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CEPAL SHIPPING TECHNOLOGY OPTIMIZATION MODEL

I. INTRODUCTION

In the period since 1960, the major trade routes of the world have been subject to the progressive introduction of new shipping technologies of very high productivity in comparison with the more traditional methods that preceded them, which had remained essentially unchanged over the previous century. The new technologies are typified by the fully cellular container ship able to economize substantially on time spent in port due to the more efficient loading and unloading methods that containers permit. Container vessels also tend to have relatively high operating speeds. Their productivity in terms of cargo volume has enabled one of them to displace up to five or six conventional vessels.

Another example of the new productive technology is the specialized bulk carrier. Timber carriers equipped with high-capacity, fast-acting cranes can significantly increase the rates of loading and unloading for this difficult commodity. Specialized automobile carriers that load and unload on the roll-on/roll-off principle can have a considerable impact on the need for port services.

These new technologies were first introduced on routes offering dense flows of containerizable traffic, balanced directionally between countries having high labor costs and hence a tendency towards high cargo handling costs. They have since become more widely spread. For instance, fully cellular container ship services now link northwestern Europe with both the Caribbean area and the Republic of South Africa. Neither area has especially high labor costs, and neither exports large quantities of goods that were traditionally considered containerizable, although the definition of what constitutes containerizable cargo has become broader with the passage of time. However, except for a few

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instances such as the CAROL service between northwestern Europe and the Caribbean, and the Delta and Sealand services from the east coast of the United States to Argentina and Brazil, Latin America thus far has felt the impact of the new technologies only slightly.

Now, there is reason to believe that a change in this situation may be imminent. Maritime trade between Japan and Latin America currently employs no container vessels and only a few specialized carriers for traffic other than ores, but some of the lines carrying this trade may find it advantageous to introduce greater specialization on some routes. For example, shipyards everywhere are at present operating well below capacity due both to a worldwide slackening in trade growth and hence in demand for ships, and to the opening of a number of new shipyards. In an attempt to attract shipbuilding, some nations are offering heavily subsidized credit for the purchase of new vessels from their yards. Such is the case of Japan, which recently granted to Pakistan a loan with a three percent interest rate, a 10-year grace period before repayments commence and a repayment period of 30 years, which exceeds of the probable life of the ship. 1/

This might therefore be the moment for interested Latin American countries to take advantage of unusually favorable conditions in order to implement improvements in shipping technology. Clearly, potential benefits from such technology should be investigated thoroughly to determine that they do in fact exist, and to identify means whereby to ensure that as many as possible accrue to the region. Perhaps only under certain circumstances should Latin America accept the new technologies.

In an attempt to quantify benefits that might accrue to Latin America from new shipping technologies, CEPAL has developed a linear programming model that uses estimated flows of maritime traffic between

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1/ Seatrade, Vol. 8, Nº 9, Nov. 1978, p. 5

/a group

a group of ports as a basis for selecting itineraries, frequencies of service and vessel types that might best handle the trade. The model is capable of optimizing from the point of view of alternative interested parties such as vessel operators or certain of the countries served. Through the dual to the primal problem, it also has a limited ability to determine the incentives necessary to secure implementation of the service configuration best suited to the party of main interest, even though such a configuration would not necessarily be optimal from the standpoint of other interested parties.

The model is intended for initial application to trade between the west coast of South America and Japan. This case has been selected because, so far, the routes affected have not been subject to the introduction of the new technologies to the same extent as have the world's major trade routes and so are still open for choice among alternatives. The model has been tested in a small but realistic case study and has been found to work well. It may therefore be of considerable assistance to the nations of the west coast of the region in developing a united maritime policy for their trade with Japan.

This paper is concerned primarily with a model, but the model is merely part of a larger methodology. After goals for the region have been identified, institutional arrangements for achieving them would still remain to be determined. Such arrangements might take one of several feasible forms of cooperation between the nations of the region and Japan. For instance, it might be desirable for Latin America to receive from Japan the same kind of ship financing that Japan has recently granted to Pakistan. Ultimately, the objective is to tap Japan's capital resources for the implementation of a maritime transport system acceptable to that nation while meeting Latin America's needs for better service.

