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METHODOLOGY FOR THE MEASUREMENT OF CURRENT FERTILITY FROM POPULATION DATA ON YOUNG CHILDREN*

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RESUMEN

En la mayoría de los países las estadísticas vitales son inadecuadas, o no existen. Otros países han creado sistemas de registro, pero sus datos sobre natalidad no están suficientemente clasificados como para permitir el estudio de la fecundidad diferencial. Este artículo describe los métodos para estimar medidas refinadas de la fecundidad y para medir los diferenciales de fecundidad; estas estimaciones son casi tan precisas como las tasas efectivas.

Los métodos utilizan la información censal referente a los hijos propios menores de 5 años de edad que se ha recogido en los Estados Unidos, y que en algunos países puede obtenerse mediante la tabulación de una muestra y que en otros puede recogerse en el futuro. La relación entre los ninos propios menores de 5 años de edad y las mujeres por edad, corregida por la subenumeración censal de la mortalidad de los niños y de las mujeres, y de los niños que no viven con sus madres, constituye una tasa de fecundidad por edad acumulativa de 5 años.

Para derivar las tasas medias anuales de fecundidad por edad de las relaciones corregidas entre los hijos propios menores de 5 años de edad y las mujeres por edad, se desarrollan los multiplicadores de interpolación osculatriz. Se analizan los procedimientos para derivar las tasas bruta y neta de reproducción. Los métodos pueden aplicarse fácilmente a los datos censales de otros países. Para estimar las tasas de fecundidad por edad, se formulan ecuaciones de regresión basadas en los datos de los Estados Unidos. Se explican aplicaciones a otras medidas de la fecundidad. El artículo examina la aplicabilidad y las limitaciones de los métodos y formula algunas recomendaciones.

I. INTRODUCTION

The principal obstacle to the study of the growth component of the world population has been the absence or unreliability of vital statistics in the majority of nations. Many of these nations with poor or nonexistent vital statistics have recently been conducting population censuses, and this census information must be used to measure the growth components of the population in the absence of adequate vital statistics. Even in nations where the quality of vital statistics is good, census records often offer information on fertility differentials by social and economic characteristics of the population that are not available in the vital records.

This paper presents certain long-existent techniques and more recent refinements for deriving a variety of fertility measures from (1) commonly available data on the population by age and sex and (2) less commonly available data on women by number of own children under 5 years old present in the home. It is hoped that the methodology presented here will inspire a fuller exploitation of both types of data and especially that more nations will tabulate data of the latter type in the future.

Information on own children present in the home does not involve any new question on the census schedule but can be obtained by performing a recoding operation on the existing census schedules, as is explained in Section III. Once the number of own children present has been coded for a given woman on the census schedule, the data can be tabulated by other characteristics of the woman and her family that may be available from the census schedule, whereas such cross-classifications are not normally available from data of type (1). The techniques presented here enhance the value of data on own children by permitting the use of a variety of fertility measures for the study of fertility

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differentials by social and economic characteristics of the population.

The measures shown here involve data on either total population under 5 years old or own children of that age present in the home and on women of associated ages, as explained below. It is well known that censuses tend to undercount young children more than women and that the data are affected by mortality among children and women since the birth date of the children. Since not all young children live with their mother, data on own children present in the home are further curtailed as compared with the total population under 5 years old. As will be seen from supplementary data presented in this paper, adjustments for these shortcomings are usually possible and are often minor in effect. Even for countries where certain data are lacking or are of poor quality, it should be possible to establish through high or low adjustments, the range within which the true adjusted values probably fall. For example, an appendix cites some sources of life tables for various countries of the world, from which one may select a table for a country thought to have similar mortality to one under consideration that has no life table of its own. Reference is also made below to the Bogue-Palmore regression equations which contain implicit allowances for mortality and other characteristics and require no life tables at all.

The present article does not deal with longitudinal data for real cohorts of women who may be subject to age-specific birth rates of different magnitude than those prevailing in the five-year birth period here considered as the women pass through life. The relation between trends in period fertility and cohort fertility is complex, and the former are not always indicative of the latter.[1] Nonetheless, the analysis of period or current fertility is valuable in its own right. The article is not intended to discourage the collection of other and often superior information on fertility through special questions which may, for example, obtain the complete fertility history of each woman.

II. RATIOS OF POPULATION UNDER FIVE YEARS OLD TO WOMEN

Associated ages of women.—One crude measure of fertility, often shown in reports of the Bureau of the Census, the United Nations, and many other sources, is the so-called fertility ratio, which some writers refer to as the "child/woman ratio." Typically, this measure is computed by dividing the number of persons under 5 years old in the population by the number of women 15–49 years old or, less often, by the number of women of more restricted age ranges, such as 15–44, 20–44, or 18–44. Perhaps a brief discussion of what is involved in the associated ages for women will lead to a greater consensus on an age range for common use in such ratios and thereby improve the comparability of ratios in various reports. Of course, the choice of an age range for women is sometimes forced by the nature of the available age detail.

The choice by the Bureau of the Census of the 15–49 age range for computing fertility ratios is based largely on material like that shown in Figure 1. It will be observed from the figure that ratios of own children under 5 years old to women are about equal for women in the age ranges 15–19 and 45–49; this pattern implies that an unbalanced age treatment would be involved if the base of the fertility ratio were limited to women 15–44 years old. There are some population groups with appreciably higher fertility at both ends of the age distribution than in the example shown and even some in which women in age group 45–49 have higher fertility in terms of young children than women in age group 15–19, as in data from the 1910 Census. In order to encompass most mothers of children under 5 years old, the use of the range from 15–19 years is preferable to any narrower range.

[1] Norman B. Ryder, "Problems of Trend Determination during a Transition in Fertility," *Milbank Memorial Fund Quarterly*, XXXIV (January, 1956), 5–12.

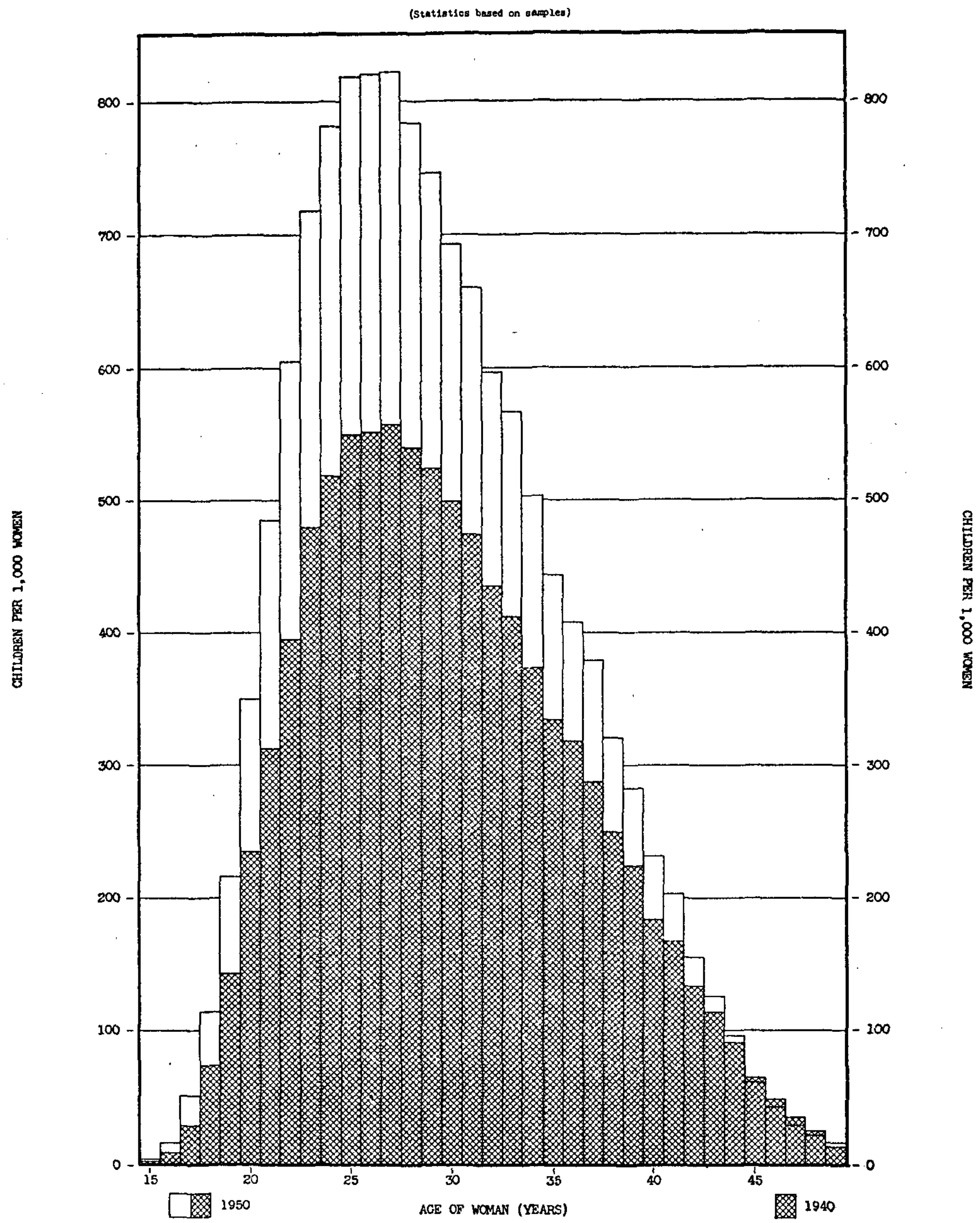

FIG. 1.—Number of own children under 5 years old per 1,000 women 15–49 years old, by single years of age of woman, for the United States—white women, 1950, and native white, 1940.

