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ECONOMIC-DEMOGRAPHIC MODEL

A case study for certain Latin American Countries

by

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40.

I kindly request that the colleagues who will receive a copy of this work provide useful and free criticism or suggestions, because the author believes in this manner he will be facilitated better.

Thank you!

Stylianos C. Athanassiou

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CHAPTER I: FIELD OF RESEARCH

1. Introduction

The analysis of a socio-economic phenomenon leads us to a set of hypotheses. These hypotheses can be expressed in mathematical forms which are called models. During the last thirty five years, the construction of such models has advanced rapidly in the socio-economic sciences. Thus, a great number of models have been developed by means of which an economy or an area of social activity is described. Furthermore, it has been proved that there are many interrelationships between demographic and economic factors and in general, the increase of national output can be attributed in economic, demographic and social factors. Therefore, in the last few years, an effort is made to relate all the aforementioned factors in a model. This may w ll allow us to see the influence of each factor in the all over economic growth of a country. Finally, a model is suitable for statistical fitting and it can be used for forecasting on a scientific basis. Thus, a model is an instrument of planners and policy makers.

More details about the models as regards their construction, mathematical form, use, etc. are mentioned in the relevant chapter 4.

2. The investigation of economic-demographic relationships

The investigation of the existing relationships between economic and demographic factors, in long-run term can be realized from two points of view. The first will refer to the influence of population changes upon the growth of national output. Of course, in addition to population changes, other socio-economic factors are associated with the increase of national output. The second is the reverse viewpoint, i.e. the influence of economic factors, and mainly of national output on population development.

The first case, that is the impact of population on the growth of national output is of great importance on account of the efforts being made by all the countries -developed and less-developed- on one hand for their economic growth -of which population is the main factor but also the goal- on the other hand, for increase or decrease of their birth rates.

3. The purposes of the present work

The main purpose of this study is the construction of an economic-demographic model. This model will consist of a number of equations belonging to a mutually interdependent system. The equations will describe the existing relationships between the crucial economic and demographic variables, whose parameters will be statistically estimated on the basis of empirical observations. Even though the central variables of the model will be the national output and labour force, however, it can be characterized that it follows one direction, i.e. the impact of demographic factors upon the national output. This is because, the factor population enters in the model as an exogenous variable. In more details, the purposes of the present work can be analysed as follows: The determination of economic and demographic factors, which on one hand, contribute to the increase of national output and on the other hand, determine the development of supply and demand for labour force as well as the estimation of the influence of these determinative factors on the aforementioned central variables of the model by applying an interdependent system of equations. Thus, by means of the system of equations -model- we will acquire a quantitative knowledge of the interlinking between economic and demographic variables. Of course, the full treatment of all the possible factors or relationships between economic and demographic factors is an exceedingly difficult task to be completed under present conditions. This is because, on the one hand, we do not have available adequate statistical data which are required by the model; on the other hand, the time available within which this project must be completed, can be considered limited. In view of these limitations we will try so that the model to have a catholic character, in the meaning of the possibility of its re-adjustment from the point of view of variables and its application in the majority of Latin American countries. Furthermore this model will be used forecasts as far as the development of the endogenous variables, national output and labour force in the projected period 1971-75. The model will refer to the national level and probably, to the secondary sector of the economy, if statistical data are available. Furthermore, we will investigate the production function which is the first equation of the system in the industry by branch on the basis of cross-section data -the second purpose of this study. This investigation will allow us to see the contribution of the production factors, capital and labour force to the increase of the output and consequently we will have a clear picture of the industrial sector of the economy. Finally, we will try to estimate basic economic factors not included in the model to be applied but they are related to the population and national output as for instance final consumption and per capita income on the results obtained from the model. This effort particularly as regards the per capita income is of great importance because it will allow us to know the development of the main "dimension" of the degree of economic growth in the coming years. These are the purposes of the work to be carried out here. In this opportunity it is advisable to mention in this section that the estimation of the factors which influence the exogenous variable of the model, participation rates of labour force, will be done by applying a numerical model. This numerical model is described in a separate chapter of the present work.

4. Usefulness of the study. The selection of certain Latin American countries for the application of the model

Population is the goal but also a factor of economic growth of a country. As we said in the introduction of this chapter, there are many relationships between economic factors and population changes. It is known that the size and the distribution by age and sex of the population determines the size and composition by age and sex of the labour force which is one of the main factors of the increase of national output, while the productivity depends on the labour force, its structure from educational point of view, etc. All the developed countries try to investigate these relationships in mathematical models by means of which the interdependence of demographic and economic factors appears clearly and their statistical estimation is achieved. From this general view, the construction of such a model -economic and demographic- is of great importance for Latin American countries where the economic growth is the main problem for solution due to the low level of per capita income and the high rate of population development. Furthermore, the economic development plans are based on the results of demographic and economic research and their targets are determined in accordance with the economic and population changes. Therefore, data referring to the existing demographic and economic relationships, expected changes in population and economic factors are necessary in the planners and policy makers. Latin American countries are at the stage of economic development and prepare long-term economic plans. This means that they need of such data and results derived from the model and from this point of view the model will be useful.

As regards the investigation of the industrial sector of economy, it will allow us to see the contribution of the factors, capital and labour force to the increase of industrial production as a whole and by industrial branch and it will lead us to useful economic results in order to determine the possibilities for a further industrial development, exercising policy indicated, etc. Finally, the interlinking relationships between demographic and economic variables in an interdependent system of equations on the basis of data of Latin American countries take place for the first time, and this can be considered as an important point of this work.

The selection of Latin American countries, for the application of the model will be made on the basis of what has been mentioned in the previous section as regards the adequacy of statistical data and time available. Although all the Latin American countries can be considered as less-developed countries on the basis of the per capita income, however, there are considerable differences between them, both as to the level of per capita income as also as regards other features of less-developed countries.[1] In this instance, an effort will be made to include countries of different level of economic growth for application of the model. Thus, on the basis of the main indicator of the degree of economic growth -per capita income- and available statistical data, the countries, Chile, Mexico and have been selected for the application of the model.

5. Sources of statistical material and the structure of the work

The sources of quantitative information about the demographic and economic data used in this study are the official publications of the following organisations and institutions:

- United Nations (Yearbook of National Accounts Statistics, etc.)
- Economic Commission for Latin America/U.N. (Statistical Bulletin for Latin America, etc.)
- Centro Latinoamericano de Demografía/U.N. (Boletines Demográficos, etc.)
- Instituto Nacional de Estadísticas. Chile. (Statistical Yearbooks, National censuses, etc.)
- Oficina de Planificación Nacional. Chile. (Plan for Economic Development 1960/70, etc.)
- International Labour Organisation. Geneva. (Regional Programme of Labour Force for Latin America, Situation of Labour Force in Panama, Projections of Labour Force for Latin America, etc.)
- Banco Central de Chile
- Ministerio de Fomento. Venezuela (Statistical Yearbooks, National censuses, etc.).

1/ The per capita income in Paraguay was US$ 149 during the year 1960 while in Venezuela it was US$ 859 in the same year. The typical features of less-developed countries are: low per capita income, unequal income distribution, surplus labour force, small size of internal market and lack of an organized capital market. (Contos, A., <u>The Application of Macro-Economic Models to Development Planning: Peru</u>. Iowa State University. Iowa, U.S.A., 1966. pp. 1-3).

But, a study as the present one, requires detailed and completely revised statistical data. For serious reasons, the obtaining of such data is not possible for many countries in Latin America. Therefore, the limitations to the present work imposed by the statistical material as this we said elsewhere, are heavy.

The work consists of nine chapters. In the first chapter, we give a general picture of the study from the point of view of its purposes, usefulness, etc. In the second chapter, we describe in general the problem of economic growth in Latin American countries, the population development, etc. In the third chapter, the hypotheses and the basic functional relationships of the main economic and demographic factors are given. The theoretical model and the model to be applied together with its technical and statistical problems are included in the fourth chapter, while in the fifth chapter, we present the numerical model which will be used for estimation of the factors affecting the participation rates of labour force. Furthermore, in the chapters sixth and seventh, we realize the empirical analysis of the model as a whole, we present the statistical results obtained and we investigate the existing situation of industrial branches by estimating the production function. In the eighth chapter, we develop the form of the model for making forecasts and we adopt some assumptions with respect to the predetermined variables. Furthermore, we use this model for long-term prediction as regards the development of the endogenous variables in the coming years and for the countries in which the model was applied and we compare the results obtained by the two statistical methods (TSLS and OLS). Finally, in this chapter we estimate the consumption function and the per capita income in this projected period 1971-75 on the basis of the results obtained from the model and other data. In the last chapter (ninth chapter) a review of findings is made and a summary of general conclusions is presented.

CHAPTER 2. ECONOMIC GROWTH. POPULATION

1. In general

As it appears from the aim of the present study, the model to be built will include economic-demographic relationships. By these relationships, changes in basic economic and demographic variables as for instance, national income and labour force, will be determined as well as effects of the production factors to the increase of national income will be estimated. Speaking broadly, this model will be model of economic growth. From this point of view, it is considered advisable to refer to certain fundamental concepts as regards the meaning of economic development, its importance for less-developed countries, the realized rate of growth in Latin American countries, etc. It is known that the man is the objective scope of economic development. Furthermore, the size of population is a component of the indicator by which the level of economic development is expressed. Therefore, a general picture of the evolution of the population of Latin American countries as a whole, its distribution by age and with respect to residence is required to be given in this chapter. Finally, we will mention briefly the international migration.[1/]

2. Economic growth as problem. Its meaning

Economic growth, as an objective of applied social sciences, has engaged since the end of World War II. During the period from that time up to now, economic development has become the main problem for solution in all countries, regardless of the socio-economic progress they have achieved, and also the fundamental target of economic plans and state policy.

Of course, this problem appears differently from intensity point of view, results, etc. in each country. This is obvious, because the socio-economic and population reality, technological progress, etc. are not the same in all the countries.

1/ The analysis of the variables related to the model is carried out in the relevant chapters for the countries in which the model is applied.

Referring to the meaning of the term of economic growth, we meet a great deal of disagreement, or lack of clarity on this point, and this because economic theory does not give the same explanation to the term "economic growth".1/ Here, we shall not deal with the new points or the existing differences regarding this term. We think that it is enough to repeat the definition which was given by Professor Kanelopoulos, A.: "Economic development is the long-term procedure, during which an economy as a whole, realizes an increase and differentiation of the results of its productivity, in conjunction with the structural changes, leading to an increase of the real per capita income of an increasing population".

3. The main dimension of the degree of economic growth

For the quantitative determination of economic growth there are indicators. The most suitable indicator for measuring the degree of economic development is the per capita income of a country's population.2/ This indicator has the advantage of being acceptable to statistical measurement. In spite of the general acceptance of this indicator, as a measure of the degree of economic development, and the facility of its statistical estimatiom, it still has some weakness and difficulties as to its accurate determination. These difficulties refer both to the national income as also to the population, that is to those elements which form this indicator.3/ In accordance with this main dimension of the degree

1/ Many writers define the meaning of economic development by its causes, and others by its results. Economic development, however, is not either the cause or the result, but the procedure itself (Kanelopoulos, A., "Economic Development", Vol. I, Athens, Greece, pp. 82-86).

2/ TEMPO Population Growth and Economic Development. General Electric's Center for Advanced Studies. California, U.S.A. Kanelopoulos, A., Economic Development, Vol. I, Athens, Greece. Demeny, P. and López, A., Construction of General Models of Economic and Social Development. United Nations, Economic Commission for Latin America, Santiago, Chile.

3/ Except for grants and borrowings, Gross National Product (GNP) as ordinarily defined, determines national income, Y. The national income, Y, divided by the country's population, P, gives the per capita income, $Y/_P$. Furthermore, this indicator of economic development, can be used for international comparisons between countries, as regards the level of their economies.

of economic growth, less-developed countries are considered to be those economies whose per capita income does not exceed 25 per cent of the corresponding per capita income of the United States.[1/]

Of course, this dimension must be related also with other economic variables, such as the final consumption, the distribution of income, employment, the educational level, etc. in order to have a complete picture of economic development of a country. Finally, economic development as a process in time will be characterized and measured by increase in per capita income as we said in the previous section on the basis of the definition of economic growth.

4. The problem of economic growth and the less-developed countries

The problem of economic growth appears more intensively in the less developed countries. This is obvious, because these countries are suffering under economic misery and poverty. These countries are more than two-thirds of inhabitants of the earth, and they are countries in Africa, Asia and Latin America. The appearance of the problem of economic development in these countries is attributable to two reasons. The first reason is the awakening of the peoples -the great awakening, according to Professor Gunnar Myrdal that is, in other words, the conscious comprehension by these people of what condition they are in, and their belief that they can emerge from this unfavourable situation. A second reason, is the belief of the economically developed countries, regarding the imperative need of providing material and moral aid to the effort being made by the less-developed countries. This conception came from the fact that the industrially advanced countries saw that the future of humanity will depend upon the less-developed countries, and specially on the manner in which these peoples will try to alter their existing socioeconomic situation. Furthermore, if the existing gap between the developed and less developed countries is not reduced, this will have an

1/ Kanelopoulos, A., Economic Development, op. cit. pp. 105-107.

unfavourable effect upon the investors of developed countries in these latter countries and a drop in trade in the industrially developed countries. In addition to these reasons, there are others, such as cultural, political, etc. The efforts, however, of the less-developed countries for their economic development as also the efforts on the part of the developed countries, though the granting of both moral and material aid had a result in the less-developed countries achieving considerable economic progress and the rise of their standards of living. As an example it is mentioned that during the last decade, 1960-1970, the rate of growth of the poorer countries rose to the satisfactory level of 4 percent on the average.[1/] This rate of growth exceeds significantly that which they had obtained during the immediately preceding decade (1950-1960). By this continuously increasing rate of growth, the less-developed countries tend to approach the rate of the economically developed countries, and consequently the gap between them decreases. This is a most favourable point for the less developed countries, as far as realized economic development is concerned.

5. The rate of growth in Latin America

In the previous section, we said that the rate of growth in less-developed countries experienced a considerable increase during the period 1960-70. In this section, we will see to what extent the aforementioned ascertainment is for Latin America countries. On the other hand, this period of time (1960-70) is the sampling period for the analysis of development of the variables of the model to be applied and consequently, the importance of the picture of the economic growth of these countries increases in this case.

In accordance with the estimates made by the Economic Commission for Latin America (ECLA), the average annual rate of growth of the gross domestic product was 5.3 per cent during the period

1/ The rate of growth in the United States and Europe for the period 1960-1970 was 4.6 percent and 4.7 percent respectively. (United Nations, Yearbook of National Accounts, 1971. New York, U.S.A., 1972).

