Conference Paper

USE OF MATHEMATICAL AND NUMERICAL METHODS IN THE LATIN AMERICAN DEMOGRAPHIC CENTRE CELADE. SANTIAGO, CHILE

(This Document has been prepared by Albino Bocaz S. CELADE, Santiago, Chile. The ideas and comments expressed in it are the author's exclusive responsibility and do not necessarily represent those of CELADE)

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I N D E X

This document intends to give a relatively general scope about the use of various mathematical or numerical models put into practice in the demographic work carried out by CELADE.

The application, in some cases, of a given technique has been based on the solidity in front of a demographic behaviour which is not up to the exacting restrictions of the model, thus obtaining satisfactory results from the practical point of view.

One should have in mind that in several Latinamerican countries the vital statistics coverage is just slightly over 50 per cent, and that census data are distorted by differential sub-enumerations by sex and age, making impossible the use of such figures without previous adjustments.

In other circumstances it has been possible - via numerical application - to prove that some of the methods put forward, due to the small number of parameters considered, have not enough flexibility to conform to different demographic behaviours by age (children under one, for instance).

Besides, the fact that the parameters do not show any definite trend through time makes impossible the use of the model to extrapolate (this is the case of demographic rates projections), and that there is not any theory available to evaluate the variability of those parameters in terms of the sample size used in the demographic survey or in the census tape.

It must be pointed out that CELADE has always paid attention to the development of new models and their testing through its research programmes as one of the preliminary steps before putting them into practice.

It may be noted as well that the presentation of these new techniques in the various courses and seminars carried out every year. Lab exercises are developed in these courses making numerical applications of these techniques to data from Latinamerican countries in order to critically analize their results and, eventually, adopt the method.

The activities carried out in CELADE are so varied in nature that has not always been possible to advance in assessing the advantages of a technique based on a given mathematical model (for instance, the markovian processes of renewal, used to model human reproduction) because of lack of enough time and financial resources. This same circumstance has hampered the development of new procedures or the improvement of techniques already developed by CELADE's researchers.

In order to get an approximate idea about the way in which these models are used in CELADE, whether demographic, probabilistic or numerical, is advisable to have in mind the following types of works carried out in this centre:

1) Population Projections.
2) Analysis of past and recent mortality.
3) Analysis of past and recent fertility, taking into account the effect of family planning programmes.
4) Analysis of internal migration.
5) Analysis of the population participation in the economy.
6) Inter-action models between demographic and economic variables.
7) Teaching activities.

This document does not consider the use of models related to the works done in the fields referred to in the points 4) and 5) because of the limited time available. The efforts made in teaching about the analysis and criticism of the models currently used in demography will not be mentioned either (point 7). It can be said, though, that some of the techniques (elaboration of life tables, establishment of multiple regression models) used in the other works are applied to the points mentioned above. This is done with the purpose of explaining changes in the corresponding demographic indicators or incorporating the regression models into the simulation models. Population is thought to be used as an endogenous variable in these simulation models.

When pointing out the use of some model or technique, the research in which that particular model or technique has been used will be mentioned. This does not imply that the same technique has not been used in other works undertaken in CELADE.

1) Population Projections.

The availability of base population by sex and age at a national level is the starting requirement for every population projection. The basic data to elaborate this initial structure are provided by the nearest census to the projection starting date. It is well known that censuses are hindered by inadequate coverage of population by age which is more likely to be produced by household omissions than by omissions of respondents within the households.

Census structures must be adjusted adequately and in this process the stable population theory plays a part of paramount importance.(1) The use of semi-stable populations is of lower importance.(2) At the time the stable and semi-stable population theory began to be used in CELADE (1958), there were not manuals such as "United Nations A39" nor exhaustive studies about this theoretical model; there were not tabulations for population structures and the tabulations for deaths considered by that manual.

The theory of semi-stable populations was not, at that time, sufficiently developed and tabulations about this kind of theoretical population were not available, though, later on, they were elaborated by Coale-Demeny.