The pattern in Figure 1 is different in many ways from that which would be shown by age-specific birth rates for any one year. Data on children under 5 years old reflect five-year cumulations of births at successively older ages of women, with the result that the older ages assume increased importance as compared with births for only one year. In data on children under 5 years old, women 15–19 years old have relatively low fertility as compared with those 20–44 years old, whereas in terms of age-specific birth rates for one year, women 15–19 years old sometimes have a rate as high as that for women 25–29 or 30–34 years old and far higher than that for even older women. In data on own children under 5, the peak occurs at a considerably higher age of women than in age-specific birth rates.

Meaning of the fertility ratio.—The ratio of the population under 5 years old to women 15–49 years old is sometimes regarded as a measure of effective fertility or of fertility remaining after the bulk of infant mortality has occurred. As may be seen from mortality data by detailed age, the bulk of deaths in the first five years of life occurs in the first few hours, days, or weeks of life. For example, in the United States in 1960, 79,733 deaths occurred in the first month of life as compared with 6,272 in the next month and 974 in the eleventh month after birth. Most of the population under 5 years old are past the neonatal period and the associated relatively high death rates. At the present time, in the United States, death rates for the population 1–4 years old as a group are no longer higher than for women of childbearing age.

Because the fertility ratio makes use of the total number of women in a broad age range, such as 15–49 years, it may happen that one population has a higher fertility ratio than another population simply because it has relatively more women at the most fertile ages within the age range used and fewer at other ages. If desired, fertility ratios for several populations may be indirectly standardized for age of women to eliminate the effect of varying age distributions.

Replacement quotas for fertility ratios.—Replacement quotas for use with ratios of children under 5 years old to women 15–49 years old can be computed from life tables and data on population by age and sex. These provide a means of taking mortality of women and children and age distributions of women into account and for evaluating the magnitudes of fertility ratios in terms of replacement needs. Some writers simply use the ratio of a life table stationary population under 5 years old to a life table stationary population of women 15–49 years old to obtain a replacement quota. Such a ratio does not allow for the effect of the often very different distribution of women by age in an actual population as compared with that in a life table stationary population, and comparisons of the ratio in an actual population with a replacement quota so computed reflect the effect of the different age distributions and bias the replacement tendencies. An illustrative example of a computation of a replacement quota that allows for the age distribution of women is given in the accompanying tabulation.

According to the above table, 10,654,592 children under 5 years old would be required for replacement of the women shown in column (d) as determined by the application of the ratios in column (c) to the numbers of women shown in column (d). Division of the 10,654,592 "replacement" number of children by the 36,810,103 women 15–49 years old yields the replacement quota of 290 children under 5 per 1,000 women age 15–49 shown on the line for "replacement needs." This is the ratio needed for replacement given the *present* age distribution of women in the United States population. It is sometimes called a "temporary replacement quota," because the quota may vary as the age distribution of the women changes over time. The replacement index of 1.63 shown in column (d) is roughly similar to a net reproduction rate, except that it gives more or less weight than the latter

PROCEDURE FOR COMPUTING REPLACEMENT QUOTA

Age and Sex	1958 Life Table Stationary Population for Whites $_nL_x$ (a)	Own Children under 5 per 1,000 White Women. U.S., 1960 (b)	Column (b) Adjusted to Replacement Magnitudes* (c)	U.S. White Population, 1960 (d)
	Children under 5			
Males				
Unweighted	(458,967)	...	...	...
Sex-weighted†	514,153	...	...	...
Females	489,086	...	...	...
Total under 5	1,003,239	...	...	...
Actual population under 5	...	...	...	17,365,558
Population under 5 needed for replacement of population Σ(c)(d)	...	...	...	10,654,592
	Women			
15–19	485,700	94	59	5,772,421
20–24	484,366	826	515	4,822,381
25–29	482,785	1,070	667	4,839,985
30–34	480,748	726	453	5,379,640
35–39	477,830	416	259	5,708,906
40–44	473,363	171	107	5,298,277
45–49	466,445	38	24	4,988,493
Total, 15–49	3,351,237	...	...	36,810,103
	Children under 5 per 1,000 Women			
15–49				
Actual	...	...	...	472
Replacement needs	...	...	...	290
Replacement index (472 ÷ 290)	...	...	...	1.63

* Adjustment proportion applied to rates in column (b) to obtain those shown in column (c):

$$\frac{\text{Life table population under 5 (see col. a): } 1{,}003{,}239}{\Sigma\text{ (a) (b): } 1{,}608{,}792} = 0.6236 .$$

The figure of 1,608,792 is the number of children under 5 the life table women in column (a) would have if subject to the rates shown in column (b). The factor of 0.6236 applied to rates in column (b) adjusts the ratios to magnitudes needed to yield the life table stationary population under 5 (in this case, 1,003,239 of column [a] or the replacement magnitudes, column [c]).

† Obtained by multiplying the life table unweighted value of 485,967 by 1.058. This adjusts the male life table to a proper level relative to the female life table; among whites, the sex ratio at birth is about 105.8 male births per 100 female births.

Sources of data.—Col. (a): *Vital Statistics of the United States, 1958,* I, 5-4; col. (b): *United States Census of Population, 1960,* PC(1)-1, p. 1-485; col. (d): *United States Census of Population, 1960,* PC(1)-1, p. 1-199.

to various ages of women and it is not strictly interpretable in terms of a synthetic cohort of women passing through life. As will be seen later, it is as easy to compute a net reproduction rate as a replacement index.

III. OWN CHILDREN UNDER FIVE YEARS OLD

Coding operations.—This section is intended mainly to indicate the relatively simple operational principles involved, in the hope that other countries will be encouraged to develop data on own children, which at present seem to exist only for the United States. These data can be obtained merely by performing a coding operation on existing census schedules and do not require the addition of a special question to the schedules. The coding operation takes advantage of the fact that the census enumeration is carried out *by households*, so that family members are grouped together on the schedules. If the schedules contain information on surnames of persons, age, relation to head of household, and marital status, the relationships of the family members to each other are reasonably clear. Consider, for example, the type of entries for persons enumerated in one household as shown in the accompanying tabulation.

Name	Relationship	Age	Marital Status
Brown, James.....	Head	47	Married
Brown, Sarah.....	Wife	44	Married
Brown, Susan.....	Daughter	29	Never married
Brown, Sam......	Son	4	....
Jones, John.......	Son-in-law	28	Married
Jones, Edna......	Daughter	26	Married
Jones, Mary......	Grandchild	3	....
Smith, Eva.......	Lodger	27	Divorced
Smith, William....	Lodger	3	....

In the above example, Sarah Brown evidently has three own children present (Susan, age 29; Sam, age 4; and Edna, age 26), one of whom is under 5 years old. She would therefore be credited with one own child under 5 years old. In the same household, Edna Jones would likewise be credited as having one own child under 5 years old (Mary), as would Eva Smith (William). The bulk of households contains only parents and children and offers few coding problems for coding number of own children under 5 present. In some, there may be persons under 5 years old present who may be disregarded when the mother is obviously not in the household, as when a household contains only grandparents and grandchildren. In a very few, there may be doubt as to which of the two women present is the mother of a child under 5 years old; here, the child probably should be assigned arbitrarily to one woman or the other, with preference given to the woman who is closer to, say, age 24. Even in populations with very high fertility, relatively few (less than 7 percent) married women age 25–29 are likely to have as many as three own children under 5 years old. A coding scheme that provides for 0, 1, 2, 3, and 4 or more (mean value about 4.1) own children under 5 years old per woman probably would be ample.

In countries where customs are such that a fair proportion of unmarried women have children by men not living with them, it may be desirable to code such women by number of their own children present; this probably can be done without much difficulty, as young children generally live with their mother. In the United States Census of 1940, very few single (never married) women could be identified as mothers of children under 5 years old present in the home, and it is thought that most mothers of illegitimate children were reported in the census as having been married. In subsequent censuses, the coding operation was simplified by limiting it to women reported as married, separated, widowed, or divorced, with no discernible effect on the quality of results.

Diversified detail possible.—Once the number of own children under 5 has been determined and inserted on the statistical record for each woman, the data can be tabulated in association with such other characteristics of the woman and her family as are also noted on that record. Tabulations made for the United States include such characteristics of women as age, marital status, race-nativity-parentage, education, and labor force status, as well as family income and occupation of husband.