1960-70 and it showed a slight increase if we compare with the previous decade 1950-60 (4.6 per cent).[1] This rate of growth is higher than the average rate of the less-developed countries as a whole in the same period. The rate of growth in Latin America varies in each country. There are countries whose rate of growth is at low level as for instance Haiti (0.5 percent), Uruguay (1.3 per cent) and countries as for instance Panama (8.0 per cent), Mexico (7.2 per cent), whose rate of growth reaches or exceeds the rate of the developed countries. More specifically, the rate of growth for the countries in which the model will be applied, is shown in the following table for the past period 1960-70.

Table 5.1. Average rate of growth of the gross domestic product at market prices

Country	1960-70
Argentina	3.7
Chile	4.3
Mexico	7.2
Panama	8.0
Peru	4.9
Venezuela	5.8

<u>Source</u>: Economic Commission for Latin America. United Nations, New York, 1972.

The internatinnal development strategy provides that the rate of growth should be at least 6 per cent for the developing countries. For some countries in Latin America this target does not seem possible to be achieved while for the others it can be said that they will reach or exceed the rate of growth of 6 per cent. Here, it is worth to mention the conclusion of the Economic Commission for Latin America that the rate of growth of 6 per cent can not be considered enough for Latin American countries as a whole to overcome the serious socio-economic problems.

1/ United Nations, ECLA. <u>Economic Survey of Latin America, 1970</u>. United Nations, New York, 1972. pp. 9-11.

6. The population in Latin American countries

In this section, we shall give a short picture of the population of Latin American countries as a whole, from the point of view of size, rate of growth and sex distribution. Latin America at present has the highest annual population increase. During the 1960's the average annual growth rate for the region as a whole was 2.86 percent. This rate of growth followed an upward trend, if it is compared with the rate of growth of the two last decades, 1940-1950 and 1950-1960, which was 2.34 percent twenty countries of Latin America was 275 million persons in the year 1970, compared with 138 million persons which was the population of these countries in the year 1945.2/ This high rate of growth had as a result a doubling (98.7 per cent) of the population of those countries furing the last twenty five years (1945-1970).3/

Considering the population of these twenty countries as a whole, and disregarding its regional distribution, the sexes are evenly balanced, with a small surplus of males which, however, is being reduced from census to census.

1/ United Nations, ECLA. Economic Survey of Latin America. United Nations, New York, 1972. pp. 12-14.

2/ Latin American Demographic Centre (CELADE). Boletín Demográfico. Year VII, N° 13, Santiago, Chile, 1974. pp. 6-7.

3/ In accordance with the estimates made by CELADE, the population in Latin America will reach 612 million inhabitants in the year 2000, while the population of less-developed countries whose population rate of growth is 2-4 per cent on the average will be 5 050 millions approximately in the same year. Bulletin of Labour Statistics (1974 World Population Year). International Labour Office, Geneva, 1974.

Table 1. Percentage distribution of sexes during the period 1950-1970

Sex	Years		
	1950	1960	1970
Male	50.41	50.24	50.12
Female	49.59	49.76	49.88

Source: Latin American Demographic Centre (CELADE), Boletín Demográfico N° 13. Santiago, Chile, 1974.

Based on data of the last census (1970), the number of males and females varies, in the first two five-age groups (0-9) and this can be attributed to the infant mortality. As regards the distribution by age, it can be said to be normal, in the sense that there exists in the population the proportion between the young and old age groups. Finally, the population distribution by region of residence -urban and rural- showed the following changes during the period 1950-1970. The urban population was 149 million persons (54.41 percent) in 1970 compared with 98 million persons (47.27 of the total population) in 1960 and with 62 million persons (39.22 percent) in the year 1950. Consequently, the urban population, increased by 142.1 percent and 52.1 percent in 1970, in comparison with the years 1950 and 1960 respectively. The rural population showed a drop during the aforementioned period 1950-70.

In conclusion, we can say that the population development in Latin American countries is characterized in the main by a high natural increase. Furthermore, the increase of population did not cause considerable changes in the age and sex structure of the population, in the twenty Latin American countries under survey. On the contrary, the urbanisation of these countries showed a considerable increase during the two last decades, 1950-1970.

7. International migration

The emigration movement between the countries of Latin America can be considered very small, although its evolution showed an upward trend since the year 1950 up to day. The number of emigrants was 952 thousands, in the year 1960 and forteen countries of Latin America. This size of external emigration was .84 percent of the population of those countries.[1/] The emigration flows are mainly among neighbouring countries such as, for example, from Bolivia, Chile, Paraguay, to Argentina, etc. The reasons for these international migrations are mainly economic. Unfortunately, the lack of statistical data, which in the instance is great, does not allow us to expand more on the demographic variable.

1/ Morales, J., Panorama de la Migración Internacional entre Países Latinoamericanos. Latin American Demographic Centre (CELADE), Series A, N° 121, Santiago, Chile, 1974. pp. 10-13. The forteen countries are: Argentina, Colombia, Costa Rica, Chile, El Salvador, Honduras, México, Nicaragua, Panamá, Paraguay, Perú, República Dominicana and Uruguay.

CHAPTER 3. BASIC HYPOTHESES. FUNCTIONAL RELATIONSHIPS

1. Per capita income changes

As it is known, per capita income which, as we have said, is the main dimension for measuring the degree of economic growth, it is high in some countries and low in others. Furthermore, per capita income changes in a given period of time, in each country. By definition, per capita income is the ratio of national income to the population of a country. Consequently, the changes in per capita income are attributable to the changes which occur to the two variables, national income and population. As a result, it is considered advisable to see which factors influence the changes of national income and the size of population. Thus, we will have the factors which cause the per capita changes.

In the model to be built -which is the main purpose of the present project- the main variables will be national income and population features. Furthermore, economic and demographic factors which are related to the national income will be included in the model. Thus, we would be able to determine the national income changes and to give a complete answer to the questions as regards the factors which contribute to the increase of national income -secondary scope of this work- and cause the per capita changes.[1/]

Therefore, in this chapter, we will try to determine the factors of population development, of labour force changes and the factors of the increase of national income which will be the central variable around which the model will be developed. In addition to this effort, we will try to relate the main factors of the model and their determinative factors in a system of equations. All the relationships to the formulated, on the basis of the existing theory, will be the hypotheses for testing by the model as a whole.

2. Population development

In accordance with the demographic theory, population development is influenced by natural increase and migratory movement. The natural increase is defined in the general manner of crude birth rates minus crude death rate, while external migration,

1/ Plans for economic development have mainly two objectives (1) maximization of national income and (2) maximization of employment.

which appears in the form of two opposite flows, the incoming inmigrants (input) and the outgoing emigrants (output), can be determined as the difference between these flows.1/ In the case that the number of incoming migrants exceeds the number of outgoing emigrants, we have net immigration which can be characterized as positive migration. In the opposite case, it is clear net emigration or negative migration.2/

Thus, we have

$$d(P) = N^m \pm M^n \qquad (1a)$$

or

$$d(P) = B - D \pm M^n \qquad (1b)$$ 3/

where

$d(P)$	:	Change of population
N^m	:	Excess of births over deaths
M^n	:	Net migration
B	:	Births
D	:	Deaths

The equality (1b) indicates that population change depends on births and deaths and net migration if international migration exists. The number of births and deaths depends on the size and

1/ Friedlander, A., Labour Migration and Economic Growth, The M.T.I. Press, Massachusetts, U.S.A., 1965. pp. 13-14.

2/ Frangos, D., Greek-English Dictionary. Athens, Greece, 1967. pp. 82-88.

3/ For certain countries of Latin America, in which the economic-demographic model is applied, we have negative external migration, that is, net emigration and consequently we use the formula

$$d(P) = B - D - M^n$$

where the symbols

$d(P)$, B, D and M^n as in the relationship (1b).

age distribution of population and upon fertility and mortality rates by age and sex.[1] Finally, external migration, which as we have stated, influences the size and age distribution of the population, mainly depends on economic and secondarily on social and political reasons.

3. Growth of domestic output

Growth of output of a country is dependent upon its population and other factors.[2] The "other" factors are capital and natural resources.[3] This can be written, in a mathematical form as follows

$$Y = f\ (P,\ K\ Q) \qquad (1a)$$

or

in explicit form

$$f(Y,\ P,\ K\ Q) = 0 \qquad (1b)$$

where

Y : Domestic output
P : Population
K : Capital
Q : Natural resources
f : Symbol of function

The functional relationships (1a-b) express the influence of population and other factors, such as capital and land, on changes in the domestic product.

However, output is directly dependent on the working population, rather than on total population. In fact, a fraction of the population contributes to the productive procedure, the labour-force and especially the employed labour force. Of the other factors, capital can be considered the most important factor,

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1/ General Electric's Center. Population Growth and Economic Development. TEMPO Press. California, U.S.A. pp. 3-5.
2/ Spengler, J., Points of Contact between the Growth of Population and the Growth of National Production. United Nations, Vol. IV, U.N., New York, 1965.
3/ Kanelopoulos, A., Economic Development. Vol.I.Athens, Greece. pp. 265-267.

while the factor of land in a production function such as (1b) function, which refers to the whole economy of the country, can be considered as constant and consequently, it is omitted among the independent variables.[1/] As a consequence, functional relationship (1b) can be written in the following form

$$Y = f\ (L^e, K) \qquad (2a)$$

where

L^e : Employed labour force
K : Capital
Y,f: As in relationship (1b)

Furthermore, in the case that capital is fixed in the short-run, but the employed labour force changes, then the domestic output will be a function of labour force variable, as follows:[2/]

$$Y = f\ (L^e) \qquad (2b)$$

The same can be said in the case that the amount of labour force is constant in a period of time and capital increases, then we have

$$Y = f\ (K) \qquad (2c)$$

Finally, the above production function (2a), based on theoretical considerations, has been specified by Gobb-Douglas in the following form:[3/]

$$Y = a_o L^{a_1} K^{a_2} \qquad (3)$$

where

Y, L^e, K : As in relationship (2a)
a_o : Arbitrary constant
a_1, a_2 : Coefficients

Function (3), similarly to function (2a) expresses a relationship between output, employed labour force and capital for a given level of technology and socio-economic organisation.

1/ Koutsogianni-Kokkova, A., Production Functionsin Greek Economy. KEPE. Athens, Greece, 1964. pp. 23-25.
2/ Brennan, M., Preface to Econometrics. South Western Publishing Co., Ohio, U.S.A., 1965. pp. 59-63.
3/ Gobb, C. and Douglas, P., "A Theory of Production". American Economic Review, Vol. XVIII, March, 1928.

Finally, the production function (2a) can be called "national" production function because it refers to the national level.

4. Labour force

4.1. Determinative factors

Labour force of a country, as we saw in the "national" production function, enters as a main independent variable. Labour force is that part of the total population which contributes by the supply of work to production process for economic goals and services and it is usually composed of individuals aged 15-64 able to and desiring to work.1/ The size of the labour force is a dynamic demographic "dimension", that is, it changes over time, and it depends mainly upon demographic factors.2/ The size of the population is a determinative factor of the size of labour force. An increase of population, will cause an increase of the age 15-64 groups, that is, the labour force. As a sequence, the growth of the labour force of a country is an increasing function of its population, in the form

$$L^a = f(P) \qquad (1)$$

where

L^a : Total labour force
P : Population of a country

Another determinative factor of the size of the labour force is the degree of participation of the age groups (15-64) in the labour force.3/ An increase of participation rates will increase the size of labour force. Thus, the relationship (1) can be written as follows

$$L^a = f(P, R) \qquad (2)$$

1/ The age limits differ in each country; for this reason, mention will be made of each country whose labour force is investigated in the present work.

2/ Siampos, G., Demographic Trends in Greece. Ministry of Co-ordination. National Press, Athens, 1969. pp. 185-188.

3/ Hatzoylou, S., Participation Rates of Greek Population. Athens, Greece, 1966. pp. 8-10.

where

L^a, P : As in the relationship (1)

R : Participation rates of labour force

Finally, certain factors, as for instance, education system, military service, etc., influence the participation rates of some age groups (15-24 aged) and consequently these factors cause changes in the size and distribution by age and sex of labour force of these age groups.

4.2. Changes in employment and unemployment

The labour force of a country, which as we said in the previous section, is the "supply of work", is classified in two groups. The employed -demand for labour force- and unemployed labour force.1/ The growth of demand for labour force is determined by the available labour force and economic factors, for instance the level of production, capital, etc. Thus, an increase of output, if labour force is not increasing as rapidly as output, creates more jobs and consequently will cause an increase of employed labour force. This relationship, under the above constraint, can be expressed as follows

$$L^e = f\ (L^a,\ Y) \qquad (1)$$

where

L^e : Employed labour force

L^a : Total labour force

Y : National output

By the same way, we can think of regarding the factor capital, the increase of which will cause an increase of demand for labour force.2/ In these two cases, an increase in labour force, with the level of production, or capital constant, will cause an increase of unemployment.

1/ United Nations. Application of International Standards to Census Data on the Economically Active Population. United Nations, New York, 1951. pp. 12-14.

2/ General Electric's Center. Population Growth and Economic Development. TEMPO Press. California, U.S.A. pp. 3-5.

Here, it is considered advisable to mention the function used by Ferber, R. and Verdoorn, R., as regards the estimation of demand for labour force in industrial branches.[1] They use as independent variables the level of production at time, t, and lagged one period, t-1, as follows:

$$L^e : f(Y, Y_{t-1}) \qquad (2)$$

L^e : Employed labour force

Y_t : Production, at time, t

Y_{t-1} : Production lagged one year, t-1

Furthermore, a similar to above (a) function was introduced by Gary, F. and Klein, L.[2] They use as explanatory variables of demand for labour force, the employed labour force lagged one year, L^e_{t-1} instead of Y_{t-1} and the "time" as follows:

$$L^e = f\ (Y, L^e_{t-1}, t) \qquad (3)$$

where

L^e_{t-1} : Employed labour force lagged one year, t-1

t : Time

Y : As in the relationship (2)

Finally, another similar function to above (2) and (3) was used by Professor Klein, L., and Goldberger in the econometric model of the United States 1929-52 as regards the demand for labour force.[3] Both functions (2) and (3) are indicated for

1/ Ferber, R. and Verdoorn, P. Research Methods in Economics and Business. MacMillan Publishing Co., New York, 1962. pp. 374.