Therefore, it was necessary to elaborate a lengthy document with a whole set of numerical procedures to make use of stable population characteristics in evaluating basic demographic data from defective censuses and vital statistics. This research and demographic divulgation document, available for students and researchers, was completed with a wide set of tabulations on stationary, stables and semi-stables population models.(3)

In countries with more than one population census, population structures must be adjusted to make the adequate survivorship and sex ratios comparable, provided the population is considered as a close one.(4)

Lacking the adequate survivorship ratios, model survivorship ratios have been used as the ones given in the United Nations model mortality tables or the Coale-Demeny regional tables system. If it is thought that these models are not adequate enough, even if the level is changed following the variation of age intervals, specially made mortality tables may be used in agreement with the particular conditions of each country where elaborating them should not be a very difficult problem.

In actual working conditions in CELADE it has been shown that in most cases those mortality models are not adequate enough to cover the variations of mortality by age. Empirically, it has been proved that mortality under age 15 is not strictly correlated with mortality between age 15 and 49, as the correlation is given in the model tables even though, these would be regional model tables. The same is true for age 50 and over.

In spite of all these discouraging circumstances the survivorship models allow us to establish certain confidence limits within which the values of the studied population can be found. Finally, by allocating the survivorship observed values within these ranges (some of them higher than 1) the most plausible values can be determined.

The use of various hypotheses about the behaviour of natality and mortality makes possible to establish a finite number for the probable values of the total population in a quinquennial group of ages. J. Somoza has called them "reference values" (5) and the quantity of numerical values is determined by the type of data available and the researcher's confidence on the reliability of some of the hypotheses. The reference values are supposed to give an interval within which is expected to find the most probable value for the population considered.

As the final step the researcher must select one of the values already estimated or rather calculate a new value in terms of the available values.

Up to now, the crucial problem of the method is to determine the degree of confidence that each value is worth of, in order to establish then, the nearest value to the population value sought after.

Once the adjustment process of the population structure by sex and age (which is the starting point for the population projection) is finished, the problem about the future evolution of demographic rates arises.

If the case is that of a population projection by sex and age at a national level, with a practically negligible international migratory movement, the only problem to worry about is the future value of natality and mortality.

Three types of approaches are used in the case of fertility projections:

1) Change of the fertility distribution function by a decreasing exponential function. This function refers solely to age. So that if f_{xt} is the fertility distribution function at age (x) in year (t) its value in (t+5) will be $f_{x\ t+5} = \exp(-k_t x) \cdot f_{xt}$ provided the new values $f_{x,t+5}$ would be consistent with overall fertility level estimated for (t+5) years. (6)

2) Adoption of fertility distribution function models (7) or the one corresponding to developed countries estimated for (t+n) years.

3) Projection of fertility rates by quinquennial age groups via logistic function of two asymptotes. (This procedure has been also used in the projection for central

mortality rates). This logistic function is based on two pre-determined values and the use of two asymptotic values or otherwise on three pre-determined values and one estimated asymptotic level.

For the survivorship ratios projection it is assumed a linear variation of the variable log ($l_x/(1-l_x)$), using the values (l_x) for the year in which the projection is begun, and as asymptotic values the limit mortality values.

$$l_x^H = 98734(0.999738)^{1.103735^x} \quad \text{con} \quad l_1^H = 98700$$
$$l_x^M = 99125\ (0.999792)^{1.105005^x} \quad \text{con} \quad l_1^M = 99100$$

which correspond to a Gompertz type variation of the line (l_x) plotted agains mortality data in Norway.

It has already been pointed out (8) that both Gompertz and Makeham laws described adequately the survivorship line (l_x) for age 20 and over. As the values for (l_x) at ages lower than 20 get close to 100.000 due to a very low mortality level, Gompertz law can be applied to even lower ages down to age 1, as Bourgeois-Pichat has proved it (9).

This situation has led to project the Gompertz law parameters for age 20 and over instead of projecting mortality rates via logistic interpolation. For ages under 20, if the hypothesis is valid, the mortality decrease under 20 will reach the levels in developed countries.

2) Analysis of Mortality

2.1 Under-registration of mortality

If two successive population censuses and a death distribution by age (mortality structure) are available, assuming the population, as stable, is possible to establish, by means of a system of linear equations, the census and mortality under-enumeration (10). Thus the overall mortality level can be estimated via the nomogram ratio

$$m = -23.45 + 2.490\, d_{5+} + 0.606\, r$$

where m = crude mortality rate

d_{5+} = death at age 5 and over

r = population growth rate

This method has been applied in the Dominican Republic by de Lancer (11), where the coverage for mortality registration was 50 per cent and that for natality, 75 per cent. It is thought that these are adequate values for the period 1950-1960. The same method has been applied in Peru by J. Salazar (12), and in Honduras by I.R. Díaz (13).