II. FORMAL STATEMENT OF THE MODEL

1. Primal problem

In mathematical terms, the objective of the model is to minimize

$$Z = \sum_{l} b_l y_l + \sum_{ijkl} c_{ijkl} x_{ijkl} + \sum_{jikl} c_{jikl} x_{jikl}$$

subject to the constraints

$$- y_l + \sum_{ijk\}l} x_{ijkl} \leq 0 \qquad \text{for all } l \tag{1}$$

$$- y_l + \sum_{jik\}l} x_{jikl} \leq 0 \qquad \text{for all } l \tag{2}$$

$$\sum_{l} x_{ijkl} = q_{ijk} \qquad \text{for all } ijk \tag{3}$$

$$\sum_{l} x_{jikl} = q_{jik} \qquad \text{for all } jik \tag{4}$$

$$\sum_{l\}ijk} y_l \geq F_{ijk} \qquad \text{for any } ijk \tag{5}$$

$$\sum_{l\}jik} y_l \geq F_{jik} \qquad \text{for any } jik \tag{6}$$

where i is a port in Latin America;
j is a port in Japan;
k is a type of cargo;
l is a shipping service consisting of a particular type of ship plying a particular round voyage itinerary;

Note: The character } is used to mean "is an element of".

/b is the

b is the long-term round-voyage operating cost per shipping ton of a vessel while at sea;

c is the cost per shipping ton of loading, discharging and operating a vessel while in port;

F is a minimum frequency of service, measured in shipping tons per time period;

q is a quantity of cargo offered for carriage, measured in shipping tons, that must be transported;

x is the quantity of cargo, measured in shipping tons, that the model assigns to a particular service;

y is the capacity, measured in shipping tons, that the model assigns to a particular service in order to transport all of the cargo quantities assigned to that service.

Note that according to the definition of a shipping service, each "l" corresponds to a particular set of "i", "j" and "k". Ports "i" and "j" must be on the itinerary of that service, and cargo "k" must be transportable by the vessel type used for the service.

Note also that the order of "i" and "j" indicates the direction of travel. When written "ij", the direction is Latin America to Japan, whereas "ji" specifies Japan to Latin America. Treatment of the two directions separately is necessary to ensure that the assignment of cargo to a service does not exceed available one-way capacity.

(a) Objective function

The first term of the objective function represents the long-term operating costs for each service "l", which are the product of the unit operating cost of the vessel while at sea times the vessel capacity. The second and third terms represent the total in-port costs associated with the shipment of cargo "k" from port "i" to port "j" or from port "j" to port "i", respectively, via service "l". These costs are the

/product of the

product of the sum of the ship's cost in port plus cargo handling costs times the amount of cargo assigned to the service.

(b) Constraints

Constraint equations (1) and (2) ensure that the capacity offered by each service "l" is at least sufficient to transport all types of cargo "k" offered for shipment via that service from port "i" to port "j" or from port "j" to port "i", respectively. Since services consist of round voyages while capacity is used by any given traffic in only one direction at a time, one constraint for each direction is required to completely represent a given service.

Constraint equations (3) and (4) ensure that all types of cargo "k" offered for carriage from all ports "i" to all ports "j" or from all ports "j" to all ports "i", respectively, are transported by the combination of services "l" whose itineraries cover those ports. Regardless of shipping costs, no cargo will be left in port.

Constraint equations (5) and (6) are optional, and should be used only if a service "l" must be provided with greater than a certain minimum frequency to transport some cargo "k" from some port "i" to some port "j" or from some port "j" to some port "i", respectively. The time period must be consistent with the period during which the quantity of cargo "x" is generated, since the model contains no explicit expression of time.

Note that the model does not constrain the capacity offered by any one service to be less than some maximum level. Capacity is determined strictly by the amount of cargo assigned by the model to each service on the basis of constant unit operating costs corresponding to a hypothetical vessel of a particular configuration. If the capacity requirements calculated by the model are such as to make a larger or smaller vessel a more reasonable configuration for costing, unit costs should be revised and the model rerun until there is reasonable correspondence between assumed configuration and assigned capacity.