Proportion of population under 5 years old living with their mother.—Once the total number of own children under 5 years old has been obtained by tabulation, this number can be compared with the total population under 5 years old, to determine the over-all proportion of children of this age counted as living with their mothers. In the United States, in 1950, for example, 98.2 percent of the white population under 5 years old was identified as living with white mothers ever married 15–49 years old. Age-specific ratios of own children under 5 to women of all marital classes combined (including single) can be multiplied by the over-all ratio of the population under 5 to own children under 5, to allow in rough fashion for children not living with their mothers. More refined adjustments may be possible by methods that are beyond the scope of the present article.

IV. COMPUTATION OF GROSS AND NET REPRODUCTION RATES FROM CENSUS DATA WITHOUT THE INTERMEDIATE STEP OF FIRST ESTIMATING AGE-SPECIFIC BIRTH RATES

The present section has much wider application than those that follow, and therefore it is presented first. Many countries have data on population by age and sex, but few have age-specific ratios of children under 5 to women. The techniques presented in the present section are developed in terms of such age-specific ratios but, as will be illustrated in an example, later on, when such age-specific ratios are not available for a given population, use can be made of data adapted from some other population thought to have similar age-patterns of fertility. Sections V and VI assume that age-specific ratios are available for a given population and may not otherwise be applicable.

General theory and development of formulas.—Gross and net reproduction rates are conventionally computed from vital statistics for a single year by the well-known formulas:

$$GRR = \Sigma M_a ,$$

$$NRR = \Sigma M_a \cdot P_a ,$$

where

GRR = the gross reproduction rate;
NRR = the net reproduction rate;
M_a = an age-specific birth rate, or the number of daughters born in the year to women age a, divided by the number of women age a in the population;
P_a = the life table probability of survival from birth to age a, or the probability that daughters will live to their mothers' ages to replace women of that age.

The summation is taken over all child-bearing ages.

The number of daughters born to women age a may be denoted as D_a, and the number of women age a as W_a. Then

$$M_a = \frac{D_a}{W_a}.$$

Using life table probabilities of survival, we would expect a census or survey taken i years after the birth of the daughters to show $D_a \cdot P_i$ surviving daughters age i. Similarly, among the women formerly age a, the number of survivors would be $W_a(P_{a+i}/P_a)$, and they would be age $a + i$. The census or survey taken i years after the birth of the daughters would show the following ratio of surviving daughters to surviving women:

$$R_{a+i} = \frac{D_a \cdot P_i}{W_a \dfrac{P_{a+i}}{P_a}}.$$

Since $M_a = D_a/W_a$, it follows that $R_{a+i} = M_a[(P_i \cdot P_a)/P_{a+i})]$ and $M_a = R_{a+i}[P_{a+i}/(P_i \cdot P_a)]$.

We may now replace the M_a values in the conventional formulas for gross and net reproduction rates with the mathematically identical values $R_{a+i}[P_{a+i}/(P_i \cdot P_a)]$. This yields

$$GRR = \frac{1}{P_i} \Sigma R_{a+i} \frac{P_{a+i}}{P_a},$$

$$NRR = \frac{1}{P_i} \Sigma R_{a+i} \cdot P_{a+i},$$

and the summation is to be taken over all ages for which $R_{a+i} > 0$.

The above formulas involve daughters of a single age, i. They can be adapted for use with ratios of daughters under 5 years old to women by age in a manner which is illustrated in terms of the *NRR*, as the formula is simpler than the one for the *GRR*. For convenience, we shall make minor changes in the symbols used, to introduce age at census and life table stationary population L_x values.

For data involving daughters under one year old, we may adapt the equation for the *NRR* as follows:

$$NRR = \frac{1}{L_0} \Sigma R_A{}^0 \cdot L_A,$$

where L_0 is a female life table stationary population under one year old, expressed on a unit-radix basis. A is the age of women at the census date, $R_A{}^0$ is the ratio of daughters under one year old to women age A; L_A is the female life table stationary population age A, expressed on a unit-radix basis.

The above can be manipulated into $L_0 NRR = \Sigma R_A{}^0 \cdot L_A$. In a general extension of this procedure to daughters of successive ages under 1, 1, 2, 3, and 4, we have

$$L_0 NRR_0 + L_1 NRR_1 + L_2 NRR_2 + L_3 NRR_3 + L_4 NRR_4 = \Sigma R_A{}^{0-4} \cdot L_A,$$

where NRR_0 is the net reproduction rate associated with daughters under one, NRR_1 is the rate associated with daughters age 1, and so on. $R_A{}^{0-4}$ is the ratio of daughters under 5 years old to women age A. When the ratio combining all daughters under 5 is all that is available, instead of separate ratios for daughters of each single year of age, the NRR_0 to NRR_4 values cannot be separately figured. We can, however, obtain the following weighted average $\overline{NRR}$:

$$\overline{NRR} = \frac{L_0 NRR_0 + L_1 NRR_1 + L_2 NRR_2 + L_3 NRR_3 + L_4 NRR_4}{L_0 + L_1 + L_2 + L_3 + L_4} = \frac{1}{\sum_0^4 L_i} \Sigma R_A{}^{0-4} \cdot L_A.$$

The $\overline{NRR}$ so obtained will differ from a straight arithmetic average of the five individual but unknown *NRR* values only insofar as these vary from year to year. Weighted high values of the individual *NRR*'s for some years within the five-year period are more or less balanced by weighted low values for other years. As may be determined by experimentation with life table weights and assumed single-year values of *NRR*'s, the $\overline{NRR}$ usually varies from a straight arithmetic average in only the third or fourth digit. It is thus an acceptable average.

Space limitations prevent a similar demonstration of the development of the formula for the $\overline{GRR}$, which formula is somewhat more complex to derive. The formula is

$$\overline{GRR} = \frac{1}{\sum_0^4 L_i} \Sigma R_A{}^{0-4} \cdot \frac{L_A}{L_{A-2\cdot 5}}$$

In this formula for the $\overline{GRR}$, it is assumed that women age A at the census date were, on the average, two and one-half years younger at the time their daughters under 5 years old were born. That, obviously, is an incorrect assumption for very young women who have mostly daughters under one year old and for women of advanced ages who have mostly daughters four years old; but errors at some ages of women are largely compensated for by errors in the opposite direction at other ages, and over-all the net error in the $\overline{GRR}$, as determined by experimentation, tends to be less than $\frac{1}{2}$ of 1 percent.

When census ratios are in the form of ratios of children of both sexes combined to women of age A, it is possible to apply a sex ratio to the data, to derive ratios of daughters to women. This can be done in any of several ways:

1. Make all computations for the $\overline{NRR}$ and $\overline{GRR}$ in terms of ratios of children of both sexes and apply a sex ratio at birth to the end product. If this is done, use life table survival proportions for both sexes under 5 years old and life table proportions for women. The sex ratio of children at birth tends to be nearly constant for most populations. In the United States, about 105.8 boy babies are born per 100 girl babies in the white population, and among Negroes the ratio runs around 102 to 104 boy babies per 100 girl babies.
2. Determine from population data by age and sex the ratio of female children under 5 years old to children of both sexes and then apply that ratio to the census child/woman ratios. If this is done, use life table values for females under 5, as well as for women.
3. For a third procedure, similar in principle to method 2, see the accompanying illustrative computation. If age-specific ratios of *own* children to women for a *given* population (instead of similar ratios taken from an "other" population) are used, an adjustment operation similar to the one in the illustrative computation not only allows for the sex ratio but also for children not living with their mothers.

V. DERIVATION OF AVERAGE ANNUAL AGE-SPECIFIC BIRTH RATES FROM AGE-SPECIFIC RATIOS OF YOUNG CHILDREN TO WOMEN

With the aid of life table survival proportions, age-specific ratios of children under 5 years old to women can be adjusted to "restore" deaths among the children and the women since the birth date of the children. (See the intermediate formulas in Section IV.) The data then become the equivalent of birth rates cumulated over a five-year period of time, by age of women at the end of this five-year period. Various procedures are possible for manipulating the cumulated birth rates in order to derive average annual birth rates by age of woman at child births. This section presents two procedures, one based on a special adaptation of Sprague's fifth difference osculatory interpolation formula and the other based on regression techniques.

Adaptation of Sprague's formula.—In terms of ordinates rather than age groups, Sprague's fifth difference osculatory interpolation formula is

$$\begin{aligned} Y_z = Y_{n+x+2} = Y_n &+ \frac{x+2}{1!}\Delta Y_n \\ &+ \frac{(x+2)(x+1)}{2!}\Delta^2 Y_n \\ &+ \frac{(x+2)(x+1)x}{3!}\Delta^3 Y_n \\ &+ \frac{(x+2)(x+1)x(x-1)}{4!}\Delta^4 Y_n \\ &+ \frac{x^3(x-1)(5x-7)}{4!}\Delta^5 Y_n, \end{aligned}$$

where $z = n + 2 + x$, the symbol n denotes any integral number, including 0, and x is any fraction less than unity. This is the osculatory formula for writing values of the function which lie between the integral values $n + 2$ and $n + 3$ in terms of Y_n and its five leading integral differences.