On the other hand, use has been made of the stable population theory and with the information about the population structure by sex and age, and the mortality structure available at a given time, the ideas included in Manual A39 (chapter 4) have been used to make those structures consistent.

2.2 Mortality Projection via Logistic Model

J. Somoza (14) has analyzed the possibility of elaborating mortality projections for Argentina for the period 1960-2000 making use of logistic modification, on the basis of mortality tables of 1914, 1947 and 1960. This method was developed by Brass, and analytically, means the logistic conversion of a model mortality table, -specific for each country- into another mortality table in which the values (T_x) be the nearest to the values observed in (t).

From an analytical point of view, it can be said that the model mortality table is represented by a vector (v_e) of equal elements to log $(l_i^e / (1-l_i^e))$ which is applied to the linear conversion.

$$v_t = a_t + D_t\, v_e$$

where a_t is a vector of the same magnitude of (v_e) whose elements are all of them equal to (a_t), D_t is a diagonal matrix whose elements are equal to (b_t), and (v_t) is a vector of equal elements to log $(l_i^t / (1-l_i^t))$ and so alike as possible to those of a life table of (t).

Even though Somoza has not made a critical analysis of the results obtained, it can be said that the method is not all the satisfactory that one could expect because of two important difficulties:

a) the establishment of the model mortality table.

b) the fitting of the trend values of the regression ratios (a_t) and (b_t).

From a strictly analytical perspective it is not the conversion which is inadequate but the extreme stiffness of the vector (a_t) and the matrix (D_t). It is much easier to use the Gompertz law and the analysis of the trend of its parameters.

2.3 Projection of the mortality structure by means of a markovian chain

Following the Damiani's idea (15), J. C. Lerda (16) has attempted to summarize the mortality structure variation (Function d_x from mortality table) via a matrix equation

$$d_{t+5} = T\, d_t$$

where d_t = vector of the values of (d_x) at year (t)

d_{t+5} = vector values of (d_x) for (t+5)

T = triangular matrix of elements (t_{ij}) equals to $a_i p_j$ and stochastic in vertical direction.

Two types of triangular matrixes can be used, (upper and lower ones) so that Lerda has analyzed the differences produced in the values projected, drawing the conclusion that the best value for vector d_{t+5} is an average of the projections. The procedure was first applied to model mortality tables with relatively satisfactory results. Eventually the research would have continued with the elaboration of tables for Latin American countries. Unfortunately this was not possible because the researcher left CELADE.

Anyway the procedure is much more flexible than the one mentioned in 2.2 due to the fact that a transitional matrix is available in which the elements are more closely related to age.

2.4 Multiple decreasing life tables
(Cause death differentials)

With the elaboration of two life tables, one for the Capital Federal and the Province of Buenos Aires, (Table A), and the other for the rest of the country (Table B) M.J.E. Cerisola (17) has determined, for 1960, the increase produced in life expectancy at age (x) as a consequence of the elimination of certain groups of death causes. For the sake of analysis death causes have been grouped in five broad categories. It is known that the main problem in this type of analysis is the poor quality of data available, in particular when the proportion of medical certificates is rather low and the immediate cause of death is not clearly stated, along the existence of a great amount of death certificates in which the death cause is badly defined.

The conclusion arrived at in this document -which is the expected one beforehand- is that the elimination of certain death causes makes the mortality levels to get closer to the mortality differentials by areas, or in other words those causes are actually contributing to mortality differentials.

The work intended to continue with the analysis of the mortality structure trend in those five groups with the purpose of elaborating future mortality projections. This is a subject of the most relevant interest, not only from a descriptive viewpoint but from the perspective of its use in population projections and simulation models.

2.5 Construction of Life Table through Chain Linear Regressions

Various studies carried out in CELADE have used a chain linear regression system to elaborate life tables, applied to survivorship ratios.

This is not a quite original idea because the principle followed has been used to elaborate model mortality tables. The advantage is that, in most applications, is much better to directly use the survivorship ratios (${}_5P_x$) rather than previously having the values for (l_x).