2/ Gary, F. and Klein, L., The Brookings Quarterly Econometric Model of the United States. North-Holland Publishing Co., Amsterdam, 1965. pp. 699-702.

3/ Klein, L. and Goldberger, A., An Econometric Model of the United States, 1929-52. North Holland Publishing Co., 1955. pp. 16-17.

$$L^e = \left(\frac{W_o}{W}\right) (a_o + a_1 Y_t + a_2 Y_{t-1} + a_3 t)$$

where

Y_t, Y_{t-1}, t : As in the relationships (2) and (3)

W_o : Value of production

W : Wages per man employed

empirical analysis and forecasting.

As regards the unemployment we can say that the changes in its size are affected by the available labour force and the increase of demand for labour force. Thus an increase or decrease of employed labour force will cause a decrease or increase of unemployment respectively.

So we have

$$L^u = f\ (L^e) \qquad (4)$$

where

L^u : Unemployment labour force

L^e : As in the relationship (1)

5. Relationship capital and savings

In Section 3, we adopted the hypothesis that output is a function of capital, i.e.

$$Y = f\ (K) \qquad (1)$$

where

Y, K : as in the relationship (2a) of the 3rd section

If we assume that all savings are converted into productive capital and foreign capital is exogenously determined, then we have the following identity:1/

$$S \triangle K \qquad (2)$$

where

S : Savings

$\triangle$: Symbol of identity

Substituting the identity (2) in the function (1), we get

$$Y = f\ (S) \qquad (3)$$

1/ This hypothesis was assumed by Friedlander, L., in his model. Labour Migration and Economic Growth. The M.I.T. Press, Massachusetts, U.S.A., 1965. pp. 13-14.

The relationship (3) expresses that national output, Y, is an increasing function of savings, S. Finally there exists a relationship between Capital and Savings in the sense that capital, K, increases annually by the amount of domestic savings, S, plus possible assistance from abroad.[1]

6. International migration

In Section 2, we said that external migration is a factor of population development, particularly in migratory countries. Emigration, apart from changes in the size of the population, also causes changes in its structure, by age and sex, in the critical age-groups of 20-44.[2] The reason for this is that emigrants generally belong to these age-groups. These age-groups (20-44) are a fraction of the labour force of the country. As a result, it can be said that external migration, which is a movement of the labour force from one country to the other, influences the labour force, both as regards its size and its distribution by age and sex.[3] These movements of the labour force

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1/ This hypothesis was assumed by Friedlander, L., in his model. Labour migration and economic growth. The M.T.I. Press, Massachusetts, U.S.A., 1965. pp. 13-14.

2/ In the case of immigration, we have an increase of the labour force, whilst in the case of emigration, we have a decrease in the number of individuals in the critical age-groups (20-44 aged).

3/ External emigration influences the natural increase of population, by means of a change in the birth rate because the emigrants belong to the reproductive age groups, and by a change in the death rate.

are mainly attributable to economic reasons, such as wage-rates, unemployment, other conditions (housing, work, etc.), etc. which are incorporated in the factor per capita income. Therefore, international migration can be considered as a function of the differentials of per capita income between two countries where an emigration flow exists.1/

$$d\ (M) = f\left({}^{Y^a}/_{P^a} - {}^{Y^b}/_{P^b}\right) \qquad (1)$$

where

$d\ (M)$: The change of emigration

${}^{Y^a}/_{P^a}$: Per capita income of country, a

${}^{Y^b}/_{P^b}$: Per capita income of country, b.

7. Technological progress

We have already determined the main factors, demographic and economic, which contribute to the increase of national output. With the passage of time we see a quantitative change in relationship between the factors which consist of the "national" production function. This change is the result of technological progress, which is achieved in the meantime by the application of innovations in the economy.2/ Therefore, we can say that the increase of national product is also attributable to the technological progress.3/ In the statistical estimation of the aforementioned function, we shall see in what way the factor, technology, enters to it.

1/ Friedlander, L., Labour Migration and Economic Growth. The M.I.T. Press, Massachusetts, U.S.A., 1965. pp. 21-28.

2/ Kanelopoulos, A., Economic Development. Vol. I, Athens, Greece. pp. 149-150 and 265-266.

3/ In the relationship between inputs and outputs, technological progress influences as follows: an increase of output with given inputs (neutral change) or unchange of output with the decreased inputs (biased change). Sarantides, S., An Introduction to Economic Analysis. Karaberopoulos, S. Publishing Co., Piraeus, 1971. pp. 274-275.

8. Socio-economic organisation

Socio-economic organisation is a fundamental factor for economic development in general. As it is known, the demands of the economy and particularly those of its industrial sector, increase in manpower. For the coverage of the needs in labour force, population movements (from rural to urban areas), training of new workers will take place. The population movements, as it is known, create many problems, for instance, housing, education, pollution, transportation, etc. while the general and professional education is a problem of organisation. Furthermore we have to meet the problems of the new living conditions of emigrants, that is, family, social problems. On the other hand, innovations, the result of the great inventions, enter to the economy and the application of these innovations necessitate training.

To meet all the aforementioned problems, an organisation, administrative, social and economic, is necessary. We shall not extend ourselves on this factor of socio-economic organisation, because the importance of this factor is self-evident as regards the effort being made by State to increase national output.

9. Consumption

In addition to the factors which affect the changes in domestic output, the analysis of which, from the point of view of functional relationships we realized in the previous sections of the present chapter, there are also economic factors which are affected by the main dimensions of the economic and demographic part of the model, that is, of domestic output and population respectively. Among these economic variables, consumption can be considered the most significant.

Total -private and public- consumption consists of all production not used for net capital accumulation, that is, it consists of all the goods produced and consumed. The principal factors of the consumption functions are the following:[1/]

1/ Pavlopoulos, P., A Statistical Model for Greek Economy 1949-59. North-Holland Publishing Co., Amsterdam, 1966. pp. 36-39.

a) income

b) population factors

c) liquid assets

d) the influence of past standards of living.

In accordance with national income theory, aggregate consumption is a function of income. In this case we refer both to the magnitude of income and its distribution. In the less-developed countries, the income can be considered the most important factor of the level of consumption due to the relatively low standards of living in these countries, etc. Furthermore, it can be said that the actual income is itself only an approximation to the true casual variable of the consumption function, which may be called normal or permanent income.1/

Thus, the consumption function in the simple form is

$$C_t = f(Y_t) \qquad (1)$$

where

C_t : Consumption, at time t

Y_t : Income at time t

As regards the distribution of income, we can use the agricultural income and urban income as separate variables.

Consumption changes, apart from the income, depend on population changes. Such changes refer to the size of population, the age distribution, the distribution of population by region, family size and the occupational distribution.2/

Thus, the functional relationship (1) can be written as follows:

$$C_t = f(Y_t, P) \qquad (2)$$

where

C_t, Y_t : as in relationship (1)

P : population

1/ Friedman, M., A Theory of the Consumption Function, Princeton University Press, 1957.

2/ As regards the population distribution by region, we use the rural and urban population. This distribution corresponds to the aforementioned income distribution. Moreover, the urban per capita income is higher than the rural income. This means that the consumption is greater in the urban areas.

The consumption function (2) is assumed to be a linear function as follows:1/

$$C_t = a + b\ Y_t + c\ P_t \tag{3}$$

The liquid assets is also a determinant of the level of consumption.2/

In the language of functional relationships, we have

$$C_t = f\ (Y_t, P_t, A_t) \tag{4}$$

where

A_t : Total national liquid assets of households, at time t.

The use of the variable, liquid assets, A, as an explanatory variable in the consumption function is very limited. This is because in many economies the market for liquid assets is very small.

Finally, the past standards of living influences the size of consumption. The habits accumulated through time, will exercise an influence at the present time. In this case, the variable consumption lagged one year, C_{t-1}, is introduced in the function under study, as follows:3/

$$C_t = f\ (Y_t, C_{t-1}) \tag{5}$$

where

C_t, Y_t : As in relationship (1)

C_{t-1} : Consumption lagged one year

1/ TEMPO: Description of the Economic-Demographic Model. TEMPO. General Electric's Center for Advanced Studies. Santa Barbara, California, U.S.A., pp. 19-21 and 26.

2/ Brennan, M., Preface to Econometrics. South-Western Publishing Company, Ohio, U.S.A., 1965. pp. 345-346 and Suits, D., An Econometric Model of the Greek Economy. KEPE. Athens, Greece, 1966. pp. 23-40.

3/ Pavlopoulos, P., A Statistical Model for Greek Economy 1949-59. North-Holland Publishing Co., Amsterdam, 1966. pp. 40-41.

CHAPTER 4. AN ECONOMIC-DEMOGRAPHIC MODEL

4.1. Population as an endogenous variable

Many scientists have stated theories regarding population development.1/ In certain of these theories an effort was made to explain the changes in population in conjunction with the socio-economic growth of peoples.2/ Certain of these theories were not verified by ensuing reality, while others were considered as being obsolete. Thus, one of the most important theories, the Malthusian theory, has been given up, as having many drawbacks, from many points of view,3/ while the more modern theory concerning population of W.S. Thompson and F.W. Notestein, which also refers to the socio-economic level of countries, has had a small application.4/

On the basis of the above thoughts, the increase of population, in the studies of socio-economic phenomenon was considered as an endogenous variable.5/ Therefore, the population factor was the main dimension in the models of classical economists. The models developed since then, considered economic and demographic variables independent of one another -neoclassic theory. Thus, many neo-classical growth models, demographic variables, were assumed to be determined exogenously.6/ However, the recent economic literature in the field of economic development, inherited much

1/ United Nations, "History of Population Theories", included in The Determinants and Consequences of Population Trends, United Nations, Population Division. New York, 1953.

2/ Theories regarding population a) that of Professor of Political Economy Thomas R. Malthus, "The Principles of Population" b) that of W.S. Thompson and F.W. Notestein, "Theory of Demographic Transition", 1950.

3/ According to the Malthusian Theory, the increase of agricultural production was not foreseen, resulting from the improvement of production methods, as also the high standard of living, as a result of industrialization, the development of transportation and communication, and the decrease of birth rates, on account of birth control.

4/ In recent years it has been supported that neither the European countries developed exactly as foreseen by the theory of "transition" nor have the economically developed countries developed on the basis of the aforementioned theory.

5/ Nelson, R., "Theory of the Low-Level Equilibrium Trap in Underdeveloped Countries", American Economic Review, December 1966. pp. 894-908.

6/ The explanation of demographic behaviour,"was left to demographers, sociologists and biologists, has stated J. Schultz. "The Value of Children, An Economic Perspective". Journal of Political Economy, Vol. 81 March/April 1973.

from the theory of the classics, particularly as regards the importance of population, and the viewpoints of Malthus concerning population, hold a noteworthy position in present-day theory.[1/] We therefore have a return to the classical treatment of population development, as an endogenous variable.[2/] This is obvious, because the existence of interrelationships and interactions between demographic and economic growth has been proved and this has been acknowledged by the makers of demographic and economic policy.[3/] A great number of theoretical and empirical analyses have incorporated population evolution as an endogenous variable, affecting and being affected by economic growth.[4/]

1/ Kanelopoulos, A., Economic Development, Vol. I, Athens, Greece. pp. 17-21 and 272-291.

2/ Friedlander, S., Labour Migration and Economic Growth. The M.I.T. Press, Massachusetts, U.S.A., 1965. pp. 10-12.

3/ Adelman, I. "An Econometric Analysis of Population Growth", The American Economic Review, Vol. 23, 1963. pp. 315-339.

4/ Here we give some of the main and the most recent studies on this subject. Beckmann, M., "Income Growth and Population Growth", European Economic Review, Brussels, Belgium, 1970, pp. 546-552. Brandt, W., Some Observations on the Relationships between Population Increase and Economic Growth. United Nations, World Population Conference, 1965. Choudhry, Nand Kotowitz, V. Some Simple Economic-Demographic Relationships, United Nations, World Population Conference, 1965. Habakuk, J., Population Growth and Economic Development, Leicester University Press, Leicester, 1971. Orstenko, V., Influence of Social and Economic Factors on Demographic Characteristics, United Nations, World Population Conference, 1965. Ronald, B.,"Steady-State Paths in an Economy with Endogenous Population Growth", Western Economic Journal, December 1970. Horvat, B., Relation between the Rate of Growth and the Level of Development. Development Research Centre, Bloomington, 1972. Lloyd, R., "A Growth Model with Population as an Endogenous Variable", Population Studies Review, Vol. XXVII, November 1969. López, T., Population Factors in Latin American Economic Development, Universidad de los Andes, Bogotá, 1967. Sudhakar, G., "Interaction between Population Growth and Some Socio-Economic Indicators, Journal of Institute of Economic Research, Dharwar, January, 1968. Thirewall, A., "A Cross Section of Output and Per Capita Income in a Production Function", Manchester School of Economic and Social Studies, N° 14, December 1972. General Electric's Center, Population Growth and Economic Development, TEMPO, General Electric's Center for Advanced Studies, Santa Barbara, California, U.S.A. The International Labour Office is making an effort to investigate the demographic-economic characteristics in conjunction with educational system, for Asian countries. World Employment Programme, Geneve, 1973. Stone, R., Demographic Accounting and Model Building, OECD, Paris, 1971 is a successful effort of coordinating, within a system, a great variety of demographic and social statistics.

2. Theoretical basis of the model

2.1. Interdependent systems

Models are frequently used in economic analysis for solving quantitative problems. They can be expressed by means of an equation or a system of equations. The equations -structural equations- express the theoretical causalities and interactions which are to be tested against the statistical data. More specifically, they describe the relationships which exist between the sectors of the economic activity or to other non-economic parts and which must be estimated statistically from empirical observations.1/ It is possible, an interdependence between variables of a system of equations to be. This means that the dependent variable of an equation to be explanatory variable of another equation of the same system of equations and vice versus. In this case we have a system of regression equations with interdependent variables. This system is called interdependent system. Models of more than one equation and the aforementioned type appear in a compound form, on account of the interactions and interdependences of the several variables to be predicted. Such models have two classes of variables: endogenous variables -those explained by the model- affecting one or more variables, but also being affected by one or more variables, and the exogenous ones, which are not explained by the model, they affect variables of the system, but are not affected by the variables of the system.2/ These latter variables are also called predetermined variables.

Furthermore, it is conventional, the variables to be predicted by the model, i.e. the endogenous variables to be denoted as Y's and the predetermined variables, as Z's, each with suitable subscripts.