An Illustrative Computation, Making Use of Age-Specific Ratios Taken from an "Other" Population than the One for Which the Computation of Reproduction Rates Was Done

Step 1.—Adaptation of Ratios Taken from an "Other" Population, To Obtain Estimates of Such Ratios for a Given Population

Age	Given population: white females, Ohio, 1960 (a)	"Other" population: own children under 5 per 1,000 women, Illinois, 1960[1] (b)	Estimated females under 5 per 1,000 white women, Ohio, 1960 (c)= (b) adjustment factor[2]
Under 5.....	501,054		
15-19.......	319,372	93	47
20-24.......	278,629	821	413
25-29.......	273,347	1,080	543
30-34.......	307,400	757	380
35-39.......	325,096	443	223
40-44.......	296,436	180	90
45-49.......	268,255	49	25

[1] Ratios shown in column (b) involve own children of both sexes and thus are about half the magnitude that would be expected if they involved only daughters.

[2] Adjustment factor= $\frac{\text{White females under 5 in Ohio: } 501{,}054}{\Sigma(a)(b)\text{: } 996{,}893} = .5026$

(The figure of 996,893 is the number of children (both sexes) under 5 years old, women in Ohio would have if they were subject to the ratios shown in column (b).)

Step 2.—Computation of GRR and NRR for the White Population of Ohio, 1955-60

Age	Estimated females under 5 per 1,000 white women, Ohio, 1960 [1] (a)	Life table survival proportions, multiplied by 5[2]: $\frac{5}{\sum_0^4 L_i} \frac{L_A}{L_{A-2\frac{1}{2}}}$ (b)	Life table survival proportions, multiplied by 5[2]: $\frac{5}{\sum_0^4 L_i} \cdot L_A$ (c)
15-19.....	47	1.0213	.9931
20-24.....	413	1.0209	.9903
25-29.....	543	1.0207	.9870
30-34.....	380	1.0202	.9829
35-39.....	223	1.0192	.9770
40-44.....	90	1.0175	.9678
45-49.....	25	1.0147	.9537

$\overline{GRR} = \Sigma(a)(b) = 1{,}756$

$\overline{NRR} = \Sigma(a)(c) = 1{,}694$

For comparison with the above results, the $\overline{GRR}$ computed from actual vital statistics for Ohio is 1,800.

[1] Taken from last column of table in Step 1.

[2] Multiplication by 5 was done in order to adjust for the class interval used (5-year age groups instead of single years of age).

By conventional procedures, beyond the scope of the present article, the formula can be adapted to obtain coefficients or multipliers for application to data for five-year age groups or other equally spaced groups, to subdivide the central group of five such groups into fifths or tenths or any other desired fractions. The same thing can be done with many other formulas, but the one employed works reasonably well.

The Sprague equation is actually based on *two* overlapping curves—one beginning sooner and ending sooner than the other—which are forced to have certain characteristics in common (tangent and radius of curvature) at the beginning and end of the middle interval within which the interpolation is to be made. In this fashion, the Sprague equation forces interpolations made for successive "frames" to join smoothly, so that, for example, an estimate of population age 29 from application of the formula to age groups 15–19, 20–24, 25–29, 30–34, and 35–39 will be followed by a smooth progression to an estimate of population age 30 from age groups 20–24, 25–29, 30–34, 35–39, and 40–44. Without that overlapping feature, from the implicit use of two curves, estimates made from successive "frames" might exhibit jerky or unrealistic patterns of trend when passing from one central age group to another. The Sprague equation also preserves group totals, so that estimates of the population by single years of age always add up to the given five-year group totals. The equation does have some disadvantages, though, as it is extremely flexible and in some applications, not generally involved here, may yield some minor but unrealistic "wobbles" in pattern.

The following material indicates the general principles on which the interpolation coefficients presented in Table 1 are based. The development involves several stages of interpolation, but in Table 1 these stages are consolidated into one operation, with no need to actually perform the stages separately.

1. As a first step, ratios of children under 5 years old to women in each five-year old group are multiplied by 5, so that the results will approximate the five-year age group sums of the (unknown) ratios for women in single years of age.

2. The usual Sprague multipliers for subdivision of a central five-year age group of five successive five-year age groups into single years of age are applied to the results of the first step. This yields estimates of the ratios of children under 5 years old to women by single years of age. Since the multipliers in this application are suitable only for subdividing the middle five-year age group (in this case, 15–19), the sum of the ratios is assumed to be zero for age groups 5–9 and 19–14, in order to pull out the ratios for ages 15, 16, 17, 18, and 19 from data for age groups 5–9, 10–14, 15–19, 20–24, and 25–29. Similarly, the sums of single year of age ratios for the age groups 50–54 and 55–59 are assumed to be zero for the purpose of pulling out ratios for ages 45, 46, 47, 48, and 49.

3. As a third step, rearrange the single year of age ratios obtained from Step 2 into five sets, as follows:

Set	Ages						
1..........	15	20	25	30	35	40	45
2..........	16	21	26	31	36	41	46
3..........	17	22	27	32	37	42	47
4..........	18	23	28	33	38	43	48
5..........	19	24	29	34	49	44	49

The above data are in terms of ages of women at last birthday; women age 15, for example, are anywhere between exact age 15.00 and 15.99. In what follows, cited ages will still represent years of age but are sometimes measured from a fractional beginning age. Age $18\frac{1}{2}$, for example, will mean a year of age in the range from 18.50 to 19.49.

Each of the above five sets, taken singly, reflects unduplicated ages of mothers at the births of their children during the past five years. For example, in the first set, the women age 20 at a census date were on the average half a year younger, or age $19\frac{1}{2}$, at the time their children under one year old were born, age $18\frac{1}{2}$ when their children age 1 were born, and so on. Thus, the women age 20 at the census date were of successive average maternal ages $14\frac{1}{2}$–$19\frac{1}{2}$ as they passed through the five years ending on the census date, and their cumulated fertility at the end of this period is the equivalent of a cumulated age-specific birth rate for women ages $14\frac{1}{2}$–$19\frac{1}{2}$ at the birth of the children, with each age counted only once. This cumulated rate can be divided by 5 to obtain the average birth rate for women of maternal age $14\frac{1}{2}$–$19\frac{1}{2}$ as a group.

Similarly, in the first set, the women age 25 at census reflect the birth experience of women at ages $20\frac{1}{2}$–$24\frac{1}{2}$; those age 30 reflect experience at age $25\frac{1}{2}$–$29\frac{1}{2}$, and so on. Analogous principles apply to the other sets. As another example, in the fourth set the data for women age 18, 23, 28, etc., at the census date reflect the birth experience of women at prior ages $13\frac{1}{2}$–$17\frac{1}{2}$, $18\frac{1}{2}$–$22\frac{1}{2}$, $23\frac{1}{2}$–$27\frac{1}{2}$, and so forth.

4. Again using the Sprague formula, it is possible to interpolate and recombine data from Step 3 for each set separately, so as to obtain ratios for women who were of standard five-year age intervals (15–19, 20–24, etc.) at time of birth of their children. Each of the five sets then yield estimates of birth rates by standard five-year age intervals of women.

5. The five sets of age-specific birth rates obtained from Step 4 differ from one another in respect to when a given maternal age such as age 15 was experienced during the five-year birth period. That age was achieved earlier within the five-year period by the women in some of the five sets than those in other sets. By adding together the results of the five sets and taking an average, we obtain average annual age-specific birth rates that reflect the average experience of women at a given age during all of the five years preceding the census date, instead of the experience at a particular time within the five-year period that would be implied by the use of any one set taken alone.

The coefficients presented in Table 1 are based on the various steps outlined above and were obtained by applying those for each successive step to those that went before. The derivation was in terms of R_{15-19}, R_{20-24}, as independent variables. R_{15-19}, R_{20-24}, etc., are age-specific ratios of children under 5 years old to women of five-year age groups at census as indicated by the subscripts, adjusted for mortality among women and children since the birth date of the children.

When used in the manner just described (Table 1), the Sprague fifth difference formula involves the implicit assumption that trends in ratios of young children to women in successive five-year age groups are indicative of trends in fertility at component single years of age *within* each five-year age group. Real populations are affected by characteristic patterns of age at marriage and other factors that create somewhat different trends within the five-year age groups, especially at ages under 20. In terms of comparisons of average-annual age-specific birth rates with actual age-specific birth rates for whites in various states (Tables 3 and 4), the Sprague formula yields results that generally deviate by much less than 10 percent from the actual birth rates for women in the age range from 20 to 39 years, but it does less well for those in age group 15–19, 40–44, and 45–49. Even at these beginning and end age groups, the results may be considered useful for many purposes.