The chain regressions are

$${}_5P_x = a_x + b_x \, {}_5P_{x+5} \qquad (x=35,30,\dots.\ 0)$$

They have been applied to a set of life tables elaborated for the region. The correlation observed in the survivorship ratios between contiguous quinquennial age groups is quite high (over 0,90). In spite of this situation there is the problem of "cumulation of the estimation error" acting as a multiplier and the effect of this is that the most remote survivorship ratios of ${}_5P_{35}$ will have a higher estimation variance.

The procedure has been first applied by P. Merlo (18) to the evaluation and adjustment of 1950 and 1962 population censuses taken in Ecuador as a previous step to the elaboration of the population projection for the period 1960-2000.

R. Mezquita (19), F. González and J. Dabasa (20) have applied this method to their researches on the estimation of sex mortality differentials in Cuba for the periods 1919-1931, 1931-1943 and 1943-1953.

2.6 Children Survivorship by Age of Mothers

If the total number of children ever borne to a woman (HT) and the total surviving children (HS) are known, the global survivorship ratio can be determined (HS/HT). The total number of children ever born (HT) will depend on the age of mothers at the time of the survey (x), the age of mothers at marriage (x_c) and the reproductive behaviour during the interval (x_c ,x). A certain proportion of the children will be dead (HM= HT-HS). This will depend on mortality conditions through the first year of life and the four following years. This number (HM) will vary in agreement with the existing correlation between these mortality rates and the mortality for age 5 and over.

Brass (21) has calculated the theoretical values for HT and HS and the mean age for surviving children (y) making use of model mortality tables and summarizing the nuptiality and reproduction conditions via a model fertility pattern of one parameter.

Obviously it is expected that $l_y \cong$ (HS/HT) but it has been found that the value for (y) does not correspond to an "integer" value. Because of this, Brass has moved (y) up to an "integer" age (y'), obtaining the following:

Age of mothers	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Age (y')	1	2	3	5	10	15	20

He was forced to calculate conversion factors -different than 1- in order to calculate then the global ratios HS/HT to the conventional values ($l_{y'}$).

These conversion factors depends on the shape of the fertility rates distribution -assumed to be constant through time- so that a cross tabulation can be made to look for the conversion factors. (Table V.1, Manual IV. United Nations).

Arrias and Farnós (22), based on these ideas and using survey data from URBAN PECFAL have determined the survivorship global ratios for 2 000 women surveyed in Buenos Aires, Bogotá and San José, grouping them into conventional quinquennial age groups: 20-24, 25-29, ... 45-49.

The value of ratio P_1 / P_2 has been calculated from the same survey, (this ratio defines the shape of fertility distribution) to determine the conversion factors shown in Table V.1.

It has been observed that the values for (l_y) obtained vary quite irregularly which makes mandatory to adjust them through a logistic transformation. This fitting substantially varies the values for ($l_{y'}$). Therefore, it is doubtful if those adjusted values correspond to the ages (y') adopted by Brass.

It can be argued that the irregular behaviour of ($l_{y'}$) is due to the fact that each age group is only formed by around 300 women. To avoid this problem, the reproductive experience of women has been placed into the same age group while they belonged to that age group. This has greatly increased the size of the sample on which these ratios (HS/HT) are based.

Vallenas (23) has, on the other hand, elaborated individual age specific fertility tables using a cumulative fertility model put forward by Macció (24) and an individual age specific mortality table finding out that the conversion factors (from HS/HT into $l_{y'}$) are significatively different from the ones calculated by Brass. (In the elaboration of these tables, Vallenas used data from, the 1940 population census taken in Peru). The same situation has been present when use has been made of mortality and fertility tables developed by CELADE for Latinamerican countries.

In spite of these problems, the application of Brass' technique is quite useful to obtain a set of probable values for $l_{y'}$, although it should be taken into account that the values are determined by the limits put upon the model:

a) constant fertility through time and having conventional shape.

b) model life tables (not always suitable to Latin America)

This is CELADE's attitude in regard to data from national demographic surveys carried out in Honduras, Peru, Panama and Bolivia. Besides, it is quite clear that is not possible to accept the Brass models without any discussion of their restrictions in particular when it is known that there are changing conditions in natality and mortality, specially the former because of the impact of family planning programmes. In any case it is acknowledged that vital statistics cannot be used due their poor quality.

2.7 Orphanhood Data

By means of maternal orphanhood data from the experimental census taken in Grecia (Costa Rica, 1971) mortality levels at age 20 and over have been tentatively established. (25)

According to Lotka's theory about stable populations (26) the probability for a person at age (x) of having his mother of age (x+y) alive is given by l_{x+y} / l_y.

