} - \pi^e) + a_2 (\tilde{\pi} - w) - a_1 c_2 (p - p^e) + a_4 (\hat{L} - \tilde{\pi} - \hat{K})$$

The price freeze operates "as if" expectations had been reduced immediately; if the monetary authorities succeed in reducing adequately the rate of monetary expansion, the economy will stabilize at the new rate of inflation $\tilde{\pi}_1$.

After the freeze, the wage rate will remain constant ($\hat{w} = 0$) and if the equalities $\tilde{\pi} = \pi^e$ and $\hat{L} = \tilde{\pi} + \hat{K}$ are maintained, the short-run aggregate supply curve will shift to the right as is shown in Figure 9. This makes possible a rapid reduction in the rate of inflation with a greater utilization of productive services. However, there is no guarantee that this will be the strategy that maximizes welfare in the economy.

If the initial real wage was above that of equilibrium and if at the end of the strategy when salaries are once again freed the real wage becomes fixed exactly at the equilibrium level, we would certainly have a net gain for the whole society. This means that initially we were not on the long-run aggregate supply curve since the utilization of labor was less than that corresponding to full employment. In this case, the wage freeze served not only to stabilize prices, but also aided to correct a distortion. Therefore, the measure produced a net gain for society.

If the wage rate initially was already at its equilibrium level, then by the end of the strategy real wages would be below the equilibrium point and there would be an excess demand for labor. Subsequent freeing of wages will imply a rise in nominal wages as a result of the interplay of supply and demand; thus, the benefits of the stabilization program can be jeopardized.

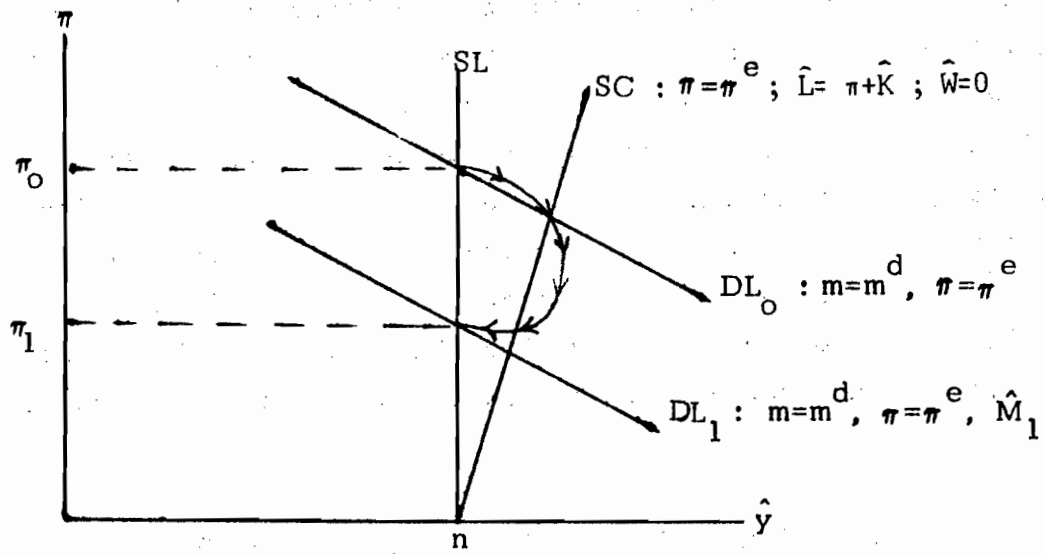


fig. 9. The gradualist treatment with wage freeze.

/The other

The other critical point in regard to this approach is that a relative success in stabilizing via price control can lead authorities into the illusion that the money supply bears no relation to the inflationary process. The rate of monetary expansion will be maintained at relatively high levels, the stocks of money and credit will grow, and the economy will begin to operate in the stage of over-employment, but long-run inflationary pressures will remain latent, finally bringing about the failure of price controls. Persistence in controlling prices with a high rate of monetary expansion will cause the rate of inflation to increase only at certain points in time when authorities are not able to resist pressures to correct administered prices.

VIII. THE TENDENCY TOWARD INSTABILITY IN THE RATE OF INFLATION

The analysis developed in the two preceding sections makes possible an alternative explanation of why many economies demonstrate tendencies toward price instability together with an inflationary bias. If the government places a priority on a high rate of growth, it can for some time raise the average rate of output growth above the natural rate. To this end, it will employ an expansionary shock treatment to increase the rate of growth: later on, when the rate of inflation reaches a level considered "unacceptable", it will use a gradualist program to reduce inflation. But since periods of deflation imply hardships, the starting inflation rate is never reached again; in this way, as the post-war experience shows, the rate of inflation has increased in the majority of Western countries.

The model can be easily understood with the help of Figure 10. The expansionary stage begins when the rate of inflation is at π_0 (point E_0) where the long-run aggregate demand and supply curves intersect. Then the government decides to accelerate the rate of growth of real output, which it can do in the short-run, if it properly raises the rate of monetary expansion trying at the same time to maintain the expected rate of inflation at the level π_0^e .