Some regression equations.—Coefficients for estimating average annual age-specific birth rates from census data on ratios of young children to women may also be derived by fitting regression equations to actual age-specific birth rates for various states, with census age-specific ratios for the same states used as independent variables. Actual age-specific birth rates reflect the effect of variations in age patterns at marriage and the many other factors which affect fertility. Regression equations fitted to age-specific birth rates themselves take better indirect account of such factors affecting fertility than does the Sprague equation. Hence, the regression equations so obtained are theoretically capable of providing more nearly accurate estimates of average-annual age-specific birth rates than those from the Sprague formula. Regression equations have been developed from data for whites in various states in the period of 1955–60. These equations also work well with data for whites in the 1940 and 1950 Censuses, as will be seen, but their usefulness for application to data for nonwhites or for countries having markedly different age patterns of fertility than among whites in the United States has not been tested. Similar regression equations might be developed for use with data for nonwhites, following or improving on principles outlined by the present pilot study.

Table 2 presents the results of various regression equations fitted to actual age-specific birth rates for whites in forty-eight states in the five-year period ending on the 1960 Census date (April 1, 1960), with adjusted age-specific ratios of own children under 5 years old to white women used as independent variables.

The age-specific birth rates used in the derivation of the regression equations were based on official vital statistics adjusted for underregistration of births but are affected by possible errors in the intercensal estimates of women by age used for the population bases of those rates. Ratios of own children under 5 years old to white women by age from 1960 Census data for each state were adjusted for mortality among children and women since the birth-date of the children with the aid of United States life tables for 1958 and were further adjusted for population under 5 not living with their mothers by multiplying the ratio of total white population under 5 years old in each state to those counted in the census as living with

Table 1.—SPRAGUE'S OSCULATORY INTERPOLATION EQUATION—COEFFICIENTS FOR ESTIMATING AVERAGE-ANNUAL AGE-SPECIFIC BIRTH RATES FROM CENSUS-BASED ESTIMATES OF TOTAL BIRTH RATES IN FIVE-YEAR PERIODS BY AGE OF WOMEN AT END OF PERIOD

Birth rate	R_{15-19}	R_{20-24}	R_{25-29}	R_{30-34}	R_{35-39}	R_{40-44}	R_{45-49}
B_{15-19}	.1378	.1109	-.0208	.0020	.0002	-.0001	...
B_{20-24}	-.0365	.1378	.1109	-.0208	.0020	.0002	-.0001
B_{25-29}	.0073	-.0365	.1378	.1109	-.0208	.0020	.0002
B_{30-34}	-.0008	.0073	-.0365	.1378	.1109	-.0208	.0020
B_{35-39}	.0001	-.0008	.0073	-.0365	.1378	.1109	-.0208
B_{40-44}	...	.0001	-.0008	.0073	-.0365	.1378	.1109
B_{45-49}	...	...	.0001	-.0008	.0073	-.0365	.1378

R_{15-19}, R_{20-24}, etc. in above table are age-specific ratios of children under 5 to 1,000 women of the 5-year age group indicated by the subscripts, adjusted for mortality of children and women since birthdate of children, for proportion of population under 5 not living with mother, and for undercount of women and children in census, so that results are total births per 1,000 women in 5-year period by age of woman at end of period.

B_{15-19}, B_{20-24}, etc. are average annual birth rates per 1,000 women by age at birth of the children as indicated by the subscripts.

The coefficients are used in the manner illustrated by the following example:

$$B_{15-19} = .1378\ R_{15-19} + .1109\ R_{20-24} - .0208\ R_{25-29} + .0020\ R_{30-34} + .0002\ R_{35-39} - .0001\ R_{40-45}$$

If a given population has
$R_{15-19} = 100$
$R_{20-24} = 878$
$R_{25-29} = 1138$
$R_{30-34} = 772$
$R_{35-39} = 443$
$R_{40-44} = 182$

the equation yields $B_{15-19} = .1378(100) + .1109(878) - .0209(1138) + .0020(772) + .0002(443) - .0001(182) = 89$

mothers 15–49 years old. The 1960 Census ratios were not adjusted for undercounting of white children under 5 years old and women 15–49 years old. Available evidence indicates that, in 1960, roughly equal proportions (slightly less than 2 percent) of the children and women were not counted, so the adjustment for undercounting would have little effect on the ratios.[2]

The forty-eight states mentioned above include all states except Alaska and Hawaii. The District of Columbia was also excluded. In Table 2, the "All States Equations" relate to those fitted to data for all forty-eight states. In an experiment to determine whether or not net migration of population from one state to another between the birth dates of children under 5 and a census date might be a feature worth taking into account, regression equations were also fitted to data for fourteen states which were determined to have relatively little net migration of white women between 1950 and 1960 ("Stable State Equations"), to data for twenty states which had relatively heavy net outmigration ("Out-Migration State Estimates"), and to data for fourteen states which had relatively heavy net in-migration. The general theory behind this experiment is that migration may be somewhat selective of persons of different average fertility than those resident in an area at the time their children under 5 years old were born. Also, this experiment takes some account of varying social and economic conditions, because states with considerable out-migration of population tend to be those which offer relatively fewer economic opportunities than others, and those which attract many in-migrants tend to be economically better off. Fertility tends to be relatively high in the states with limited economic opportunities and relatively low in those with many opportunities.

As may be seen from Table 2, the corresponding coefficients of the regression equations fitted to data for the several groups of states vary considerably from each other, as well as from those for the "All States Equations" (which in-

[2] U.S. Bureau of the Census, *1960 Census of Population*. Vol. I: *United States Summary*, Table U, p. xxxix.

Table 2.—Regression Equation—Coefficients for Estimating Average-Annual Age-specific Birth Rates from Census-based Estimates of Total Birth Rates in Five-Year Periods by Age of Women at End of Period

Birth rate	Constants	R_{15-19}	R_{20-24}	R_{25-29}	R_{30-34}	R_{35-39}	R_{40-44}	R_{45-49}
	"All States" Equations							
B_{15-19}	-.0075	.5524	.0390	...	...	...	...	...
B_{20-24}	-.0064	.1452	.0938	.1358	...	...	...	...
B_{25-29}	...	-.0322	.0154	-.0030	.3459	-.1843	...	...
B_{30-34}	...	...	-.0131	-.0169	.2005	-.0998	.1766	...
B_{35-39}	...	...	...	-.0444	.0894	.0312	.1102	.1100
B_{40-44}	-.0054	...	...	...	...	...	.1006	.0521
B_{45-49}	-.0004	...	...	...	...	...	.0026	.0241
	"Stable States" Equations							
B_{15-19}	-.0039	.5374	.0364	...	...	...	...	...
B_{20-24}	...	.1222	.1023	.1205	.0116	...	...	...
B_{25-29}	...	-.0592	.0611	-.1283	.6122	-.4104	...	...
B_{30-34}	...	...	-.0004	-.0443	.2048	-.0458	.1410	...
B_{35-39}	...	...	...	...	-.0341	.0456	.4917	-.6782
B_{40-44}	-.0064	...	...	...	...	...	.1235	-.0243
B_{45-49}	-.0007	...	...	...	...	...	.0048	.0227

TABLE 2—*Continued*

Birth rate	Constants	R_{15-19}	R_{20-24}	R_{25-29}	R_{30-34}	R_{35-39}	R_{40-44}	R_{45-49}
	"In-Migration States" Equations							
B_{15-19}	-.0345	.3874	.0892	...	...	...	...	...
B_{20-24}	...	.5810	-.1282	.5967	-.4873	...	...	...
B_{25-29}	...	-.3671	.1484	-.1321	.4749	-.2562	...	...
B_{30-34}	...	...	.0398	-.0959	.0829	.5174	-.5828	...
B_{35-39}	...	...	...	-.0092	.0056	.1199	.0233	.1346
B_{40-44}	...	...	...	...	...	-.0016	.0863	-.0003
B_{45-49}	...	...	...	...	...	.0053	-.0156	.0379
	"Out-Migration States" Equations							
B_{15-19}	-.0016	.5746	.0296	...	...	...	...	...
B_{20-24}	...	-.2896	.2581	-.2047	.3581	...	...	...
B_{25-29}	...	-.0424	.0050	-.0009	.3666	-.2081	...	...
B_{30-34}	...	...	-.0172	-.0025	.1602	-.0147	.0694	...
B_{35-39}	...	...	...	-.0309	.0314	.0701	.1966	...
B_{40-44}	-.0030	...	...	...	...	...	.0660	.1485
B_{45-49}	-.0014	...	...	...	...	...	.0045	.0368

R_{15-19}, R_{20-24}, etc. and B_{15-19}, B_{20-24}, etc. are defined in the same way as in the note to Table 1.
Example of how the coefficients are used in equations:

$$B_{15-19} = -.0075 + .5524\, R_{15-19} + .0390\, R_{20-24}$$

$$B_{20-24} = .0064 + .1452\, R_{15-19} + .0938\, R_{20-24} + .1358\, R_{25-29}$$

volved fitting to data for all forty-eight states used in the study). However, the real test of the usefulness of the various procedures comes when the equations are actually used to estimate age-specific birth rates from census data on (adjusted) age-specific ratios of children under 5 years old to women, and the results are compared with one another and with actual age-specific birth rates computed from vital statistics.

Comparison of results of application of the various methods.—Tables 3, 4, and 5 compare the results of various methods with actual annual average age-specific birth rates for the white population. The dates shown are for the period in which the children under 5 were born. The same R_{15-19}, R_{20-24}, etc., values were used for all methods.