If the persons at age (x) who have their mothers alive are taken into account, the mothers' mean age ($\bar{y}$) at birth of that particular child together with the measure of the variance (Var y) for those ages can be calculated. This can be expressed as

$$l_{\bar{y}+x}/l_{\bar{y}} = (1-h_x)/(1-e)+c \text{ Var } y$$

in agreement with a ratio developed by Henry in 1960 (27).

It can also be considered a somewhat similar equation in which instead of using the mother's mean age at birth of children with current age (x), a conventional age (B) can be used and taken into account changes produced in orphanhood correlated with change in age (x) of respondents.

The following equation can be established proposed by Brass-Hill (28)

$$l_{25+x}/l_{25} = (1-{}_5h_x) + c_x\,({}_5h_x - {}_5h_{x-5})$$

where ${}_5h_x$ = maternal orphans at ages (x, x+4)
c_x = fitting factor (positive or negative)

It is possible to prove numerically that the equations by Henry, and Brass-Hill do not lead to highly different values, with the Henry's equation having a more sharp theoretical justification.

It is observed that the values ($l_{x+B+2,5}$) chow a higher consistency with age than ($l_{y'}$) though it is necessary to smooth the values by means of a logistic transformation.

The application of the orphanhood method, both to census and demographic surveys data, has proven quite satisfactory. That is why CELADE will insist upon the including of this type of questions in the population censuses and demographic surveys carried out in each country.

The orphanhood method applied to all children at age (x) may be limited only to eldest children of age (x) or first born children (29). The possible error introduced might be that the reported eldest children would not actually be the eldest children, but one can be sure that every woman has the same probability of being included in data gathering.

The technique has been applied to the National Demographic Survey in Honduras (EDENH) in the fourth round of the Survey including retrospective questions, (RETRO-EDENH). It is expected that the variance effect on the mothers' age will be lower that if the children of age (x) are considered, regardless of their rank. Survivorship values $(l^{HM}_{B+x+2,5})$ (HM = eldest children) have been lower than expected. $(l_{B+x+2,5})$(HCO $\neq$ children regardless of rank).

2.8 Widowhood Data

The approach provided by this technique is similar to that of orphanhood (30). It has been applied, with relatively successful results in the National Demographic Survey in Honduras and in the same type of Survey (retrospective) in Bolivia. The results in this last survey have been less satisfactory.

The basic equation is

$$l_{B+x+5}/l_B = (1-{}_5v_x) + c_x({}_5v_x - {}_5v_{x-5})$$

It is observed that this is "almost" the same to that for orphanhood, in which the term "Orphanhood" becomes "widowhood". The correction factors (c_x) are different because they should deal with survivorship at other ages.

The parameter B appearing in the equation is determined by the simulation group. It would be different if women get married before age 20, B = 22,5 and if they do it after that age, B = 27,5. Only one marriage duration model must be adopted to determine c_x. Hill has done so by means of a distribution function of potential type.

The same as in the orphanhood technique, the values obtained (l_{B+x+5}) are smoothed via a logistic transformation.

2.9 Information about Grandparents

Questions about grandparents survivorship have been included in the national demographic surveys due to the excellent results obtained in estimating adult mortality from orphanhood.

The equations used in applying the method are based on the article about family formation and parenthood relationship by Goodman, Keyfitz and Pullum (31).

Somoza (32), based on these equations and using some model mortality tables, has been able to determine the theoretical values of the probabilities that a person at age (x) have

paternal granfather alive
paternal grandmother alive
maternal grandfather alive
maternal grandmother alive

These theoretical proportions have been compared with the values obtained by the pilot survey carried out in Lima-Callao in 1974.

It has been found that it is useful to continue in the same line of research because of the satisfactory results obtained.

3) Analysis of Fertility

3.1 Analytical Functions

In the case of mean parity at age (x) the following models have been used, among others:

Maccio's Model (33)

$$F_x = x(k-x)\ g_3(x)$$

where k is the extension parameter

$g_3(x)$ is the algebraic polynomial of order 3.

One of the problems of this model is to establish the precise meaning of parameter k which has been called "extension parameter" because is used to enlarge or compress the cumulative function (Fx) to make it closer to the observed values of mean parities.