1/ Suits, D., The Theory and Application of Econometric Models, Center of Economic Research, Athens, Greece, 1963 and An Economic Model of the Greek Economy, Center of Economic Research, Athens, Greece, 1964. pp. 19-20.

2/ Drakalos, C., Econometrics. S. Kloukinas Publishing Co., Athens, Greece, 1971. pp. 114-119.

Finally, a model has three aspects: its economic content, its mathematical form and its technical problems. Economic theory determines the economic aspects of the model, while the suitable mathematical form of the model will allow us the accurate estimation of the parameters. Both economic aspects and statistical criteria will verify if the adopted form of the model would be the appropriate one. The technical problems are concerned with quantitative expression of the variables, statistical properties of error terms, the problem of identification, etc. As regards the estimation of the parameters, it is made on the empirical data by applying one of the traditional methods, ordinary least squares (OLS) or two-stage least squares (TSLS). In more details, for the aforementioned technical problems, methods etc., as well as how successful is the estimation of structural coefficients, we refer to in the next sections of the present chapter and in the empirical analysis of the model applied.

2.2. The mathematical consideration of the interdependent system.

2.2.1. The structural form

Here, we will deal with the mathematical form of interdependent system of equations. In other words what is the proper form of such a model that it must take in order to estimate the parameters of the variables used.

In general, an interdependent system -model- is a system of linear equations relating the endogenous variables, Y, to the predetermined variables, Z, and the disturbance errors, e.[1/]

1/ Such models have had a wide application in the socio-economic sciences (Wold, H. The Approach of Model Building and the Possibilities of its Utilization in Human Sciences. University of Uppsala. Uppsala, Sweden, 1964).

Thus, a model of n equations with the same number of endogenous-interdependent-variables, Y, and n predetermined variables, Z, can be mathematically represented in the following manner:[2]

$$
\begin{array}{llllll}
Y_{1t} = b_{12} Y_{2t} & + & c_{11} Z_{1t} & + & e_{1t} \\
Y_{2t} = b_{21} Y_{1t} & + & c_{22} Z_{2t} & + & e_{2t} \\
Y_{3t} = b_{32} Y_{2t} & + & c_{33} Z_{3t} & + & e_{3t} \\
\cdots & & \cdots & & \cdots \\
\cdots & & \cdots & & \cdots \\
\cdots & & \cdots & & \cdots \\
Y_{n-1,t} = b_{n-1,n-2} Y_{n-2,t} & + & c_{n-1,n-1} Z_{n-1,t} & + & e_{n-1,t} \\
Y_{nt} = b_{n,n-1} Y_{n-1,t} & + & c_{n,n} Z_{n,t} & + & e_{nt}
\end{array} \qquad (1)
$$

where

$$
Y_t = \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ Y_{3t} \\ \vdots \\ Y_{n-1,t} \\ Y_{nt} \end{bmatrix} \qquad
Z_t = \begin{bmatrix} Z_{1t} \\ Z_{2t} \\ Z_{3t} \\ \vdots \\ Z_{n-1,t} \\ Z_{nt} \end{bmatrix} \qquad
e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ \vdots \\ e_{n-1,t} \\ e_{nt} \end{bmatrix}
$$

are endogenous, predetermined variables and error terms, respectively.

1/ In an economic-demographic model, the variables can be of both categories, economic and demographic variables. Furthermore, the n-predetermined variables which are in the model (1), can also be in each equation of the system. In this case, only the order of elements of the matrix, C, will change.

The system of equations (1) can be expressed in matrix form as follows:

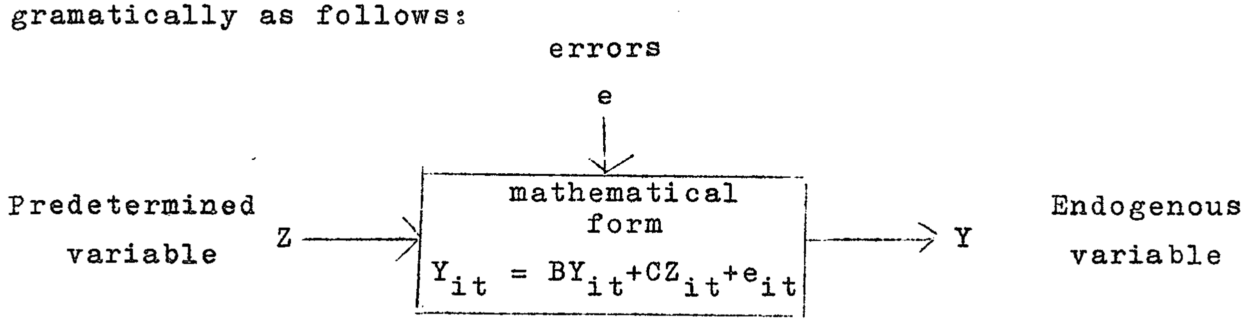
$$Y_{it} = BY_{it} + CZ_{it} + e_{it} \qquad \text{Structural form} \quad (2)$$

where:

$$B = \begin{bmatrix} 0 & b_{12} & 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{32} & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & b_{n-1,n-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{n,n-1} & 0 \end{bmatrix} \quad C = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & c_{n-1,n-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{nn} \end{bmatrix}$$

Mathematical completeness requires that there will be as many linearly independent equations as endogenous variables; hence the matrix B must be square and invertible. Furthermore, it is possible, some of the exogenous variables whose values are determined outside the system, will be lagged endogenous variables whose values are also predetermined variables.1/

Finally, the aforementioned model can be represented diagramatically as follows:



1/ Walters, A., An Introduction to Econometrics, MacMillan Publishing Co., London, 1970. pp. 180-181.

2.2.2. The reduced form

The explicit solution of the structural form (2) for Y_{it}, can be made as follows:

$$Y_{it} - BY_{it} = CZ_{it} + e_{it} \qquad (3)^{1/}$$

multiplying by the unit matrix, I, we have

$$IY_{it} - BY_{it} = CZ_{it} + e_{it} \qquad (4)$$

and

$$(I-B)\ Y_{it} = CZ_{it} + e_{it} \qquad (5)$$

Since the matrix B is square and invertible, we multiply the above equation by $(I-B)^{-1}$ and we get

$$(I-B)^{-1}\ (1-B)\ Y_{it} = (I-B)^{-1}\ (CZ_{it} + e_{it}) \qquad (6)$$

Hence

$$Y_{it} = (I-B)^{-1}\ CZ_{it} + e^{*}_{it} \qquad \text{Reduced form} \qquad (7)$$

in which

$$e^{*}_{it} = (I-B)^{-1} e_{it}$$

where

Y_{it}, Z_{it}, e_{it} as in the system (1)

and

matrices B, C as in the system (2)

The reduced form (7) is indicated for making forecast of Y_{it}, in terms of the predetermined variables, Z_{it}.

1/ Wold, H., Econometric Model Building (Forecasting by the Chain Building). North-Holland Publishing Co., Amsterdam, 1964. pp. 25-30.

3. Sequence of the variables in the model building based on the hypotheses. The schematic view of the model

In this section, we shall see the sequence of the economic and demographic variables in the construction of the model, through the relationships - hypotheses mentioned in the previous chapter.

As it is known, an economic-demographic model consists of two parts, the demographic part and the economic part. The first part includes the demographic factors, their relationships, etc. whereas the second part refers to economic variables, their function in the economic system, etc. As mentioned elsewhere, the investigation of interrelationship between these two parts can be realized by two-opposite-directions. In the present work, the impact of population on the growth of national output will be studied, i.e. we follow the direction, from the demographic to the economic part.

In the demographic part, the main dimension is the population, i.e. its size and distribution by age and sex. The factors of the increase of the population and the change of its structure, are natural increase and net migration. Fertility and mortality determine the natural movement, while immigration and emigration determine positive or negative migration, which influence the size and the distribution by age and sex of the population. Internal migration causes population changes with respect to residence (rural and urban population) and of course, these affect the natural movement of the population.

On the other hand, labour force -size and structure- mainly depends on the size of population and its distribution by age, sex, and the participation rates. Furthermore, labour force is influenced by net emigration. In accordance with the classification of labour force (previous chapter, section 4.2.), a fraction of the labour force is the employed labour force, the size of which affects and is affected by domestic output. Apart from domestic output, capital influences the number of individuals employed. As regards the first part of the model we shall not enter into further details on the demographic features, their interactions, etc. nor will we refer to the purely demographic technique used to make population forecasts. This is because population trends for coming years, in Latin American countries, have already been established[1] and, consequently, we shall use these forecasts in order to estimate the size of the labour force and especially, the employed labour force which enters in the model as one of its main variables.

As regards the second part -economic part -of the model, domestic output is the main variable. Based on the hypotheses -national production function, chapter 3, section 2- we have accepted that the employed labour force and capital determine the size of gross domestic output. On the other hand, the growth in employed labour force is determined by the growth of labour force and domestic output or capital, while these latter variables are influenced by savings. Furthermore, the technological progress and socio-economic organization are also two fundamental factors which affect the increase of national output. As regards the consumption, which is a variable to be predicted outside of the model, it is affected by the main dimensions, domestic output and population, of the system.

From the study of the sequence of the variables in the two sub-systems, demographic and economic, and the interrelationships of these variables in the whole system, we can say, in conclusion, that the variables employed labour force and capital, are the most significant for the determination of the domestic output, which together with the population consist of the main determinative factors of the consumption. Moreover, domestic output and population are the two elements by which we can determine the per capita income.

Finally, it is advisable to say that the economic part is described in terms of a set of flow variables while the demographic sub-system is stated in term of stocks. Of course, there are deviations from this general rule, and they are mentioned in

1/ Latin American Demographic Centre (CELADE). Boletín Demográfico. Year VII, N° 13. Santiago, Chile, 1974 and International Labour Office (ILO). Labour Force Projections. ILO. Geneva, Switzerland, 1971. Part III.

the relevant chapter 5, section 1 in which the quantitative expression of each variable of the model is studied.

Already the two parts have been given -demographic and economic- of the model, in which a set of demographic and economic relationships respectively, as also the inter-relationships between them, are described. Furthermore, the direction of each variable, from the point of view of its influence on the other, as same has been explained in the present section is also given.

On the basis of these, we shall below illustrate the model schematically. Figure 1 presents the model. Thus, in the left-hand half there is a collection of boxes containing demographic variables, while the right hand half consist of boxes containing economic variables. In the centre of the scheme, there is the variable, consumption, and at the top of this schematic form of the model, the per capita income. Of course, there are other factors, economic, social and demographic, which influence or are influenced by the variables of the model, but their presentation in the figure 1 was not considered necessary. This is because the figure 1 aims at giving a general picture of an economic-demographic model from the point of view of basic variables, their interrelationships etc. and in the model to be applied, a few of the aforementioned variables will enter.

4. The macro-model to be applied

In the mathematical consideration of the model, section 3, we saw that it involves a great number of endogenous and pre-determined variables. In the practice, such a number of variables in an economic-demographic model is unmanageable. It requires detailed data, which, in most cases, are not available and even if they were, we would face serious problems as regards the defining of the interrelationships, the statistical estimation of the parameters, etc. In general it is indicated that a model must refer to crucial variables so that their interrelationships can be easily determined and the statistical process of their estimation has not many difficulties.

4.2. Specification of the model

The model to be applied attempts to investigate the inter-relationships between basic economic and demographic variables. The determination of these variables, their interrelationships and the number of equations in the system to be tested against the empirical data, consist of the specification of the model. In the present case, the model attempts to interrelate the national output, available and employed labour force by means of the factors which, on the one hand contribute to the increase of national output, on the other hand, determine the development of supply of labour force and the size of employment.

The present macro-model consists of the three structural equations and one identity. The equations describe hypotheses about the interrelationship among a number of economic and demographic variables, while the identity defines the relationship -equality- that exists between demographic factors. The equations are stochastic and contain in the left hand side, the variables to be predicted by the model -endogenous variables- and in the right hand side, the predetermined variables. Moreover, in the right hand side of each equation there is an error term, on account for all the variables omitted from the functional relationship and for any deviation between the shape of the assumed function and the true relationship.[1/] This error term is assumed

1/ Walters, A., An Introduction to Econometrics. MacMillan Publishing Co., London, 1970. pp. 211-213.

to have some properties.[1] Furthermore, these equations are linear both in variables and in parameters.[2]

As regards the number of variables used by the model, it is eight and they fall in three classes: a) endogenous variables -there are four - which are explained by the model; b) lagged endogenous and c) current exogenous variables. In the system there is one lagged endogenous and three current exogenous variables. The variables of two latter classes -predetermined variables- are determined outside of the system of the equations. From a forecasting point of view, the lagged variables can be considered superior to the current exogenous variables. This is because the available information about the lagged variables is given, while values of the exogenous variables should be estimated themselves.[3] Finally, the presence of the lagged variables in a model makes it a dynamic model. In the next two sections we bring together the entire

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1/ The properties of the error term, e, refer to its probable distribution. They are the following:

a) error term, e, is a random variable with normal distribution and the mean to be zero, i.e. $E(e_i) = 0$ for $i = 12 \ldots n$

b) The variance of this variable to be fixed, i.e. $V(e_i)=S_e^2$ for $i = 1.2 \ldots n$

The properties, a and b, can be written as follows

$e_i = N\ (0,\ S_e^2)$ for $i = \ldots n$

c) The last property attributed to the e_i, is that its values are independent of one another, i.e., Covar $(e_i\ e_j) = 0$ for all $i \neq j$

(Gramer, J., Empirical Econometrics. North-Holland Publishing Co., Amsterdam, 1971. pp. 83-86).

2/ The meaning of linearity of parameters is that the parameters estimated are linear functions of the values of the variables.

3/ Pavlopoulos, P., A Statistical Model for the Greek Economy 1949-59. North-Holland Publishing Co., Amsterdam, 1966. pp. 29-32.

system of equations with a glossary of the symbols in order to obtain a picture of the model to be applied.

4.2. The equations

Based on the hypotheses, chapter 3, the sequence of the variables in the model building as it is mentioned in the third section of the present chapter and the specification of the model, the equations of the interdependent system to be tested against the data, are the following:

$$Y_t = a_o + a_1 L_t^e + a_2 K_t + e_{1t} \quad (1.1)$$

$$L_t^e = b_o + b_1 Y_t + b_2 L_{t-1}^a + e_{2t} \quad (1.2)$$

$$L_t^a = d_o + d_1 P_t + d_2 R_t + e_{4t} \quad (1.3)$$

$$L_t^u \equiv L_t^a - L_t^e \quad (1.4)$$

The equations describe the relationships which exist between the variables of the model. Thus, the first equation explains the national production.1/ The second determines the demand for employed labour force and the third equation refers to the development -supply- of the labour force. Consequently, we can say that in the model there is a production function, a demand function for employment and a supply function, of labour force. Furthermore, in the model there is an identity by which we define the unemployment. Of course, the gross domestic output and employed labour force are the central variables but also other demographic variables as available labour force, population, etc. appear in the model. Such variables are necessary in a demographic-economic model and make it more appropriate for forecasting purposes. The coefficients -structural coefficients- are the traditional constants of economic analysis, i.e. they measure the elasticities of explained variable with respect to the determinative factors of each equation.