In examining the data in Table 3, it should be kept in mind that the regression equations involved were fitted to data for individual states, not the nation as a whole, with no state given more "weight" than another. Thus, for example, Delaware with its small population contributed as much to the derivation as New York with its large population.

For 1955–60, it appears that the estimates obtained for the nation (see Table 3) by the use of the "All States Equations" shown in Table 2 and those obtained by the use of the "Stable States Equations," also shown in Table 2, gave results of approximately equal reliability for the period 1955–60, while those obtained by the use of the osculatory interpolation equations shown in Table 1 give results that were of somewhat less reliability than those from the other two methods.

The regression equations, developed from data for the period 1955–60, might be expected, *a priori*, to work less well when applied to data for earlier dates than when applied to data from the 1960 Census, because of changes that have occurred in age-at-marriage patterns of the population and in other factors affecting fertility.

Actually, they appear to give results of acceptable quality when applied to national data from the censuses of 1950 and 1940. No one method consistently gives better results at all age groups than the other methods when applied to data from the censuses of 1950 and 1940.

It will be recalled that the corresponding coefficients given in Table 2 differ widely among regression equations developed for all states from those for stable states, in-migration states, and out-migration states. Despite the widely differing coefficients, applications of the several formulas each tend to yield useful results (see Table 4). As forced, there is some improvement in results when regression equations for in-migration states are applied to states of that character, but the improvement is not great as compared with the application of the "All States Equations" to the in-migration states. Similar findings of relatively little improvement for stable states and for out-migration states may be noted when specially tailored regression equations are used instead of the "All States Equations." This, we believe, indicates in a general way that equations for "All States" may work reasonably well for application to data by education of women or other social and economic detail, for which tests against actual birth rates are not presently possible. Table 5 presents an even more severe test, when the regression equations developed from 1955–60 data, a period of relatively high fertility, are applied to state data from the

Table 3.—ACTUAL AND ESTIMATED AVERAGE ANNUAL NUMBER OF BIRTHS PER 1,000 WHITE WOMEN BY AGE, AND TOTAL FERTILITY RATES FOR THE UNITED STATES: 1935–40, 1945–50, AND 1955–60

Subject	Age of woman (years)							Total fertility rate[1]
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	
1955-1960								
Actual birth rates	82	246	194	113	57	15	1	3540
Birth rates estimated by:								
1. "All States" regression	82	244	192	112	56	15	1	3510
2. "Stable States" regression	82	247	192	112	56	15	1	3525
3. Osculatory interpolation	89	228	202	116	59	18	2	3570
Deviation from actual rates:								
1. "All States" regression	0	- 2	- 2	- 1	- 1	0	0	- 30
2. "Stable States" regression	0	1	- 2	- 1	- 1	0	0	- 15
3. Osculatory interpolation	7	-18	8	3	2	3	1	30
1945-1950								
Actual birth rates	63	185	163	106	56	15	1	2945
Birth rates estimated by:								
1. "All States" regression	68	186	159	101	57	15	1	2935
2. "Stable States" regression	69	190	163	104	59	15	1	3005
3. Osculatory interpolation	68	170	163	105	56	19	1	2910
Deviation from actual rates:								
1. "All States" regression	5	1	- 4	- 5	1	0	0	- 10
2. "Stable States" regression	6	5	0	- 2	3	0	0	60
3. Osculatory interpolation	5	-15	0	- 1	0	4	0	- 35
1935-1940								
Actual birth rates	45	124	117	80	48	19*	...	2165
Birth rates estimated by:								
1. "All States" regression	42	127	117	80	50	14*	...	2150
2. "Stable States" regression	43	132	119	82	44	13*	...	2165
3. Osculatory Interpolation	45	117	119	82	46	21*	...	2150
Deviation from actual rates:								
1. "All States" regression	-3	3	0	0	2	-5	...	- 15
2. "Stable States" regression	-2	8	- 2	2	-4	-6	...	0
3. Osculatory interpolation	0	- 7	2	2	-2	2	...	- 15

*Birth of women age 45 and over are included.

[1] Sum of age-specific birth rates for 5-year age groups, multiplied by 5.

1940 Census. The period 1935–40 was affected by the severe economic depression of the 1930's, which was characterized by some postponement of marriage to an older average age and by very low birth rates. The states shown are of various types, with respect to migration patterns and relative levels of birth rates. The results speak for themselves.

The Bogue-Palmore method.—Donald J. Bogue and James A. Palmore recently studied the inter-relationships between various measures of fertility computed from vital statistics and from census data.[3] Taking data for forty-nine countries which have good vital statistics and census data as their sample, they found that various measures of fertility computed from vital statistics—such as the crude birth rate, age-specific birth rates, reproduc-

[3] Donald J. Bogue and James A. Palmore, "Some Empirical and Analytic Relations among Demographic Fertility Measures, with Regression Models for Fertility Estimation," in *Demography*, I, No. 1, 1964, 316–38.

Table 4.—ACTUAL AND ESTIMATED AGE-SPECIFIC BIRTH RATES PER 1,000 WHITE WOMEN, AVERAGED FOR ALL STATES, STABLE STATES, IN-MIGRATION STATES, AND OUT-MIGRATION STATES: 1955–60

Subject	Age of woman (years)							Total fertility rate
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	
All States								
Actual birth rates	87	255	193	113	58	16	1	3617
Birth rates estimated by:								
1. "All States" regression	87	255	193	113	58	16	1	3621
2. Osculatory interpolation	95	238	205	117	62	20	2	3696
Average deviation, within States[1]								
1. "All States" regression	3	7	5	3	2	1	0	68
2. Osculatory interpolation	9	17	13	5	4	4	1	104
Fourteen "Stable States"								
Actual birth rates	87	264	198	116	60	17	1	3715
Birth rates estimated by:								
1. "All States" regression	88	261	199	116	59	17	1	3703
2. "Stable States" regression	87	264	198	116	59	16	1	3713
3. Osculatory interpolation	97	244	210	120	63	20	2	3782
Average deviation, within States[1]								
1. "All States" regression	3	5	4	2	1	1	0	48
2. "Stable States" regression	3	4	6	2	2	1	0	50
3. Osculatory interpolation	11	19	12	4	3	4	1	68
Fourteen "In-migration" States								
Actual birth rates	93	259	197	112	56	15	1	3662
Birth rates estimated by:								
1. "All States" regression	92	253	192	111	56	15	1	3597
2. "In-migration States" regression	93	259	197	111	56	15	1	3660
3. Osculatory interpolation	97	236	202	115	59	18	2	3646
Average deviation, within States[1]								
1. "All States" regression	4	7	6	3	2	1	0	85
2. "In-migration States" regression	3	5	5	2	1	1	0	43
3. Osculatory interpolation	6	23	8	4	3	3	1	68
Twenty "Out-migration" States								
Actual birth rates	83	245	187	112	59	17	1	3516
Birth rates estimated by:								
1. "All States" regression	84	251	190	113	59	17	1	3581
2. "Out-migration States" regression	83	245	187	113	59	17	1	3528
3. Osculatory interpolation	93	236	203	117	63	21	2	3671
Average deviation, within States[1]								
1. "All States" regression	3	8	3	3	1	1	0	71
2. "Out-migration States" regression	3	7	3	2	2	1	0	55
3. Osculatory interpolation	11	11	16	5	4	4	1	154

[1] Deviation of estimated birth rate from actual birth rate for each State, summed regardless of sign and averaged for all States in each group. This is the average gross deviation; to obtain the average net deviation, subtract the estimated birth rates shown above from the actual birth rates.

tion rates, and so forth—were highly correlated with census data on the ratio of population under 5 and 5–9 to women ages 15–49 and the percent of population under 5 and under 14. Using statistics on the infant mortality rate and on the proportion of women ever married by age, they found that they could raise these correlations even higher. They generated a system of multiple-regression equations for the computation of age-specific birth rates and other measures of fertility from the commonly available types of census data and infant mortality rates. Their system relies heavily on correlation of one type of fertility measure with another to derive the various measures. It does not involve the use of life tables but rather makes implicit allowance for mortality, as infant mortality rates tend to be corre-

Table 5.—ACTUAL AND ESTIMATED AVERAGE ANNUAL AGE-SPECIFIC BIRTH RATES PER 1,000 WHITE WOMEN, FOR SPECIFIED STATES: 1935–40