The use of the model has, in some cases, modified the location of the higher mean fertility rate moving its position to the following quinquennial age group. These are the cases of Ecuador (1955), El Salvador (1961) Guatemala (1950 and 1964), Nicaragua (1963), Peru (1960) and the Dominican Republic (1950). It is estimated, though, that these changes would be partly justified in terms of the good quality of the data available.

The use of this model allows to easily find out individual age specific fertility rates the Beers Multipliers to the function F_x / x(k-x).

The estimation of individual age specific fertility rates together with the estimation of individual age specific mortality tables make possible the development of population projection by individual age and year, avoiding the inconsistencies produced when breaking down the quinquennial projections.

Bocaz Model 1 (34)

$$F_x = x(k-x)\ g_3(x)$$

which is similar to the one developed by Maccio but the implicit idea is to produce 2 conversion matrixes to convert the mean parity vector (r) into mean fertility rates vector (f) or vice versa.

If (r) and (f) are respectively the mean parity and fertility vectors in the (7) quinquennial age groups of women in their reproductive period (15-50), and A and B are the conversion matrixes we have

$$f = A\ r$$
$$r = B\ f \text{ con } \quad B = A^{-1}$$

If adequate values of r are available is possible to estimate (f) by means of

$$\hat{f} = (B'B)^{-1}\ B'r$$

Bocaz Model 2 (35)

$$F_x = F_w \Big/ \left\{ 1+[(x_s-x) / (x-x_I)]^m \exp(-a+bx) \right\}$$

where

x_s = upper limit age of reproductive period
x_I = beginning age of reproductive period
F_w = completed family size

The above equation is more easily expressed as

$$\log(p_F/q_F) = m \log(p_x/q_x) + a - bx$$
$$p_F = F_x/F_w \qquad q_F = 1-p_F$$
$$p_x = (x-x_I) / (x_S-x_I) \qquad q_x = 1-p_x$$

hence the name "modified biologistic function" as it is known in CELADE.

From the above equation we can derive the fertility rate function

$$F_x/F_w = p_F q_F \left[m/(x_S-x_I)p_x q_x - b\right]$$

By means of the projection of the trend followed by the parameters m, a, and b the fertility projection can be elaborated.

Up to now this function has been applied to cumulated fertility by cohort in the Brazil censuses obtaining a very good reproduction of the cumulative frequency curve.

There are still the following types of analysis to carry out:

a) adequate interpretation of parameters

b) determine the causes why some of the parameters do not present a clear variation trend.

3.2 Linear Models

Rothman (34) has tried to explain the variability in the number of children born to surveyed women in Buenos Aires on the basis of data from PECFAL Urbano, according to

a) age and schooling level

b) age and economic participation

in separate analysis, using a simple additive model (without interaction)

$$\bar{y}_{ij} = \mu + a_i + b_j$$

In the first analysis he has found that age explain the 84 per cent of schooling, the 14 per cent of the variation. In the second analysis, the figures are: 78 per cent for age and 19 per cent for economical participation.

Rothman has not considered, at the same time, the variables and a model which takes into account the interaction.

In CELADE, the group of researchers mainly devoted to the fertility analysis through data from PECFAL Urbano and Rural has made various applications of the linear multiple regression models and the technique known as path analysis.

This group has also studied the use of first class Beta function (Pearson Curve Type I) to summarize the fertility of women surveyed in cities covered by PECFAL Urbano following the suggestion made by Henry about its use.(35) The moments method has been tried to determine the parameters such as has been suggested by Potter, proving empirically the low efficiency of the method. On the other hand, Bocaz (36) has developed an iterative procedure to determine the parameters when the maximum of likelihood principle is used for punctual estimation, which has been considered more suitable than the one suggested by Sheps-Majumdar. In that study, the establishment of parameters for using in case the above function would be acting as a fertility rate distribution function has been searched for.

J. Arêvalo (37) has tried the Henry's method to determine human reproduction by means of probabilities in family enlargement. Birth rank data obtained in Chile for the period 1952-1959 have been used in the study. The information differentiates the legitimate from illegitimate births.

According to Henry's method (38) if $M_{j-1,t-z}$ represents women of parity (j-1) in the year (-z), and a total number of women ($a_j M_{j-1,\ t-z}$) will have a higher parity (j or over). From this total ($\alpha\ a_j M_{j-1,t-z}$) women will have parity (j) within (z) years, so that ($\sum_{z=0}^{w} \alpha_z a_j\ M_{j-1,t-z}$) will be equal to all birth (B_{jt}) of rank (j) at year (t).