A trial application of the model is presented in Appendix A.

/2. Dual problem

2. Dual problem

The dual of this model is especially difficult to interpret, but it is also especially worthwhile to run because of the insight it gives into the quality of the optimal solution. For instance, one result is the amount to which costs would increase if a particular cargo shipped between a certain pair of ports were carried by the second best service, rather than the best as determined by the primal problem. This is important information if for any reason implementation of the best service is not considered feasible.

The objective of the dual is to maximize

$$W = \sum_{ijk} t_{ijk} q_{ijk} + \sum_{jik} t_{jik} q_{jik} + \sum_{ijk} p_{ijk} F_{ijk} + \sum_{jik} p_{jik} F_{j}$$

subject to the following constraints:

$$r_l + s_l + \sum_{ijk\}l} p_{ijk} + \sum_{jik\}l} p_{jik} \leq b_l \qquad \text{for all } l \qquad (7)$$

$$- r_l + t_{ijk} \leq c_{ijkl} \qquad \text{for all } ijkl \qquad (8)$$

$$- s_l + t_{jik} \leq c_{jikl} \qquad \text{for all } ijkl \qquad (9)$$

where p is the shadow cost per shipping ton of providing a required minimum service frequency;

r is the shadow cost per shipping ton of providing capacity for a service in the direction Latin America to Japan;

s is the shadow cost per shipping ton of providing capacity for a service in the direction Japan to Latin America;

t is the shadow cost, at sea plus in port, of carrying one shipping ton of cargo via that service which minimizes transport costs.

/All other

All other elements of the equations retain the significance given them in the primal formulation.

(a) Objective function

The first two terms of the objective function represent the total shadow cost of shipping cargo "k" from port "i" to port "j" or from port "j" to port "i", respectively, via that service which the model finds will result in the lowest transport cost. The unit costs "t" in general are not equal to the sum of the corresponding "b" and "c" from the primal. Cost "b" is the marginal cost per ton of added capacity, whereas "t" is the total shadow marginal cost of transport, whose at-sea component may be zero in one direction if excess capacity is provided to meet the requirements of carriage in the other direction. The possibility that the at-sea component of "t" may be zero is recognized explicitly in constraint equation (7).

The second two terms of the objective function represent the shadow cost of providing the desired minimum frequency of service to transport cargo "k" from port "i" to port "j" or from port "j" to port "i", respectively.

(b) Constraints

Constraint equation (7) ensures that the shadow costs of providing each service "l" in each direction plus the shadow costs of providing a minimum level of capacity for each cargo type "k" from port "i" to port "j" and from port "j" to port "i", where ports "i" and "j" are on the itinerary of "l", do not exceed the vessel operating costs at sea as specified for the primal.

Constraint equations (8) and (9) ensure that the shadow cost of the service offering the minimum transport cost for cargo "k" from port "i" to port "j" or from port "j" to port "i", respectively, does not exceed the specified in-port cost for the corresponding

/movement plus

movement plus the shadow cost of providing capacity in the direction specified. Only in the case of a service actually used in the optimal solution will the shadow cost "r" or "s" be equal to the corresponding cost "b" of the primal. For services not used, the model sets "r" or "s" to the level such that those services would just begin to attract traffic.

Note that in the solution of the dual, a considerable number of nonzero slack variables are present. Appendix B presents a trial application of the dual and explains the significance of its results.

/III. APPLICATION

III. APPLICATION OF THE MODEL

1. Inputs

For its application, the model requires as inputs data concerning traffic flows and costs.