Age of women and state	Actual birth rate	Method of estimation			
		All States regression		Osculatory interpolation	
		Estimated rate	Deviation from actual birth rate	Estimated rate	Deviation from actual birth rate
15 to 19 years					
Alabama	73	80	7	72	-1
California	44	46	2	48	4
Delaware	43	33	-10	42	-1
Florida	62	66	4	62	0
Illinois	32	28	-4	33	1
Kansas	41	31	-10	37	-4
Massachusetts	19	13	-6	22	3
Nebraska	35	29	-6	38	3
New York	20	16	-4	23	3
Oklahoma	73	76	3	66	-7
Pennsylvania	35	29	-6	36	1
South Carolina	66	71	5	61	-5
Tennessee	73	71	-2	66	-7
Wisconsin	31	25	-6	36	5
20 to 24 years					
Alabama	163	167	4	150	-13
California	118	118	0	111	-7
Delaware	113	120	7	113	0
Florida	132	144	12	133	1
Illinois	104	108	4	100	-4
Kansas	127	129	2	119	-8
Massachusetts	90	91	1	84	-6
Nebraska	127	127	0	118	-9
New York	88	88	0	81	-7
Oklahoma	156	163	7	147	-9
Pennsylvania	115	116	1	107	-8
South Carolina	154	162	8	144	-10
Tennessee	162	162	0	148	-14
Wisconsin	128	130	2	121	-7
25 to 29 years					
Alabama	139	146	7	140	1
California	98	97	-1	98	0
Delaware	108	117	9	113	5
Florida	105	101	-4	110	5
Illinois	101	110	9	111	10
Kansas	117	129	12	132	15
Massachusetts	107	111	4	109	2
Nebraska	124	120	-4	128	4
New York	101	104	4	101	0
Oklahoma	122	114	-8	129	7
Pennsylvania	111	119	8	119	8
South Carolina	135	136	1	143	8
Tennessee	139	123	-16	133	-6
Wisconsin	128	139	11	138	10

TABLE 5—*Continued*

Age of women and state	Actual birth rate	Method of estimation: All States regression: Estimated rate	All States regression: Deviation from actual birth rate	Osculatory interpolation: Estimated rate	Osculatory interpolation: Deviation from actual birth rate
30 to 34 years					
Alabama...........	102	111	9	109	7
California........	57	58	1	61	4
Delaware..........	71	76	5	75	4
Florida...........	67	64	-3	72	5
Illinois..........	67	72	5	74	7
Kansas............	80	89	9	83	3
Massachusetts.....	79	80	1	80	1
Nebraska..........	84	85	1	91	7
New York..........	68	69	1	70	2
Oklahoma..........	81	79	-2	82	1
Pennsylvania......	77	83	6	82	5
South Carolina....	97	100	3	97	0
Tennessee.........	99	90	-9	97	-2
Wisconsin.........	90	97	7	93	3
35 to 39 years					
Alabama...........	73	77	4	68	-5
California........	29	32	3	31	2
Delaware..........	39	44	5	39	0
Florida...........	40	39	-1	42	2
Illinois..........	36	41	5	39	3
Kansas............	49	51	2	46	-3
Massachusetts.....	43	50	7	44	1
Nebraska..........	51	54	3	55	4
New York..........	33	40	7	34	1
Oklahoma..........	53	51	-2	53	0
Pennsylvania......	46	51	5	46	0
South Carolina....	70	66	-4	60	-10
Tennessee.........	70	62	-8	63	-7
Wisconsin.........	54	59	5	51	-3
40 to 44 years					
Alabama...........	27	27	0	34	7
California........	8	6	-2	10	2
Delaware..........	14	10	-4	15	1
Florida...........	12	11	-1	14	2
Illinois..........	11	8	-3	13	2
Kansas............	18	15	-3	21	3
Massachusetts.....	13	12	-1	17	4
Nebraska..........	18	15	-3	19	1
New York..........	10	8	-2	12	2
Oklahoma..........	20	18	-2	22	2
Pennsylvania......	17	14	-3	19	2
South Carolina....	25	24	-1	30	5
Tennessee.........	27	21	-6	26	-1
Wisconsin.........	20	18	-2	23	3

lated with mortality at other ages. Age patterns of fertility also come from correlations, instead of being figured either directly from census data on own children (which are not used) or from modifications of data for some other population thought to have similar age patterns of fertility as the population under consideration. Cho has made use of their procedures to derive fertility measures for many countries of the World.[4] While the short-cut methods given by Bogue and Palmore are useful for many purposes, we have found by subsequent experimentation that they tend to be considerably less reliable in application to data for the United States than the methods outlined in the present article, which involve the direct imputation of mortality via the use of life tables

[4] Lee Jay Cho, "Estimated Refined Measures of Fertility for All Major Countries of the World," *Demography*, I, No. 1, 1964, 359–74.

TABLE 5—*Continued*

Age of women and state	Actual birth rate	Method of estimation: All States regression — Estimated rate	All States regression — Deviation from actual birth rate	Osculatory interpolation — Estimated rate	Osculatory interpolation — Deviation from actual birth rate
45 to 49 years					
Alabama	3	2	-1	6	3
California	1	0	-1	2	1
Delaware	1	1	0	1	0
Florida	1	1	0	3	2
Illinois	1	1	0	1	0
Kansas	2	1	-1	2	0
Massachusetts	1	1	0	1	0
Nebraska	2	1	-1	2	0
New York	1	1	0	2	1
Oklahoma	2	2	0	4	2
Pennsylvania	2	1	-1	3	1
South Carolina	3	2	-1	6	3
Tennessee	3	2	-1	5	2
Wisconsin	2	2	0	4	2

and the direct imputation of age-patterns of fertility either by computations from age-specific ratios of young children to women or by adaptation of data taken from some other population that is assumed to have an age pattern of fertility that is similar to the one under consideration.

Derivation of average annual age-specific birth rates for women ever married or for women married and husband present at a census date.—As noted in Section III of this article, data on ratios of own children under 5 to women may be specific not only for age of women but also for their marital status and for other characteristics of the women and their families. The interpolation coefficients given in Tables 1 and 2 were designed for use with data on women of all marital classes combined, including single women. They assume that women of each single year of age within a five-year age group contribute in roughly equal proportions to the fertility ratio for the five-year age group. Obviously, this is not the case for data involving women ever married. For example, the 1960 Census shows the following numbers of white women ever married at selected single years of age for the United States:

Age	White Women Ever Married
15	26,917
17	149,343
19	413,306
21	651,460
23	766,972

We have experimented with various weighting procedures to allow for the effect of continuing accessions to the population of women ever married for successive ages within each five-year age group. Work done thus far indicates that it is possible to develop indices that allow for these accessions. The R_{15-19}, R_{20-24}, etc., values for women ever married can be modified by the use of these indices, to adapt them for use with coefficients of the type given in Table 1, with results of useful quality. Space limitations of the present article prevent an exposition in detail.

VI. EXTENSION TO OTHER MEASURES OF FERTILITY

Section II of this article discussed the fertility ratio (ratio of population under 5 to women ages 15–49) and the computation of a matching replacement quota for use with that ratio. An explanation of how to compute gross and net reproduction rates from census data without the intermediate step of first computing age-specific birth rates was given in Section III. Procedures were also given for the computation of average annual age-specific birth rates from age-specific ratios of own children under 5 to women in Section IV. The "total fertility rate" was also shown, in Tables 3 to 5.

Once age-specific birth rates have been obtained, they in turn can be used for the computation of a variety of fertility measures, by utilizing procedures conventionally employed with vital statistics, and outlined in various text books. Among the more obvious applications are the computation of gross and net reproduction rates, and Lotka's parameters for a stable population or the intrinsic rates of birth, death, and natural increase, and the mean length of a generation. Since the procedures are well known, they will not be treated here. Where those procedures call for age-specific birth rates involving daughters instead of births of both sexes, the former type may be obtained by applying a sex ratio to the birth rates involving both sexes.

The total number of births in a population may be figured by application of the age-specific birth rates to numbers of women at corresponding ages, with the