From Arevalo's findings we may observe that there is a significant difference between the growth probabilities of legitimate and illegitimate births. This difference is more remarkable if one thinks that it leads to 5 legitimate births per woman and 1 illegitimate birth per woman. But this results are distorted by marriages produced after childbearing, thus legitimating the children born.

3.2 Stochastic Processes

Argott (39) has made use of the equations found by Perrin-Sheps (40) (correlating the human reproduction process to a markovian renewal process) to try to evaluate the effects of family planning programmes upon some fertility indicators.

The information used by Argott was drawn from the fertility survey carried out in Mexico City in 1964, as part of PECFAL Urbano.

The following indicators have been used:

0 = fecundability period (exposure to risk of pregnancy)

1 = pregnancy

2 = abortion

3 = stillbirth

4 = live-born children.

With these indicators, the author has used the following equations:

1) Time elapsed between marriage and first pregnancy

$$E(T_{o1}) = (1-\rho)/\rho$$

It is used to determine the fecundability (ρ) based on time elapsed (T_{o1}) between marriage and first pregnancy. Women have been broken down into two groups: a) those who never practiced contraception and

b) those who have practiced contraception.

No adjustments have been made for different periods of use or different efficiency of methods used.

2) Interval between marriage and first birth

$$E(T_{o4}) = v_4+(p_2t_2+p_3t_3+\frac{1-\rho}{\rho})/p_4$$

where p_2, p_3, p_4 are respectively, the proportions of pregnancies ending in abortion, still birth and live born children.

t_2, t_3, t_4 are: infertility post abortion, post stillbirth and post live born children.

v_4 is the mean duration of pregnancy ending in a live born child

3) Interval between successive births

$$E(T_{44}) = t_{40} + E(T_{o4})$$

where t_{o4} is the mean time between the ending of pregnancy and the beginning of fecundability (exposure to risk of a new pregnancy). The theoretical intervals were always longer than the observed ones. Argott thinks that this bias is partly explained by the fact that the values for (t_j) are broader than the actual values in the pregnancy history.

4) Total number of pregnancies at time (t). (tables 20 and 21)

$$E[N_1(t)|J_o=0] = Ni/2+t/m_{11}+c_{11}^2/2-m_{o1}/m_{11}$$

where :

m_{11} = mean interval between pregnancies

c_{11}^2 = relative variance of the same intervals

m_{o1} = mean interval between marriage and first pregnancy.

Values were calculated for t = 12, 24, ... 360 months, obtaining, at the end of the reproductive life, 13, 34 pregnancies for women not practicing contraception, and 11, 85 for those ever practicing contraception.

5) Total number of live born children up to the moment (t). (Tables 22 and 23)

$$E\left[N_4(t) \mid J_o = o\right] = 1/2 = t/m_{44} + c^2_{44}/2 - m_{04}/m_{44}$$

where

m_{44} = mean birth interval

m_{04} = mean interval between marriage and first live born

c^2_{44} = relative variance of birth-interval

Women not using contraception gave a total of 11,39 live born for a t = 360, and 9,64 for women ever using contraception.

6) Fertility rates at moment (t)

$$E\ N_4(t-1) - E\ N_4(t)$$

Rates for women not using contraception vary from 0,402 for age group 20-24 to 0,387 for age group 45-49. Rates for women ever using contraception are practically constant at 0,33.

3.3 Efficiency Table for Contraception

With the experience obtained in the use of the Depo-Provera (injectable progestion 300 mg.) (Consultorio de Fertilidad, Departamento de Obstetricia y Ginecología, Hospital J.J. Aguirre), J.G. Voget (42) has elaborated efficiency tables in the use of this type of contraceptive. He has followed the procedures suggested by Potter to elaborate multiple decreasing life tables (dropping out from programme due to different causes) (43).

Results show a very high rate of follow up in this type of injectable progestion. The follow up rate was 93,59 per cent up to 12 months and 85,16 per cent up to 24 months. The cumulative pregnancy rate is respectively 2,24 and 2,98 per cent.

Besides, Voget has elaborated Age-specific Contraceptive Efficiency Tables, to study this differential. He has found out that cumulative failure rates(non expected pregnancies) up to 12 months is rather higher for younger women.