/Figure 10

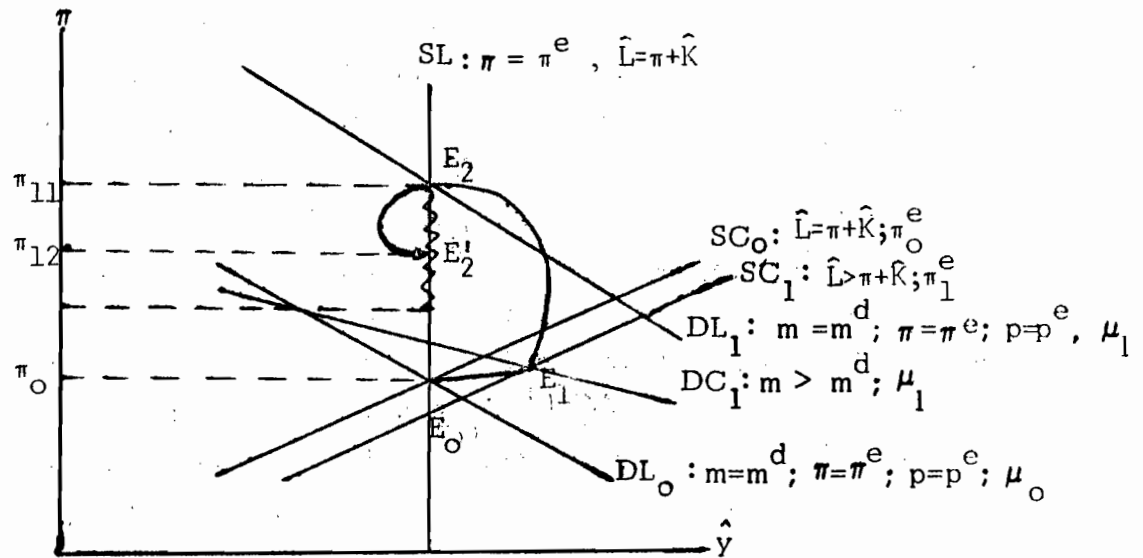


fig. 10. The dynamics of output and prices and the inflationary bias.

1 The demand

The demand curve will shift immediately to the position DC_1 and since the rate of monetary expansion will be greater than that of inflation, even though no changes occur in the composition of the monetary base, rate of growth of the stock of real credit will increase, thus shifting the aggregate supply curve to the position SC_1 . The rate of inflation may be slightly higher than, equal to, or even less than the initial rate; the final outcome depends upon the relative shifts in the short-run demand and supply curves. But certainly the first positive impact will be upon the rate of growth of real output. (Point E_1 , Figure 10.)

If the rate of monetary expansion is maintained equal to \hat{M}_1 for some time, the economy will find its new point of equilibrium at E_2^1 where it will be growing at the natural rate with inflation at π_{11} greater than π_0 . If through these increases in demand and supply a higher rate of investment is obtained, then by the end of the process there will be a gain in terms of output growth. If this higher growth of output generates an increase in investment demand (through technological change or through the substitution of physical capital for assets whose rate of return falls with an increase in inflation), the natural rate of growth itself may increase for a time. However, instability in the rate of inflation is a source of uncertainty for the economy and will probably reduce the degree of allocative efficiency so it is impossible in the context of the model presented here to determine the direction in which the rate of capital accumulation and the long-run rate of growth of output will move.

The arguments developed in Section 6 show that if the rate of capital accumulation is not changed, the level of real output at the end of the process will be exactly equal to the one which would have been obtained had the rate of inflation remained stable at π_0 . In this case, the rate of growth will have to be less than the natural rate for a period of time long enough to make real output

/return to

return to the level it would have reached without the inflationary shock. Hence the economy would go through a period of stagflation (inflation and little or no growth). But let us assume that a higher rate of growth has actually been obtained so that by the end of this process the economy is growing at the same rate and has a higher level of income than that which would have existed without the shock, although the rate of inflation will be higher, as indicated by π_{11} (or π_{12}).

The only concrete result of this strategy would be that for some time a greater rate of growth than the natural rate is achieved. It is possible, then, that governments with a myopic vision of the problem will be tempted to follow this strategy, myopic since this larger output represents presumably a smaller level of welfare.

At the point in time when the "positive" part of the expansionary process (accelerated growth) is concluded and only the negative part (a higher rate of inflation) remains, authorities will become interested in reducing the rate of inflation. A stabilization program that minimizes output losses must include: the use of escalator clauses, a policy of price guidelines, a policy of austerity in expenditures, and a corresponding reduction in the rate of monetary expansion.

If correctly implemented, the stabilization plan ought to have very small costs in terms of real output. The rate of inflation would diminish along the long-run supply curve with the rate of growth of the economy being maintained close to n , until the desired rate of inflation is reached. Regretably an adequate policy is very seldom adopted or there is rarely enough patience to wait until the required measures bear fruit; the result is that plans are abandoned before reaching the initial rate of inflation and this is, we think, the origin of the inflationary bias in present processes of inflation.

The discussion in Section 5 showed that the success in using announcement depends upon the degree of credibility that individuals have in the targets announced by the government. When a rate

/of inflation

of inflation π_0 is announced, but actually an acceleration of inflation to a level π_1 is provoked, the government is losing credibility and the possibility of using the gradualist approach later on diminishes. Could it be for this reason and because individuals can now adjust faster to changes in aggregate demand that governments find it more and more difficult to increase the growth rate with shock treatments?

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