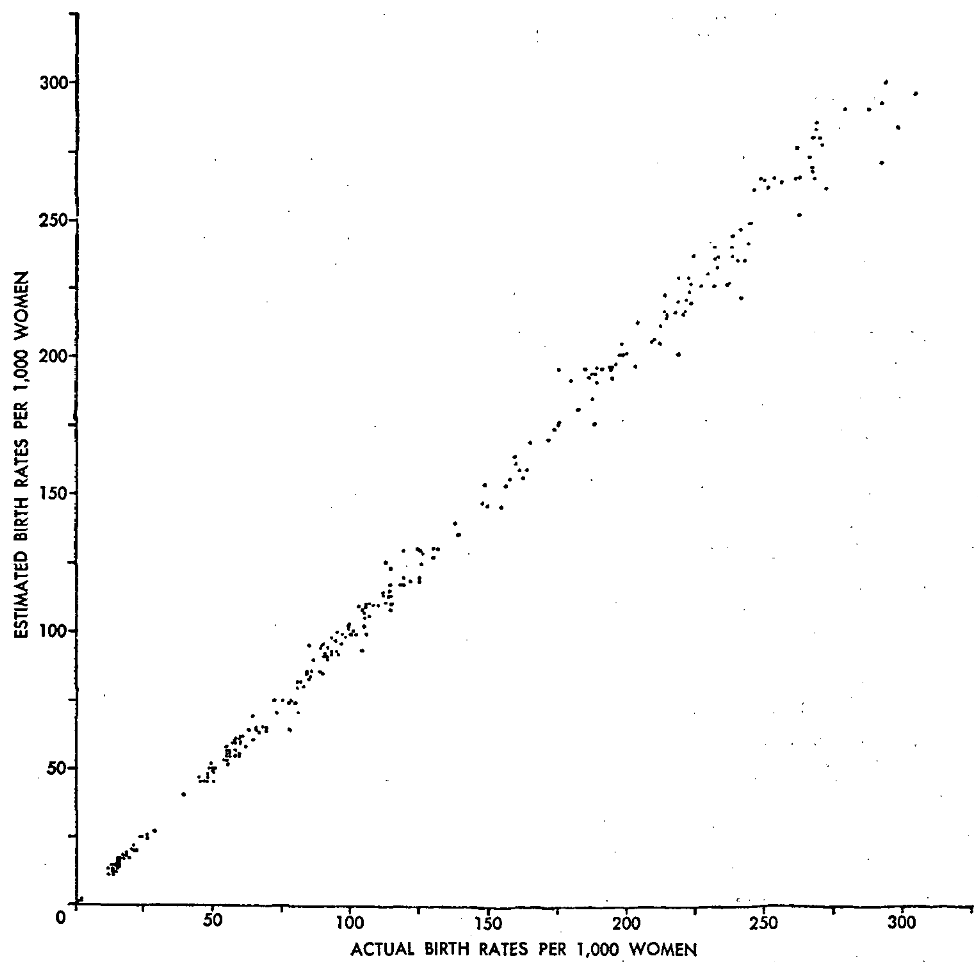

FIG. 2.—Scattergram of age-specific birth rates estimated by "All States Regression Equations" plotted against actual rates from vital statistics.

products summed. This total number can in turn be divided by the female population ages 15–44 to obtain the well-known "general fertility rate" or by the total population to obtain the crude birth rate.

Because age-specific ratios of own children under 5 to women can be in terms of marital status of women, education, duration of marriage, and many other characteristics when available from census schedules, a very wide range of new kinds of fertility measures might be developed should future investigators make the necessary effort.

Age of Woman	Average Annual Births per 1,000 Woman by Age at Birth of Child*	Estimated Ratio of Children under 5 per 1,000 Women by Age at Census, Assuming no Deaths†	Own Children under 5 per 1,000 White Women, 1960‡
15–19...	82	154	94
20–24...	246	856	826
25–29...	194	1,165	1,070
30–34...	113	760	726
35–39...	57	415	416
40–44...	15	171	171
45–49...	1	30	39

* Computed from vital statistics, for white women in the United States, 1955–60.

† Obtained by applying coefficients given in Table 6 to preceding column.

‡ From 1960 Census reports.

VII. PROCEDURES FOR APPROXIMATING AGE-SPECIFIC RATIOS OF YOUNG CHILDREN TO WOMEN FROM AGE-SPECIFIC BIRTH RATES

Many of the techniques outlined in this article involve the use of age-specific ratios of young children to women. At present, such ratios exist for very few countries. In the absence of age-specific ratios of young children to women for a given country, it may be desirable to take the age-specific ratios from some other country thought to have similar age patterns of fertility, and, using the easily available data on the total population under five years of age and women by age, these ratios of the latter country can then be adapted for the given country. Or, more likely, it may be desired to manipulate more commonly available age-specific birth rates from a similar population, so as to approximate age-specific ratios of young children to women, and then adapt the ratios to a given population. Such a procedure makes possible the use of many of the techniques outlined in this paper, even when a country does not have age-specific ratios of children to women.

A variety of procedures exists or can be developed for interpolation and recombination of conventional age-specific birth rates, to convert them into estimates of age-specific ratios of children under 5 to women. Table 6 presents coefficients or

Table 6.—SPRAGUE'S OSCULATORY INTERPOLATION EQUATION—COEFFICIENTS FOR ESTIMATING BIRTH RATES CUMULATED FOR A FIVE-YEAR PERIOD BY AGE OF WOMEN AT END OF THIS PERIOD, GIVEN AGE-SPECIFIC BIRTH RATES FOR ONE YEAR

Cumulated rate	B_{15-19}	B_{20-24}	B_{25-29}	B_{30-34}	B_{35-39}	B_{40-44}	B_{45-49}
R_{15-19}	2.7903	-.3292	.0389	...	...	...	...
R_{20-24}	2.7903	2.7903	-.3292	.0389	...	...	...
R_{25-29}	-.3292	2.7903	2.7903	-.3292	.0389	...	...
R_{30-34}	.0389	-.3292	2.7903	2.7903	-.3292	.0389	...
R_{35-39}	...	.0389	-.3292	2.7903	2.7903	-.3292	.0389
R_{40-44}	...	...	.0389	.3292	2.7903	2.7903	-.3292
R_{45-49}	...	...	...	.0389	.3292	2.7903	2.7903

Example of use:

$R_{15-19} = 2.7903\ B_{15-19} - .3292\ B_{20-24} + .0389\ B_{25-29}$

multipliers which can be applied to age-specific birth rates to accomplish that purpose. The coefficients are based on Sprague's fifth-difference osculatory interpolation formula.

An example of the use of Table 6 is given in the tabulation on page 71, together with a comparison with ratios observed in the 1960 Census.

With allowance for mortality, absence of children from home, and so forth, discussed later, a closer degree of agreement can be achieved. For the purpose of obtaining approximate age patterns of age-specific ratios of population to women from data for one population to use in computations for another population, it probably is unnecessary to adjust for mortality and so forth.

Once a set of approximate age-specific ratios of young children to women has been obtained, it can be adapted to data for a given population, as in the example given for Ohio in Section IV.

APPENDIX

LIFE TABLES

Many countries regularly publish national life tables, and also mortality statistics that can be used to compute life tables for states or other component areas. Given accurate vital statistics and census data, life tables can easily be derived and used to adjust the basic census data on own children under 5 years old and mortality rates for women and to estimate fertility measures described in this paper. For a detailed treatment of mortality tables and the procedures of constructing life tables, the reader may refer to *Population Statistics and Their Compilation*, by Hugh H. Wolfenden (The University of Chicago Press, 1954). *The Handbook of Statistical Methods for Demographers*, edited by A. J. Jaffe (U.S. Bureau of the Census, 1951), also gives a concise description of the mechanics of life table construction.

There are many countries where census data exist which are sufficiently accurate to be useful for demographic analysis, even though there are only very defective vital statistics or none at all. In such countries it is possible to make estimates, by use of census statistics alone, of many demographic indices which are normally obtained with the aid of statistics from vital registration. Techniques have been developed and have made important contributions to demography, most notably in the series of life tables for India covering almost half a century's mortality experience for almost one-fifth of the world's population. Dr. Giorgio Mortara undertook a coordinated investigation of the most important demographic characteristics of the population of Brazil. The various aspects of this work are reported in *Methods of Using Census Statistics for the Calculation of Life Tables and Other Demographic Measures (with Application to the Population of Brazil)*, by Giorgio Mortara ("Population Studies No. 7," Department of Social Affairs, United Nations, 1949). The first chapter of this report describes the techniques for computing life tables, using only census data on sex and age distribution.

For those nations with poor census data, or none at all, life tables may usefully be approximated by referring to *Age and Sex Patterns of Mortality: Model Life-Tables for Underdeveloped Countries*, published by the United Nations ("Population Studies No. 22," Department of Social Affairs, United Nations, 1955). The series of forty model life tables presented in this report, which covers the entire range of mortality variations that can be found today, provides a very economical method of approximating the most probable mortality level, by sex and age groups, for any population for which the infant or the early childhood mortality rate is known with a certain degree of accuracy.

UNDERCOUNTING OF YOUNG CHILDREN AND WOMEN

Censuses of population usually miss a small proportion of the population, as when some people move from an area not yet visited by an enumerator to another in which the enumeration has been completed or as when some die between the starting date of the census and the date the enumerator calls. In the United States, for example, various tests indicate that roughly 2 percent of the population may be missed.[5] The proportion missed varies some-

[5] Donald S. Akers, Bureau of the Census, "Estimating Net Census Undercount in 1960 Using Analytical Techniques" (paper presented at the annual meeting of the Population Association of America, May 1962).

what by age, color, and other characteristics. If both children under 5 and women of associated ages are missed in equal proportions, ratios of young children to women would not be affected by any adjustment for this undercount. Birth rates computed from vital statistics, in contrast, might be overstated if the births are more nearly completely counted than the numbers of women used as bases for those birth rates. (In many countries, however, birth registration is relatively less complete than the population censuses.) The count of young children tends to be more nearly complete, when age is determined from a question on date of birth rather than from a direct question on age. Some respondents report ages of children in terms of an approaching birthday instead of at last birthday, so there is some loss to the count at age under 5 when children almost 5 years old are misreported as age 5.

Various techniques exist for measuring the undercount of children under 5, but it is more difficult to measure the undercount of women. A variety of techniques for the former is given in a 1950 Census monograph, together with measurements of the net undercount in various censuses.[6]

[6] Grabill, Kiser, and Whelpton, *The Fertility of American Women* (New York: John Wiley & Sons, 1958), pp. 406–413. (A 1960 Census monograph)