6) Interaction models between demographic and economic variables

A. Fucaraccio and C. Arretx have developed a very simple model to evaluate the effects that changes in the population income have upon certain segments of population (44).

The point is to observe how an income increase exerts influence upon the labour market and the population participation in the economy.

Income increasing modifies the family standard of living thus having an influence upon the completed family size. This change in the completed family size, which is not observed up to year (t), will enable us to determine the corresponding age specific fertility rates, which, in turn, will reduce the crude birth rate, because of its reducing effect upon the live born children.

A lower number of children will allow the mother to participate in economic activities, and at the same time she will be able to contribute to saving and investment at family level.

The same policy of higher wages for the population will reduce the male participation in the limits of the age intervals in which the individual would be at school or acquiring a specialized training, or retiring from work because the availability of a certain amount of savings or a retiring allowance adequate to cover his expenditures.

The equations considered in this model are:

$$1)\quad I_{t+1} = (1+r_t)\, I_t$$

This indicates a geometrical growing of income, with varying rates in the different quinquennial periods considered in the projection.

$$2)\quad C_t = a_1 + b_1 I_t + c_1 OV_t$$

That is to say that the expenditures vary linearly with the income and some "other variables" indirectly considered in the model.

$$3)\quad F_t = a_2 + b_2 C_t$$

or that total fertility varies linearly with income.

$$4)\quad F_t = a_3 + b_3\, I_t + c_3 OV_t$$

derived from 3) after replacing in it equation 2).

$$a_{MU}^{t} = a_4 + b_4\, b_t$$

in which is assumed a linear relationship between urban female participation and the crude birth rate for that group.

The basic equation of inter-relationship between a demographic and economic variable is given in the equation 3), with a reflection equation as 5), in which the urban female economic participation is determined by changes produced in the natality overall level.

The regression ratios were determined through data from urban fertility surveys carried out in Buenos Aires, Bogotâ, Mexico, Panama and Caracas in the period 1963-1964.

The model considers Latin America as one unit, without making any difference between urban and rural mortality, though it does consider age mortality differentials.

Rural-urban migration has been considered as exogenous with volume and net specific rates which keep up in the projection the degree of urbanization pre-established in CELADE's projections (page 47 of the document).

For the numerical handling of the model, the authors have considered suitable to determine, in the first place, the overall levels of natality, mortality and economic activity and then, determine the age specific rates necessary for the population projection, based on relative distributions of specific rates.

The projection has been made for the period 1970-2000 using the following hypotheses about the evolution of income:

<u>Hypothesis 1</u>

The trend observed will continue within the economic conditions defined by the use of a constant income growth rate of 2 per cent per year.

<u>Hypothesis 2</u>

The income growth will vary substantially, with an increasing growth from 2 per cent to 5 per cent up to 1980 and staying on that level until 2000.

The most outstanding results are the following:

1) Population Structure

Age groups	Year 1970	Year 2000 Hypothesis 1	Year 2000 Hypothesis 2
0-14 years	42.4%	40.0%	38.4%
15-64	53.8	56.1	57.6
65 and over	3.8	3.9	4.0

This indicates an aging slightly higher than the one stated by the hypothesis.

2) Economically active population, with constant male participation.

Hypothesis about Urban Female Participation. Income.	Expected active population for 2000 Males	Females	Total
1 constant	170 482 600	47 055 700	217 538 300
variable	170 482 600	55 359 900	225 842 500
difference	--	8 304 200	8 304 200
2 constant	169 698 900	46 744 000	216 442 900
variable	169 698 900	57 295 200	226 994 100
difference	--	10 551 200	10 551 200

This shows a difference of 8 304 200 women in the hypothesis 1, considering that this income growth will contribute, by all means, to a higher female participation. In hypothesis 2 the growth is still higher (10 551 200) because an even higher income growth is expected.

2b) Economically Active Population, with male and female participation differentials.

Hypothesis about Economic Activity. Income Males	Females	Expective Active Population Males	Females	Total
1 const.	const.	170 482 600	47 055 700	217 538 300
2 variable	variable	163 888 700	57 295 200	221 183 900
difference		-6 593 900	10 239 500	3 645 600

There is an increase of 3 645 600 people because a significant growth of female participation (10 239 500) and because of a reduced male participation (6 593 900).

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