Finally, the number of variables used by the model is eight of which four are endogenous -affected or being affected, by other variables-. Thus, the system has as many endogenous as there are equations in the system. From this point of view this system of equations can be considered complete.

1/ Koutsogianni-Kokkova, A., Production function of Greek Industry. KEPE. Athens, 1964. pp. 61-62.

4.3. List of Variables

The variables in the model by category and in order of appearance are:

Endogenous variables

1. Y : National output (value added), at constant prices (in millions)

2. L^e : Employed labour force (number of workers in thousands)

3. L^a : Available labour force (number of workers in thousands)

4. L^u : Unemployed labour force (number of workers in thousands)

Predetermined variables

5. K : Capital of the economy, at constant prices (in millions)

6. L^e_{t-1} : Employed labour force lagged one year, t-1 (number of workers in thousands)

7. P : Population (in thousands)

8. R : Participation rates of labour force (in percentage)

a_1, a2, d_1
b_1, b_2, d_2 : Structural coefficients
c_1, c_2, d_2

a_o, b_o
c_o, d_o : Constants

e : Random disturbance term

t : Time

Superscripts : Number = base year

Subscripts : They refer to the year, t or lagged one year, t-1.

5. Technical problems of the model

5.1. Quantitative measurement of the variables of the model

After the determination of the model to be applied, the next task is to define the quantitative expression of the various variables which enter in it, The first is production. Ideally, it should refer to the material units of standard type. This is difficult, because the units of product are heterogeneous. As a consequence, the monetary measure of output is that most indicated.[1/] Furthermore, the model refers to the national level and in this case, the value-added of the economy is the suitable.[2/] The variable "Labour" is also measured approximately, and this is so because we do not have "standard labour units" with no variation in skill or intensity, at our disposal. Furthermore, we do not have statistical data for used man-hours of work in the production. For these reasons, in the greater part of this work, we have to use the figures for the total number of individuals in employment.[3/] We meet most of the difficulties in the measurement of the capital variable. Capital is a stock concept and relates to a point of time. If we accept the idea of capital stock at any point of time, we will not have, in this manner the real contribution of the factor "capital" in the model under study. This is because, as argued by Professor Robinson, the capital stock at any time consists of a collection of heterogeneous capital equipment at various stages of its life cycle and various degrees of obsolescence.

Nor it is easy to provide as she points out, a price measure, because the unit measurement varies with the rate of profit and the relative prices of equipment are determined by the future profit expectations.[4/]

1/ The value of output must be deflated by a price index. In this case, we must also confront the "indices" problems.

2/ Koutsogianni-Kokkova, Production Functions in Greek Economy. KEPE. Athens, Greece, 1964. pp. 25-26 and 69-7 and Sarantides, S., An Introduction to Economic Analysis. Karaberopoulos, S. Publishing Co., Piraeus, 1971. p. 263.

3/ In this case, "Labour" appears in the sense of a stock concept, and not of a flow concept, as same is observed in the output variable (See Sarantides, S., p. 264).

4/ Robinson, Joan, "The Production Function and the Theory of Capital", The Review of Economic Studies, 1953-54.

Furthermore, in the case of underemployment of fixed capital, the estimation of the model -production function- will not give the accurate relationship between output-capital.[1/]

Finally, production is a flow concept relating to a period of time, in contrast with what we accepted for capital. If we accept the idea of capital services consumed over a period of time,[2/] although this viewpoint corresponds to reality, it introduces the problem of variation in the stock over time and variation in the rate of utilization. The lack, however, of statistical data on same does not permit us a statistical estimation of capital on the basis of the aforementioned concept. On the existing statistical techniques for estimating gross fixed capital, the most usual one is the "inventory method".[3/] An inventory is maintained of the capital stock at the base year prices by adding gross investiment and by deducting capital consumption. On these elements there are available statistical data.

As regards the variables, population, participation rates of labour force and private consumption expenditure there is not problem for their statistical estimation because we have available statistical data.

Finally, the influence of factor "technological progress" on the increase of output, can be measured by introducing in the model -production function- a simple time-trend or a multiplicative trend term.[4/] Of course, this is a manner of entry of the variable into the model under study, but both its analysis and the estimation is a difficult task in the macroeconomic time series data.

1/ See Sarantides, S., op. cit. pp. 263-264.

2/ In this case we have the flow concept of capital, i.e. capital consumption during the productivity procedure of a period of time.

3/ Barna, T., "The Replacement Cost of Fixed Assets in British Manufacturing Industry, in 1955". Journal of the Royal Statistical Society, 1975.

4/ Walters, A., An Introduction to Econometrics, MacMillan and Co., Ltd., London, 1970. pp. 300-304.

5.2. Economic time series data

For the estimation of the model we will use time series data, that is, observations on the economic variables, such as output, employment, consumption, etc. at different points of time. The use, however, of time series data, as it is known, creates some problems of a statistical nature. These problems, as also their confrontation, are mentioned below. The values of the variables in time series are usually given at current prices. The use of indices for the deflation of the time series, that is the expression of the values of the variables at constant prices creates, as elsewhere mentioned, various statistical problems.1/ Fortunately, the statistical data of the variables used in the model, are given at constant prices and, consequently the deflation procedure was not necessary, however, the statistical problems are involved in same. Furthermore, the presence of a trend in time series is another serious problems. It introduces a dependence between successive observation, and the elimination of this trend, from the time series is not possible in certain cases.2/

Among the problems which arise, in the use of the time series data, as regards the technological progress we can say that it is a large and important nuisance variable in macro-production function. This is because part of the output increase, during a period of time, is attributable to the development of the technology noted in this period. Addition to this, the rate of technological advance differs between the productivity units as also between the sectors of the economy. We shall not enter into other details regarding the time series data used, because we refer to them during the analysis of the statistical results obtained.

1/ Groxton, F.E. and Gowden, D.J., Applied General Statistics. Prentice Hall, New York. pp. 273-275.
2/ Brennan, M., Preface to Econometrics. South Western Publishing Co., Ohio, U.S.A., 1965. pp. 359-362.

5.3. The problem of identification

As it is known, the economic functional relationships -equations- are formulated on the basis of the existing economic theory. The structural equation of a system, which expresses one and only one economic relationship which has been determined by the theory, is called "identified" equation.[1/] Furthermore, we distinguish the "just identified" and the "over identified" equation.[2/] In other words, in the first case, it is possible to obtain unique values for all structural coefficients of the equation, whereas, in the second case, the estimation of such values can be made only under restrictions.[3/] Here, it is useful to reexpress the two conditions -necessary and sufficient- for identification and to investigate the system of equations which is applied from this point of view.

The necessary condition for the identification of an equation of the system is: the number of variables excluded from this equation but contained in the system, should be at least one less than the number of equations of the system. By n, we denote the number of equations, by G, the total number of variables[4/] and by H the number of variables in the equation in question.

Thus

if $G - H = n - 1$, then the equation is just identified

if $G - H > n - 1$, then the equation is over identified.

Based on the aforementioned condition, the model is over identified, because all the structural equations are over identified.[5/] On the other hand, the sufficient condition for the

1/ Drakatos, C., Econometrics. Kloukinas, S. Publishing Co., Athens, Greece, 1971. pp. 119-128.

2/ In the case of "not identified" equation (under identified), the entire set of structural equations can not be estimated statistically, a fact that means that the statistical values obtained can not be interpreted from economic point of view.

3/ Brennan, M., Preface to Econometrics. South-Western Publishing Co., Ohio, U.S.A., 1965. pp. 384-399.

4/ In the number of variables, the endogenous and predetermined variables are included.

5/ For instance, in the first equation of the system, the number of variables which are excluded from this equation and they are included in the model, i.e., G-H, is more than n-1. The same can be said for the remaining equations of the system.

identification of an equation belonging to a linear system of equations is: the construction of at least one non-zero determinant of n-1 order from the coefficients of the variables excluded from the equation under study but included in the other n-1 equations.

Here, we tested, as an example, the first and third equations of the system and we constructed the following determinants:

$A_1 = a_2c_1$ for the first equation

and

$B_1 = a_2b_1$

$B_2 = - b_2$ for the third equation

Thus, the first equation in order to be identified, the value of the determinant should be non-zero, $A_1 \neq 0$, while for the identification of the third equation, one of the two determinants should be non-zero, i.e., $B_1 \neq 0$ or $B_2 \neq 0$. From the statistical estimates of these coefficients we are led to the conclusion whether the sufficient condition is fulfilled in the aforementioned equations.

6. Statistical estimation of the parameters

6.1. Limitations as regards the estimates

The statistical estimation of the structural coefficients of the model applied, are based on eleven annual observations in the case of Chile. These observations cover the period of time 1960-1970 or the period of time 1959-1969 whenever a lagged variable is used in the system of equations. The same can be said with respect to the number of observations and the period of time covered by them, for the other Latin American countries in which the model is tested. The size of the sample from this point of view -eleven observations- can be considered small. This implies a limitation of the model as regards the estimates. Unfortunately, the lack of statistical data did not allow us to elongate the period of the estimation of the coefficients. Moreover, the period of time of a decade cannot be representative of the economy of a country. This means that the parameter estimates do not reflect the "average" economic behaviour of the variables in the past which is usually used for making forecasts. Apart from the statistical aspects, the period 1950-60 was rich in the variety of circumstances. Thus, in certain Latin American countries, we had severe disruption of their economies, on account of the political reasons. This makes the economic information for these years of questionable value. On the other hand, the use of even shorter period of time, in order to avoid the aforementioned difficulties, is not indicated from statistical point of view. In other countries, the rate of growth was high, over the sample period. This creates another serious problems, for instance, a) the problem of intercorrelation among the various time series that is stronger than in the usual case, a fact that reduces the accurate estimation of the effects of the changes in the explanatory variables on the endogenous variables; b) the problem of identification in the functional relationships used.

In the previous section, we said how difficult task is the elimination of the trend from the time series as well as the ascertainment whether the data identify the function under study or another function of the same system of equations.

Finally, the number of the explanatory variables is small. Two explanatory variables are introduced in each equation of the model, while, as it is known, there is a multitude of factors which influence the variable to be predicted. This imposes a restriction of the model with regard to the estimation of the structural coefficients, because it creates problems due to loss of degrees of freedom, identification problems as mentioned previously, completeness of the equations from economic point of view, etc.

However, all the aforementioned limitations do not change the nature of the model. A model is an ideal manner of systematizing available demographic and socio-economic data and making forecasts on a scientific basis, even though there are a few doubts about the information, the application of the model, etc. In this point, we can add that the forecasting as regards the development of the endogenous variables in the coming years shows a successful application of the present economic-demographic model.

6.2. Methods of estimation. Statistical criteria

The estimation of the parameters has been carried out by applying ordinary least squares (OLS) to each equation of the model and simultaneous equation methods like two-stage least squares (TSLS) to the system.[1/] The estimates of both methods are included in the relevant tables. Here, it is advisable to repeat some of the merits of these two methods.[2/] It is well known that certain economic relationships may be non-linearities. These non-linearities can be easily tackled with OLS method, but are difficult in a simultaneous equation estimate. Furthermore, it is easier to experiment with alternative variables and alternative forms of equations in the case of OLS than in the case of TSLS.

1/ Theil, H., Economic Forecasts and Policy. North Holland Publishing Co., Amsterdam, 1961.

2/ A summary on the relative merits of the different methods of estimation of the parameters has been provided by Johnston, J., Econometric methods. Mac Graw-Hill Company Inc., London, 1963.

In the present work, the least squares regression analysis helped us to test the final functional relationships before entering in the system of equations and to compare the statistical results obtained between OLS and TSLS on the basis of the statistical criteria and the merits of these two methods. Furthermore, the OLS estimates, as it is known, in an interdependent system of equations are inconsistent[1/] and biased. This bias can not be eliminated by increasing the sample size. On the other hand, it has been shown by Theil, H. that the generalized variance of ordinary least squares estimates of the parameters around their mean in a single equation is smaller than other unbiased estimates.[2/] The TSLS method possesses the merit of giving consistent estimates and small sample bias, but in a sample as small as ours (11 observations), the property of the consistency of this method may not free from bias. Finally, the simultaneous equation methods generally require the assumption of linearity for computation purposes, and in this case we have the problem of specification error, etc. The OLS method is simple as regards the calculations of the constants and coefficients of an equation. However, the choice between the aforementioned methods, the acceptance of the results obtained should be based on the overall function of the model as a whole and not in detailed weaknesses of the methods.

The statistical criteria on the basis of which we can accept or reject an equation of the system are the coefficient of multiple determination, R^2, the standard error of estimates and Von-Neumann's ratio. Thus, the coefficient of multiple determination, R^2, determines the percentage of the variance of the dependent variable explained by the independent variables. The unbiased estimation of this coefficient in the population, $\bar{R}^2$, will be

$$\bar{R}^2_{y.123\ldots k} = 1 - (1-R^2_{y.12\ldots k}) \cdot (\frac{n-1}{n-k})$$

1/ Haavelmo, T., "Methods of measuring the marginal propensity to consume". Journal of American Statistical Association, Vol. 42, 194 p. pp. 14-25.

2/ Theil, H., "Report of the Uppsala Meeting, 1954". Econometrica, April 1955.

where, n denotes the number of observations and the k denotes the number of variables of each equation.[1/] For testing the statistical significance of the coefficient of determination, $\bar{\bar{R}}^2$, we will use the "F" distribution as follows.

$$F \text{ (from observations)} = \frac{\bar{\bar{R}}^2_{y.12 \ldots k}}{1-\bar{\bar{R}}^2_{y.12 \ldots k}}$$

at a level of 5 percent or less.

The Von Newmann's criterion is used for testing of auto-correlation. As it is known, it is the ratio of the average square of the differences of successive values of error term, d^2, to its variance, S^2, corrected by the fraction $\frac{n}{n-1}$ for degrees of freedom. The letter of the operator means "Summation of". This criterion is written

$$\frac{d^2}{S^2} = \frac{\sum (e_{it} - e_{it-1})^2}{\sum e^2_{it}} \cdot \frac{n}{n-1}$$

and it is assumed that $E(e_t \; e_{t-1}) = 0$ for testing of its statistical significance. The reliability of the statistical estimates of the parameters, will be judged by the standard error. In other words, the standard error is the measure of the fit of the data of an equation. The criterion of t-student is taken into consideration for testing of the statistical significance of the parameters estimates, at a level of 5 percent or less.

Finally, as regards the estimates of regression coefficients we can say the following: Each regression coefficient measures the change of the explained variable caused by the unique change of the explanatory variable and its sign is determined by economic theory. Consequently, we will examine the consistency of the estimates with a priori expectations regarding their sign and their quantitative effects on the endogenous variables based on their magnitude. In the section which is concerned with the statistical results obtained, we will deal with the aforementioned criteria in more details.

1/ Drakatos, C., Econometrics. Klounikas, S. Publishing Co., Athens, Greece, 1971. pp. 42-47.

7. A further investigation of the model

1. The identity of the system. The state of full employment

By applying the labour force statistics which treat the classification of the available labour force, we derived the unemployed labour force as endogenous variable of the model. Indeed, in the model described in chapter 3, section 4, an identity for the labour force is included. It is obtained as the sum of the employment and unemployment.

$$L_t^a \equiv L_t^e + L_t^u$$

It is known that in Latin American countries there is not the state of full employment and consequently, the introduction of the above basic identity in the model was necessary because it reflects the existing situation from this point of view. Furthermore, it allowed us statistically to involve the third equation in the system by means of which we can estimate the development of the supply of work on the basis of pure demographic factors. Based on what we said previously, we are led to the conclusion that the model can not be applied in the case of full employment, i.e. if

$$L_t^a = L_t^e \quad \text{exists.}$$

2. Properties of the functions of the system

Although the choice of convenient functional relationships is an important part of en econometric model construction, it is useful to know which properties have the functions of the system. In this section, we will mention the basic properties of the first function of the system and analogous thoughts can be made for the remaining two functions. Thus, in the linear production function, $f(L,K)$, we specify that

a) None of the variables may take negative values and it is concave in all directions.[1/]

1/ Walters, A., An Introduction to Econometrics. MacMillan Publishing Co., London, 1970. pp. 271-276.

b) It has continuous partial derivatives which are

$$\frac{dY}{dL} = a_1 \quad \text{and} \quad \frac{dY}{dK} = a_2$$

c) The elasticity of the explained variable -output- with respect to the explanatory variables, labour force and capital is determined as follows.[1/]

$$e_L = \frac{dY}{Y} : \frac{dL}{L}$$

$$= \frac{dY}{dL} \cdot \frac{L}{Y}$$

$$= a_1 \cdot \frac{L}{Y}$$

and

$$e_K = a_2 \cdot \frac{K}{Y}$$

The elasticity is not constant at different points of the curve, but it changes. The change depends on the level used of the production factors.[2/]

1/ A mathematical function has the property of elasticity. Thus, given a function f (X, Y, Z), its elasticity with respect to the variable, X, is defined as plus or minus its logarithmic derivative, with respect to X. We denote the elasticity by $e_X f$ according to the sign.

$$e_X f = \pm \frac{d \log f}{d \log X} = \pm \frac{X}{f}\frac{df}{dX}$$

(Wold, H., Demand analysis. John Wiley and Sons, New York, 1952. pp.

2/ Koutsogianni-Kokkova, A., Production Functions in Greek Economy. KEPE, Athens, Greece, 1964. pp. 61-65.

CHAPTER 5. NUMERICAL MODEL

1. Introduction

The labour force of a country, as we said in Chapter 3, section 4, is a fraction of the total population and it consists of those individuals who contribute by the supply of their work to the productivity procedure. For the enumeration of the labour force we apply that approach "labour force" which as it is known, is based on the activity of each person during a determined period of time. The sources of statistical data relevant to the labour force in Latin American countries are the censuses and from this point of view the aforementioned approach for enumeration of the labour force is indicated.[1/] Furthermore, the ratio of labour force by age-group to the total population of the same-age group gives the degree of participation of each age-group in the labour force.

The coefficient of participation of each age-group in the labour force can be written as follows:

$$e_i = \frac{L_i^a}{P_i} 100 \qquad (1)$$

where L_i^a : Labour force, in the population age-group, i

P_i : Total population in the population age-group, i

e_i : Coefficient of participation rates of labour force, in the population age-group, i

In the model used, the size of population and the participation rates of labour force enter as the main variables. As it is known, the size of the population can be assessed for a decade with accuracy. This is because in the population forecasts only demographic factors are taken into account. In order to evaluate the labour force in the coming years as we said during the formulation of the third relationship of the model, although it is a demographic dimension, it is necessary to know, apart from the size of population, the participation rates of the labour force. The estimation of the participation rates of the labour force and especially of certain age groups depends on some factors, the analysis of

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1/ Athanassiou, S., Manpower Planning in Greece. The English University Press Ltd. London, 1973. pp.

which is realized in the next sections. In the third section of the present chapter we present a numerical model by means of which we can estimate the influence of some factors on the participation rates of labour force.

2. Factors influencing the participation rates of some age-groups of the labour force

It is known that certain groups of individuals are not included in the "civil" labour force. Such groups are the students and the members of the armed forces. The non-participation of these persons causes changes in the participation rates of the labour forces of the age-groups in which they belong. In the case of students, the non-participation of a number of persons in the labour force depends on the education system. Specifically, the obligatory education, the supply and demand for professional and university education, the duration of studies, etc. are the factors which influence the participation rates of labour force.1/ In the case of "military service" the number of the persons which are out of the labour force, depends upon the length of the period of military service which differs in each country, and the size of armed forces of a country.2/ This factor -military service- effects the size of the 15-19 and 20-24 age-groups. Both the factors, education system and military service, because of their importance in the formulation of the participation rates of certain age-groups of the labour force, enter in the numerical model which is applied for the estimation of the participation rates of these age-groups of the labour force.

1/ Of course, we have the cases of persons with dual status of activities (e.g. a working student), but the number of these persons can be considered small and, consequently, does not alter the influence of this factor in the participation rates of these age-groups.

2/ The members of the armed forces are to be included in the economically active population, according to the recommendation of the Sixth International Conference of Labour Statistics (United Nations. Application of International Standards to C nsus Data on the Economically Active Population. U.N., New York, 1951). But, as we mentioned previously in the present work, we are interested in the "civil" labour force.

3. Numerical model

Here, we will describe the model which will be applied for the estimation of participation rates of some age-groups of the labour force. In other words, the model will include the factors, education system and military service which, as we said in the previous section, influence certain critical age-groups (15-19 and 20-24 aged) of the labour force. This model is a forecasting model because it refers to the future period of time. Of course, this model can be expanded by introducing demographic and economic factors, for instance, urbanisation, external migration, industrialisation, etc. which affect the same or other age-groups. To introduce these factors in the model, we can follow a procedure similar to below approximately. As regards the aforementioned factors -school-age population and armed forces- the model can be built as follows.

a) For the school-age population

On the basis of the instructions for the enumeration of the labour force, during the census, the school-age population is considered inactive economically population. Furthermore, a fraction of the non-school-age population participates in the economic activity of the country, that is, belongs to the labour force. Thus, we have:

P_i : Total population in the population age-group, i

P_i^s : School-age population in the population age-group, i

$P_i - P_i^s$: Non-school-age population in the age-group, i

$\bar{e}(P_i - P_i^s)$: A fraction of non-school-age population in the population age-group, i

Consequently, the relationship (1) of the section 1, for the age-group, i of the labour force which includes school-age population, can be written as follows:

$$e_i = \frac{L_i^a}{P_i} 100 = \frac{\bar{e}(P_i - P_i^s)}{P_i} 100 \qquad (1)$$

The above relationship (1) expresses the fact that labour force in the population age-group, i is a fraction of the population of this age-group after the substracting the school population.

b) For the members of the armed forces

In a similar way, as for the school-age population, we can formulate the participation coefficient in the labour force, for the population group which includes the members of the armed forces.

Thus,

P_i^a : The number of the members of the armed forces, in the population age-group, i

P_i : The total population in the population age-group, i

$P_i - P_i^a$: The population after the substraction of the persons serving in the armed forces, in the population age-group, i

From the population $P_i - P_i^a$ only a fraction is labour force, i.e., $\bar{e}(P_i - P_i^a)$. Thus, in the case of the military service we will have

$$e_i = \frac{L_i^a}{P_i} 100 = \frac{\bar{e}(P_i - P_i^a)}{P_i} 100 \qquad (2)$$

The same as in the relationship (1) can be said as regards the meaning of the relationship (2).

c) Relationship expressing the influence of both the factors education system and military service in the age group, i

Given the ratios $K_i^s = \frac{P_i^s}{P_i}$ and $K_i^a = \frac{P_i^a}{P_i}$

where

K_i^s : The ratio of school-age and inactive economically population to the total population in the age-group, i and

K_i^a : the ratio of the number of members of the armed forces to the total population in the age-group, i

we have

$$K_i^s \, P_i = P_i^s \qquad (3)$$

and

$$K_i^a \, P_i = P_i^a \qquad (4)$$

By substituting the expressions (3) and (4) in the relationships (1) and (2) respectively, we have

$$e_i = \frac{\bar{e}(P_i - K_i^s P_i)}{P_i} \, 100 \qquad (5a)$$

$$= \bar{e} \, (1 - K_i^s) \, 100 \qquad (5b)$$

and

$$e_i = \frac{\bar{e} \, (P_i - K_i^a \, P_i)}{P_i} \, 100 \qquad (6a)$$

$$e_i = \bar{e} \, (1 - K_i^a) \, 100 \qquad (6b)$$

Finally, in the case where we refer to the age-group, i, which includes school-age population and members of the armed forces, the relationship (5b) is modified as follows

$$e_i = \bar{e} \;\; 1-(K_i^s + K_i^a) \;\; 100 \qquad (7)$$ 1/

The relationship (7) says that a fraction of the population of age-group, i, after the substraction of school-age population and the population serving with the armed forces, consists of the labour force of this age-group, i.

1/ National Research Foundation. Long-Term Prospects for the Greek Economy. National Research Foundation. Athens, Greece, 1968. pp. 44-49.

4. Other factors influencing the participation rates

There are many factors influencing the participation rates of the labour force. Although, it is not the purpose of this study to define all these factors, however, it is advisable to mention here the most important ones. By this way we will facilitate the probable expansion of the numerical model. Already, we have examined the factors, education system and military service, which as we said, influence the 15-19 and 20-24 age-groups. In addition to these factors, the external migratbn can be considered a significant factor for certain critical age-groups of labour force. Furthermore, urbanisation and industrialisation causes an increase of the participation rates of the urban labour force.[1/] The existing legislation as regards the limit of beginning of pension, which differs in each country, affects the participation rates of 55-59 and 60-65 age-groups. Finally, the index of accidents, the possibilities in some persons to have dual status (e.g. working person and pensioner) etc. are few factors which influence the participation rates of labour force. In the case of female married population, apart to the aforementioned factors, we can add the following. Family conditions such as e.g. the number of childeren, their age, the employment of husband, the marriage age, professional education etc. are some of the main factors which play a vital role in the formulation of participation rates of female labour force. Another important factor in this case, is the legislation which usually permits women to retire from the labour force earlier than man. Finally, the small possibilities of coming back in the same work of a person who has given up from his work previously -not for pension reasons- reduce the participation of a number of persons in the economic activity.

1/ Athanassiou, S., Urbanisation and Industrial Development in Latin American Countries. Centro Latinoamericano de Demografía, Serie A, N° 125. Santiago, Chile. pp. 40-50.

6. EMPIRICAL ANALYSIS OF THE MODEL

6.1. The case of Chile

6.1.1 Population features

The total de facto population of Chile in accordance with the last census (1970) was 8 853 thousand inhabitants, compared with 7 374 thousand inhabitants which was the de facto population of the country in 1960. The average annual rate of growth was 1.97 percent during the decade 1960-70, which can be considered high. The distribution of the population by urban and rural population is the proportions 75.1 and 24.9 respectively in 1970. The urban population, during the aforementioned period (1960-70) experienced a considerable increase (32.5 percent). Considering the population as a whole and disregarding its regional distribution (urban and rural) the sexes are evenly balanced, with a slight surplus of women (female 51.2 per cent in 1970). As regards the distribution of population by age, we can say that it is normal in the sense that there are the proportions indicated among young age-groups, economically active population and old age-groups. The mean crude population density within the territory (742 thousand Km^2) was 11.7 persons/$Km.^2$ in 1970 compared with 10 persons/$Km.^2$ in 1960. Furthermore, the labour force -main dimension of the model- amounted to 3 185 thousand persons in 1970 (35.98 per cent of the total population) and it was increased by 27.71 per cent in the period 1960-70, while employed labour force was 2 994 thousand persons and was increased by 27.71 percent in comparison with the year 1960. The unemployment was 191 thousand persons in the census year (1970) which amounts to 6 percent of the labour force. Even though, the development of unemployment showed a slight drop in the sampling period (1960-70), however, the percentage of unemployed labour force can be considered significant.1/

1/ The seasonal unemployment in the agricultural sector of the economy causes an increase of the over all unemployment rates for a period of time. Furthermore, the unemployment rates are not the same in Santiago -capital of the country- and in the rest of the country. Indeed, the unemployment rates were 8.4 per cent for Santiago and 6.7 percent for the rest of the country in 1960. Finally, the unemployment rates vary by age and sex between Santiago and the rest of the country (Herrick, B., Urban Migration and Economic Development in Chile. The M.I.T. Press. Massachusetts, U.S.A., 1965. pp. 56-61). The same can be said for the year 1970.

Table 6.1.1.1. The development of population and labour force in Chile during the period 1960-70.

(In thousands)

n/n	Composition	Years							Average rate of growth		
		1960	1962	1964	1965	1966	1968	1970	1960-65	65-70	60-70
1.	Employed labour force	2 317	2 406	2 546	2 623	2 703	2 879	2 994	2.51	2.68	2.60
	Index of change	100.00	103.84	109.88	113.21	116.66	124.26	129.22			
	1:3	92.90	92.04	92.95	93.54	93.95	95.05	94.00			
	a) Agriculture	711	688	690	710	718	736	738	-0.03	0.78	0.37
	b) Industrial	646	712	771	796	819	819	851	4.26	1.35	2.79
	c) Services	959	1 006	1 086	1 118	1 165	1 323	1 405	3.12	4.48	3.89
2.	Unemployed labour force	177	208	192	181	174	149	191	0.44	1.08	0.76
	Index of change	100.00	117.51	108.47	102.26	98.31	84.18	107.91			
	2:3	7.10	7.96	7.01	6.46	6.05	4.92	6.00			
3.	Total labour force	2 494	2 614	2 739	2 804	2 877	3 029	3 185	2.37	2.58	2.48
	Index of change	100.00	104.81	109.82	112.43	115.36	121.45	127.71			
4.	Population	7 374	7 649	7 934	8 080	8 229	8 535	8 853	1.96	1.97	1.97
	Index of change	100.00	103.73	107.59	109.57	111.59	115.74	120.06			
	3:4	33.82	34.17	34.52	34.70	34.96	35.49	35.98			

Sources:
1. Dirección de Estadística y Censos. XIII Censo de Población. Santiago, Chile, 29 de noviembre de 1960.
2. Instituto Nacional de Estadísticas. XIV Censo de Población y III de Vivienda, abril de 1970. Muestra de Adelanto de Cifras Censales.
3. Oficina de Planificación Nacional. División de Recursos Humanos.

The general picture of Chile from the demographic point of view which we described above, is shown in the previous table 6.1.1.1. As a conclusion we can say that the population of Chile is characterized in the main by a high rate of growth and a considerable enlargement of urban population. Bearing in mind the relatively small density, the normal distribution by age and sex, we can say no demographic problem arises from these points of view, and only unemployment remains a serious problem.

6.2.2. The Chilean economy in the period 1960-70

Here, we mainly deal with the development of economic factors which are included in the model during the sampling period 1960-70. We will refer to the gross domestic product by sector and the uses of national resources. Thus, at the end of the period (1970), the gross domestic product (GDP) had increased by 54.29 percent, the greater increase being realized by the secondary production sector (65.23 percent) and the lower by the primary production sector (27.32 percent). The tertiary sector had experienced a considerable increase (51.93 percent). The industrial sector increased its share in GDP from 39.11 percent to 41.89 percent while the agricultural and "services" sector showed a slight drop from 11.56 percent to 9.54 percent and 49.33 to 48.57 percent respectively. The average rate of growth of GDP was 4.43 percent in the period 1960-70. In the table 6.2.2.1. we give more details as regards the development of gross domestic product, its structural changes by sector in the period under study (1960-70). As a conclusion, we can say that important structural changes have not taken place in the GDP and the rate of growth is at the medium level among the rates of growth of the rest of the countries of Latin America.

The per capita income, which is used as a measure of degree of economic development of the country, was US$ 613 in 1970 and consequently, Chile can be considered to be less-developed country. The average rate of growth of per capita income was 1.8 percent

Table 6.2.2.1. The development of gross domestic product, its composition and rate of growth by major sectors in the period 1960-70

(In million escudos at 1965 prices)

n/n	Sector	Years							Average rate of growth		
		1960	1962	1964	1965	1966	1968	1970	1960-65	65-70	60-70
1.	Agriculture	1 625	1 645	1 827	1 784	1 929	2 114	2 069	1.88	3.01	2.44
	Index of change	100.00	101.23	112.43	109.78	118.71	130.09	127.32			
	1 : 4	11.56	10.50	10.68	9.94	10.04	10.44	9.54			
2.	Industrial[a/]	5 499	6 611	7 315	7 628	8 155	8 464	9 086	5.74	4.57	5.15
	Index of change	100.00	120.22	133.02	138.72	148.30	153.92	165.23			
	2 : 4	39.11	42.18	42.78	42.48	42.43	41.82	41.89			
3.	Services	6 935	7 416	7 957	8 544	9 137	9 663	10 536	4.26	4.28	4.27
	Index of change	100.00	106.94	114.74	123.20	131.75	139.34	151.93			
	3 : 4	49.33	47.32	46.53	47.58	47.54	47.74	48.57			
4.	Gross domestic product	14 059	15 672	17 099	17 956	19n221	20 241	21 691	5.01	3.85	4.43
	Index of change	100.00	111.47	121.62	127.72	136.72	143.97	154.29			

Source: Oficina de Planificación Nacional. Cuentas Nacionales de Chile, 1960-1971 (mimeographed). Santiago, 1972.

a/ Includes mining and quarrying; manufacturing; construction; electricity, gas, water and sanitary services.

in the period 1960-70.[1] As regards the uses of national resources, table 6.1.2.2., the gross fixed investment had increased by 65.03 percent in the period 1960-70 and it was 13.87 percent. The rate of growth was higher (5.94 percent) in the semi-period 1960-65 than the rate of growth in the semi-period 1965-70 (4.34 percent) and the period as a whole (1960-70). The percentage distribution of gross fixed investment by major sectors is given in the table 6.1.2.3. From this table, we see that 62.14 percent was absorbed by construction sector in the early years. At the end of this period (1970) the percentage distribution of gross-fixed investment by construction and machinery sector was 54.82 percent and 45.18 percent respectively.

Table 6.1.2.3. Percentage distribution of gross fixed investment by major sectors in the period 1960-70

Sector	Years					
	1960	1962	1964	1966	1968	1970
Gross fixed investment	100.00	100.00	100.00	100.00	100.00	100.00
a. Construction and other works	62.14	59.98	63.88	60.10	54.04	54.82
1. Building	34.31	29.51	33.31	32.10	29.63	27.64
2. Other works	27.82	30.46	30.57	28.00	24.42	27.19
b. Machinery and equipment	37.86	40.02	36.12	39.90	45.96	45.18
1. Imports	31.19	31.30	26.22	31.52	33.70	36.66
2. Domestic	6.68	8.72	9.91	8.38	12.26	8.51

Source: Oficina de Planificación Nacional. Cuentas Nacionales de Chile, 1960-71. Santiago, 1972.

[1] Serious efforts have been made and are made for the solution of the problem of economic development and the perspectives, in accordance with the long term economic plan, can be considered encouraging for this direction.

On the other hand, consumption had increased by 56.15 percent, its percentage distribution remained constant (72 percent) in the sampling period 1960-70 and the average rate of growth was 4.56 percent. The proportion of private consumption was 86 percent and did not change during the period 1960-70. In the following table 6.1.2.4. the distribution of private consumption by major groups of expenditures is given.

Table 6.1.2.4. Percentage distribution of private consumption expenditure by major group of expemditure

Group	Years			
	1960	1963	1966	1969
Total private consumption	100.0	100.0	100.0	100.0
1. Food, drink and tobacco	37.2	34.0	34.8	33.0
2. Clothing and footwear	13.9	16.5	16.0	12.8
3. Other manufactured goods	17.5	20.4	21.7	27.3
4. Residential building expenditure	9.3	9.1	8.0	7.9
5. Services	22.5	20.0	19.5	19.0

Source: ODEPLAN. Antecedentes sobre el desarrollo chileno, 1960-70. Santiago, 1971.

The group food - drink - tobacco consists of the 33 percent. The groups manufactured goods and services follow with 27.3 percent and 19 percent respectively. Of course, the percentage distribution of major groups changed during the period 1960-70, but the purpose of this work does not allow us to enter in more details on the changes which have taken place.

The quantitative information about the variables mentioned previously will help us to estimate statistically the system of simultaneous equations by which basic interrelationship among the economic and demographic variables are determined and to use the estimated equations of the model to forecast the development of the most important -endogenous- variables.

Table 6.1.2.2. The uses of national resources and their rates of growth in the period 1960-70

(Million escudos at 1965 prices)

Uses of resources	Years							Average rate of growth		
	1960	1962	1964	1965	1966	1968	1970	1960-65	65-70	60-70
1. Consumption	11 788	13 034	14 136	14 556	16 122	17 155	18 407	4.31	4.81	4.56
Index of change	100.00	110.57	119.92	123.48	136.77	145.53	156.15			
1 : 5	73.09	73.15	72.92	71.62	72.54	73.31	72.20			
a. Private	10 132	11 242	12 312	12 563	13 935	14 859	15 816	4.39	4.71	4.55
a : 1	85.95	86.25	87.10	86.31	86.43	86.62	85.92			
b. Public	1 656	1 792	1 824	1 993	2 187	2 296	2 591	3.77	5.39	4.58
b : 1	14.05	13.75	12.90	13.69	13.57	13.38	14.08			
2. Gross fixed investment	2 142	2 626	2 735	2 859	2 900	3 166	3 535	5.94	4.34	5.14
Index of change	100.00	122.60	127.68	133.47	135.39	147.81	165.03			
2 : 5	13.28	14.74	14.11	14.07	13.05	13.53	13.87			
3. Exports	1 938	2 106	2 366	2 515	2 662	2 875	3 034	5.35	3.82	4.58
3 : 5	12.02	11.82	12.20	12.37	11.98	12.29	11.90			
4. Changes in stock	259	53	149	395	542	204	517	8.81	5.53	7.16
4 : 5	1.61	0.30	0.77	1.94	2.44	0.87	2.03			
5. Total	16 127	17 819	19 386	20 325	22 226	23 400	25 493	4.74	4.64	4.69
	100.00	100.00	100.00	100.00	100.00	100.00	100.00			

Source: Oficina de Planificación Nacional. Cuentas Nacionales de Chile, 1960-1971. Santiago, 1972.

6.1.3. Statistical estimates of the structural equations of the model

a) Ordinary least squares (OLS) method

$^n/_n$	Statistical estimates	$\bar{R}^2$	d^2/s^2
1.	$Y_t = -8\ 409.40 + \underset{(0.96884)}{8.95744}\ L^e + \underset{(0.481869)}{0.874864}\ K_t$	0.988	1.06
2.	$L^e = 380.88 + \underset{(0.014175)}{0.040901}\ Y_t + \underset{(0.156970)}{0.591268}\ L^e_{t-1}$	0.995	1.54
3.	$L_t = -3\ 080.51 + \underset{(0.012870)}{0.332664}\ P_t + \underset{(9.2039)}{92.1847}\ R_t$	0.999	0.80

b) Two-stage least squares (TSLS) method

$^n/_n$	Statistical estimates	$\bar{R}^2$	d^2/s^2
1.	$Y_t = -8\ 470.01 + \underset{(0.83965)}{9.00817}\ L^e + \underset{(0.417971)}{0.851657}\ K_t$	0.990	1.52
2.	$L^e = 367.59 + \underset{(0.011095)}{0.039602}\ Y_t + \underset{(0.123218)}{0.605512}\ L^e_{t-1}$	0.996	2.71
3.	$L_t = -3\ 080.51 + \underset{(0.012870)}{0.332664}\ P_t + \underset{(9.2039)}{92.1847}\ R_t$	0.999	0.80

6.2.2. Statistical estimates of the structural equations of the model

a) Ordinary least squares (OLS) method

n/n	Statistical estimates	$\bar{R}^2$	d^2/S^2
1.	$Y_t = -710\ 467 + \underset{(13.9211)}{72.4017}\ L^e + \underset{(0.6452)}{1.8495}\ K_t$	0.994	1.13
2.	$L^e = -75.411 - \underset{(0.0000248)}{0.0000665}\ Y_t + \underset{(0.00282)}{1.01932}\ L^e_{t-1}$	0.999	2.54
3.	$L_t = 12493.6 + \underset{(0.012520)}{0.075175}\ P_t - \underset{(30.6)}{120.1}\ R_t$	0.999	2.30

b) Two-stage least squares (TSLS) method

n/n	Statistical estimates	$\bar{R}^2$	d^2/S^2
1.	$Y_t = -709\ 946 + \underset{(13.9550)}{72.3489}\ L^e + \underset{(0.6468)}{1.8519}\ K_t$	0.994	1.14
2.	$L^e = -87.671 - \underset{(0.0000178)}{0.0000777}\ Y_t + \underset{(0.00202)}{1.02059}\ L^e_{t-1}$	0.999	2.48
3.	$L_t = 12493.6 + \underset{(0.012520)}{0.075175}\ P_t - \underset{(30.6)}{120.1}\ R_t$	0.999	2.30

7. APPLICATION OF THE PRODUCTION FUNCTION IN THE INDUSTRIAL SECTOR OF THE ECONOMY

1. The forms of the Cobb-Douglas function

During the formulation of the basic functional relationships of the model, chapter 3, section 3, we ascertained that the production relationship is formally described by amounts of input, labour and capital represented by L and K respectively, associated with the amount of input represented by Y as follows:

$$Y = f\ (L, K) \tag{1}$$

We also said that the function (1) has been specified by Gobb-Douglas in the form

$$Y_i = a_o\ L_i^{a_1}\ K_i^{a_2}\ e_i \tag{2}$$

where

$Y \geqslant 0$, $a > 0$, $L \geqslant 0$, $K \geqslant 0$.

e : error term

i : industrial branch

The production function (2) does not have limitations as regards the size of coefficients of the explanatory variables, a_1 and a_2. This can be made in the next function (3) as follows:

$$Y_i = a_o\ L_i^{a_1}\ K_i^{1-a_1}\ e_i \tag{3}$$

That is, the sum of the parameters a_1 and a_2 is equal to unit, $a_1 + (1 - a_1)$.

Finally, we repeat here the linear expression of the production function which is also the first equation of the model, i.e.,

$$Y_i = a_o + a_1\ L_i + a_2\ K_i + e_i \tag{4}$$

The functions (2) and (3) are linear ones in the logarithms of the inputs and output.

So we have

$$\log Y_i = \log a + a_1 \log L_i + a_2 \log K_i \log e_i \qquad (5)$$

and

$$\log Y_i = \log a + a_1 \log L_i + (1 - a_1) \log K_i + \log e_i \qquad (6)$$

These are the forms of the production functions which can be applied in the industrial sector of the economy as a whole or by industrial branch. In the next section, we deal with the manner of selection of the suitable form of the function for application.

2. Selection of the suitable form of the function for application. Statistical criteria-tests

The selection of the suitable form of the Cobb-Douglas function in each industrial branch should be based on the statistical criteria mentioned in chapter 4, section 6.2 in which we also described the statistical tests. In the same chapter, section 6.2, we said that the error term of each equation must have some properties. Here, we will refer to the statistical tests of these properties. Thus,

a) As regards the distribution of the values of random variable, e_i, we computed the coefficient of correlation between the values of the explained variable, Y, and the values of error term, e_i, i.e., $R^2_{e_i \cdot Y}$

The non-correlation between the aforementioned variables, is an indication that there is homoscedasticity, i.e.,

$$V(e_i) = S^2_{e_i} \qquad i = 1, 2, 3, \ldots n$$

or the expected value of squares of e_i, $E(e_i^2)$ is equal to $S^2_{e_i}$

In this case, the estimates of parameters are unbiased, i.e. $E(\hat{b}) = b$ and their variances are small, a fact that means that we can apply the statistical tests as regards the significance of the structural coefficients of the equation.[1/]

1/ Johnston, J., Econometric Methods. Mc Graw-Hill Book Company, Inc., New York, 1963. pp. 207-211.

b) As regards the autocorrelation of the values of random variable, we can say the following: if the values of error term is not independently distributed with respect to itself lagged, i.e., the relationship

$$E\ (e_i,\ e_j) = 0 \qquad \text{where } i \neq j$$

is not valid, then the estimates of parameters are also unbiased, i.e. $E\ (\hat{b}) = b$ but their variances are large. Therefore, the valid of statistical tests as regards the significance of structural coefficients of the equation is limited and the forecasts can be considered insufficient.1/ For testing the autocorrelation we apply the Von-Newman criterion as it is mentioned in chapter 4, section 6.2 or the Durbin Watson-test.2/

c) As regards the testing of the normality of the distribution of random variable, e_i, and the expected value to be zero, i.e., $E(e_i) = 0$, properties which allow us to apply the statistical tests and to see whether the estimates of parameters are unbiased, i.e., $E(b) = b$, can be made by computing the statistical types indicated.

Finally, it is necessary to investigate if the explanatory variables are distributed independently of each other in accordance with basic statistical assumption. If this does not happen then we have the problem of the multicolinearity.3/ We will test this assumption by calculating the coefficient of correlation between the independent variables, X_1, X_2 of an equation, i.e.

$$r^2_{X_1 . X_2}$$

1/ Johnston, J., op. cit., pp. 179-182.

2/ Durbin, J. and Watson, G. "Testing for Serial Correlation in Least-squares". I and II. Biometrica, 1950 and 1951.

3/ In the case of full correlation -exact linear relationship- between the explanatory variables the values of parameters estimated are not the real ones. Under these conditions, it is not easy to know if the variance of dependent variable explains the changes of one or another explanatory variable of the equation (Drakatos, C., Econometrics. S. Kloukinas Publishing Co., Athens, Greece, 1971. pp. 65-59).

in relation to the coefficient of determination, R^2. Thus, if the following inequality exists

$$r^2_{X_1, X_2} < R^2_{Y. X_1, X_2}$$

then, the error of multicolinearity can not be considered significant. In this case, Professor L. Klein says that the functions of which the coefficient of determination, R^2, is more than 0.950 give satisfactory estimates of parameters even though the correlation of independent variables is high.1/ During the analysis of the statistical results to be obtained, we will apply these tests as it is possible, in order to have a complete picture from statistical point of view of the equations tried.

1/ Klein, L., An Introduction to Econometrics. New York, 1962. pp.

3. Properties of the Cobb-Douglas function. Cross-section data

Given the logarithmic form of the Cobb-Douglas function (2) of the section 1 of this chapter it is considered advisable to refer to its properties and the statistical explanation of its structural coefficients.[1] Thus

a) The function is homogeneous of $(a_1 + a_2)$ degree. This means that if we multiply the amount of labour by l and the amount of capital by l, i.e.,

$$a_o\ (l\ L)^{a_1} + (l\ K)^{a_2}$$

the effect on the output will be

$$a_o\ L^{a_1}\ K^{a_2}\ l^{a_1 + a_2} = l^{a_1 + a_2}\ Y \quad \text{new output}$$

b) The function is homogeneous linear if the sum of two coefficients is the unit, i.e. $a_1 + a_2 = 1$. Based on this limitation, we take the function relationship (3) of the section 1.

As regards the structural coefficients we mention the following:

a) The coefficient a_1 measures the elasticity of response of output to labour input which is constant. Thus a one per cent increase in labour, with the capital constant, will cause an increase a_1 percent to output. The same can be said for the coefficient a_2, with no change in the employed labour force.

b) If both labour and capital are increased by one percent, we will add $a_1 + a_2$ per cent to the level of output.

c) The coefficients, a_o, a_1 and a_2 are positives. This is because the increase of the amounts of the production factors is expected to increase the output.

1/ Walter, A., An Introduction to Econometrics, MacMillan and Co., Ltd., London, 1970. pp. 275-277 and Koutsogianni-Kokkova, A., Production Functions in Greek Economy. KEPE, Athens, Greece, 1964. pp. 46-48 and 61-64.

The investigation of production function for certain Latin American countries will be based on the cross-section data.[1] As it is known these data refer to a point of time and consequently they do not have the dimension of "time", a fact that creates many difficulties in the statistical estimation of the parameters of an equation. The cross-section data may come from a total census or may be a sample of the industrial units. Furthermore, they may cover a certain size of industrial units from the point of view of the number of employed workers, distribution by branch and region, etc. More details about this information will be given in the relevant sections of the present chapter where the Cobb-Douglas function is applied.

1/ Surveys have been made by statistical services of which the object is to determine the number of workers, capital and value added in a certain category of industrial and handicraft establishments.

CHAPTER 8. FORECASTING

1. In general

Forecasting as Professor Herman Wold says is "a more or less successful prediction of the future on the basis of the observed regularities in the past, i.e. the inference from the past in the future".[1] The scientists have begun to make serious efforts to look ahead in time recently, in the socio-economic area. Thus, during the last three decades, a great number of forecasting techniques have been developed in this scientific area.[2] In socio-economic sciences, a forecast does not deal with isolated events, it is rather an estimate of some quantities at one or several future points in time, as for instance, population or economic magnitudes.

On the other hand, a model should provide a forecasting capability which is of great importance and usefulness to the planners.[3] Here, we will try to examine the model used from the point of view of the capability of making forecasts as regards the development of some economic and demographic events, beyond the period of observation. The form of the model which is presented in the next section, will allow us to do this, i.e. the prediction of the endogenous variables of the model. To what accuracy, the prediction of economic and demographic variables will be, it depends upon the specification of the model which means in the broad sense how accurately we explained the past, the realisation of the assumptions as regards the development of the exogenous variables of the model and the "elongation" of the period of forecasting. Of course, the forecasting of demographic and economic magnitudes is not an easy task, there are many difficulties. In addition to the aforementioned conditions, unforseen international, local political events and situations influence the development of population and economy of a country.

1/ Wold, H., Time as the Realm of Forecasting. University of Uppsala, Sweden, 1966. pp. 2-4.

2/ Wold, H., Econometric Model Building (Forecasting by the Chain Principle). North-Holland Publishing Company. Amsterdam, 1964. p. 5.

3/ Wilson, A., Regional Forecasting (on some problems in regional modelling). Butterworth Publishing Co., London, 1971. pp. 179-181.

2. The form of the model for forecasting

In chapter 4, we described the structure form of the model and we derived its reduced form which, as we said in that chapter, is indicated for forecasting purposes. Furthermore, we estimated the structural coefficients of the model on the basis of the empirical observations. In general, a system of estimated structural equations can be written as follows:

$$Y_{it} = \hat{B}\, Y_{it} + \hat{C}\, Z_{it} \qquad \text{Structural form (1)}$$

We solve this system of estimated equations with respect to Y_{it}. To do this we follow the mathematical procedure mentioned in chapter 4. So we have

$$Y_{it} = (1 - \hat{B})^{-1}\, \hat{C}\, Z_{it} \qquad \text{Reduced form} \quad (2)$$

and

$$Y_{it} = \hat{W}\, Z_{it} \qquad (3)$$

in which

$$\hat{W} = (1 - \hat{B})^{-1}\, \hat{C}$$

where

$$Y_{it} = \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ \vdots \\ Y_{nt} \end{bmatrix} \text{ endogenous variables} \qquad Z_{it} = \begin{bmatrix} Z_{1t} \\ Z_{2t} \\ \vdots \\ Z_{nt} \end{bmatrix} \text{ predetermined variable}$$

and $\hat{B}$, $\hat{C}$ matrices of estimated coefficients.

The unknown in the above equation (3) is the vector Y_{it} of endogenous variables. By this equation we can make forecasts as the endogenous variables, Y_{it} in a projected period of time, if the values of the predetermined variables, Z_{it}, are given in that projected period.

Furthermore, the mathematical computation of the expression $\hat{W} = (1-\hat{B})^{-1}\hat{C}$ of the equation (3) which is necessary for the estimation of this equation is the following. We compute the inverse matrix $(1-\hat{B})^{-1}$ and then we multiply it by the matrix $\hat{C}$. As it is known, the computation of the inverse matrix $(1-\hat{B})^{-1}$ takes place as follows. First, we find the determinant of $(1-\hat{B})$, $|1-\hat{B}|$ and then we calculate the adjoint matrix of $(1-\hat{B})$, adj. $(1-\hat{B})$, which is the matrix of co-factors of the transpose of $(1-\hat{B})$. Finally, we divide the adjoint matrix $(1-\hat{B})$ by the determinant $|1-\hat{B}.|$

That is,

$$\frac{1}{|1-B|} \quad \text{adj.}\ (1-\hat{B})$$

So we have

$$\hat{W} = \frac{1}{|1-\hat{B}|} \ \text{adj.}(1-\hat{B})\ \hat{C}$$

and

$$Y_{it} = \frac{1}{|1-\hat{B}|} \ \text{adj.}\ (1-\hat{B})\ C\ Z_{it} \qquad (4)$$

The estimates of the coefficients of the model, $\hat{B}$ and $\hat{C}$, are different. This is because they have been made by different statistical methods (OLS and TSLS). Consequently, the values of the endogenous variables, Y_{it}, to be obtained by the aforementioned forecasting technique in a projected period of time will be different, even though, the forecasting procedure will be the same.

Specifically, for the economic demographic model used we can present the forecasting form as follows. The sampling period for estimation of the structural coefficients of the model and for certain countries is the period 1960-70. The projected period is the years 1971, 1972 and 1975. In accordance with what we said previously, the equation (3) for the year 1971 can be written as follows:

$$Y_{i\ 1971} = \hat{W}\, Z_{i\ 1971} \qquad (5)$$

and

$$Y_{i\ 1971} = \frac{1}{|1-\hat{B}|}\ \text{adj.}\ (1-\hat{B})\ \hat{C}\ Z_{i\ 1971} \qquad (6)$$

where

$$Y_{i\ 1971} = \begin{bmatrix} Y_{1\ 1971} \\ Y_{2\ 1971} \\ Y_{3\ 1971} \\ Y_{4\ 1971} \end{bmatrix} \text{ endogenous variables} \qquad Z_{i\ 1971} = \begin{bmatrix} Z_{1\ 1971} \\ Z_{2\ 1971} \\ Z_{3\ 1971} \\ Z_{4\ 1971} \end{bmatrix} \text{ predetermined variables}$$

and $\hat{B}$, $\hat{C}$ matrices of the reduced form coefficients which have been estimated on the basis of the observations of the sampling period 1960-70.

A P P E N D I X I

Table 1. DATA USED TO ESTIMATE THE VARIABLES OF THE MODEL, IN THE CASE OF CHILE, DURING THE SAMPLING PERIOD 1960-70

n/n	Year t	Y^{65} (in million Escudos) (1)	L^{e} (in thousands) (2)	L^{a} (in thousands) (3)	L^{u} (in thousands) (4)	K^{65} (in million Escudos) (5)	L^{e}_{t-1} (in thousands) (6)	P (in thousands) (7)	R (in percentage) (8)
1	1960	14 059	2 317	2 494	177	2 401	2 285	7 374	33.82
2	1961	14 929	2 349	2 553	204	2 753	2 317	7 510	33.99
3	1962	15 672	2 406	2 614	208	2 679	2 349	7 649	34.17
4	1963	16 412	2 475	2 675	201	3 027	2 406	7 790	34.34
5	1964	17 099	2 546	2 739	192	2 884	2 475	7 934	34,52
6	1965	17 956	2 623	2 804	181	3 254	2 546	8 080	34.70
7	1966	19 221	2 703	2 877	174	3 442	2 623	8 229	34.96
8	1967	19 670	2 812	2 952	140	3 132	2 703	8 381	35.22
9	1968	20 241	2 879	3 029	149	3 370	2 812	8 535	35.49
10	1969	20 915	2 921	3 075	154	3 730	2 879	8 693	35.37
11	1970	21 691	2 994	3 185	191	4 052	2 921	8 853	35.98

Sources:
1. Oficina de Planificación Nacional. Cuentas Nacionales de Chile, 1960-1971. Santiago, 1972 (mimeografiado).
2. Oficina de Planificación Nacional. División de Recursos Humanos.
3. Dirección de Estadística y Censos. XIII Censo de Población. Santiago, Chile, 29 de noviembre de 1960.
4. Instituto Nacional de Estadísticas. XIV Censo de Población y III de Vivienda, abril de 1970. Muestra de Adelanto de Cifras Censales. Santiago de Chile, 1971.

Table 2. DATA USED TO ESTIMATE THE VARIABLES OF THE MODEL, IN THE CASE OF MEXICO, DURING THE SAMPLING PERIOD 1960-70

n/n	Year t	Y^{60} (in million Pesos) (1)	L^{e} (in thousands) (2)	L^{a} (in thousands) (3)	L^{u} (in thousands) (4)	K^{60} (in million Pesos) (5)	L^{e}_{t-1} (in thousands) (6)	P (in thousands) (7)	R (in per centage) (8)
1	1960	150 500	11 071	11 253	182	30 200	10 945	34 923	32.22
2	1961	157 900	11 199	11 409	210	29 800	11 071	36 068	31.63
3	1962	165 300	11 329	11 566	237	28 900	11 199	37 251	31.05
4	1963	178 500	11 461	11 726	265	34 100	11 329	38 473	30.48
5	1964	199 400	11 594	11 889	295	39 600	11 461	39 735	29.92
6	1965	212 300	11 728	12 053	325	40 200	11 594	41 039	29.37
7	1966	227 000	11 864	12 220	356	45 200	11 728	42 835	28.53
8	1967	241 300	12 002	12 389	387	46 800	11 864	43 775	28.30
9	1968	260 900	12 141	12 560	419	48 700	12 002	45 211	27.78
10	1969	277 400	12 282	12 734	452	52 100	12 141	46 693	27.27
11	1970	298 700	12 424	12 910	486	57 300	12 282	48 225	26.77

Sources:
1. United Nations. Yearbook of National Account Statistics 1971. New York, 1973.
2. Dirección General de Estadística. VIII Censo General de Población 1960. México, D.F., 1964.
3. Dirección General de Estadística. IX Censo General de Población 1970. México, D.F., 1972.