(a) Traffic flows

Traffic can generally be aggregated by groups of cargo having similar transport cost characteristics, according to the following criteria:

1. Volume-to-weight ratio
2. Containerizability, unitizability or susceptibility to bulk handling
3. Type of packaging
4. Degree of perishability and need for climate control
5. Unit value, if the model were set up to take time costs into consideration

Goods reasonably alike in these respects are likely also to require similar amounts of shipping capacity, have similar loading, unloading and storage costs in port, and incur similar time-related costs such as interest and deterioration in transit. Containerizability is especially important, because products that have very different handling costs when treated individually may have virtually identical costs when containerized, permitting their flows to be aggregated for input to the model. Unit costs are also an important consideration in this respect, as it may be misleading to sum flows of goods that have identical handling characteristics but different time values. (As presently formulated, however, the model does not consider these costs).

In most cases, traffic flow matrixes need not be modified

/between runs

between runs of the model unless some product is included whose traffic level is highly sensitive to transport costs. Even in the absence of such a product, it may still be desirable to introduce variations in traffic flows so as to determine the sensitivity of results to assumptions about trading patterns.

(b) Costs

Two types of costs are required by the primal. The at-sea cost "b", which is the cost of providing one unit of capacity on service "l", is considered to be independent of the length of the leg over which the cargo is to be transported.

Thus if added capacity is required on any liven leg, costs for the entire service are increased by "b", even though that capacity is unused on all other legs. This is reasonable in view of the fact that the model as currently formulated does not provide for intraregional traffic to be handled, reflecting the belief that such traffic probably would not be transported by an interregional carrier.

The in-port cost "c" has two components. The first accounts for a vessel's expenses during a port call, including the operation of the vessel itself and any port fees that must be paid. The operating cost depends on the type of vessel and its length of stay in port. The stay in turn also depends on the type of vessel, together with the volume and type of cargo loaded and discharged, any extra time consumed in complying with port formalities and any delays due to congestion. The second component covers the costs of actually loading and discharging cargo, which vary with both type of cargo and type of vessel. It would also cover such cargo-related expenses as warehousing in port and inland transport.

Depending on the particular combination of port, vessel and cargo, the incidence of the fixed cost items in the overall in-port cost may be relatively high. If this is the case, the corresponding

/"c" will

"c" will not be a constant per unit of cargo volume but will decrease as the amount of cargo increases. Decreasing costs in turn will change the results of the model. Since the linear program cannot deal with variable unit costs, the model must be rerun a number of times, modifying the costs input to the new run to reflect the results of the previous run, until a steady state is reached such that results no longer vary significantly between runs.

2. Outputs

In what might be considered a base case, costs input to the model will normally be those actually perceived by users of the services - in other words, market costs. The output then will be a distribution of trade most beneficial to a liner conference operating from a monopoly position between Japan and Latin America, and will reflect the situation most likely to occur provided there were no intervention by interested governments. However, the advantage of a model such as this is its ability to provide insights into the consequences of actions that might be taken by one or more governments to favor the general public welfare in preference to private profits. Thus, for example, it would be possible to investigate the effects of a surcharge on components such as capacity costs of cargo space in vessels that constitute a foreign exchange outflow, or a reduction in charges such as port fees that earn foreign exchange. Shadow pricing strategies representing the economic interests of the various countries could also be tested in successive runs.

Although the dual formulation is substantially more difficult to understand than the primal, it avoids the need to run the model repeatedly to test shadow pricing options. If market costs are input, the dual gives as one output the amount by which nonoptimal services would have to be subsidized in order for them to be implemented, and

/hence the

hence the shadow cost of preferring such services for social or administrative reasons. The latter may be of particular importance if optimal services proved to be infeasible, say, because they would cut across the boundaries of existing conference services.

3. Trial application

A trial application of the model is described in Appendixes A and B for the primal and dual formulations, respectively. This analysis considers two types of traffic (general cargo and timber), two ports in Japan and two in Latin America, and five services following different itineraries. The services are further differentiated by vessel specialization and size (two general cargo ships able to carry timber, three timber carriers). Traffic levels and costs input to the model do not represent any real situation, but nonetheless were chosen to be as realistic as possible. Costs were built up from the individual components discussed above, in order to gain an understanding of the problems entailed by this approach.

While the small scale of the analysis permitted input cost calculations to be made by hand, it was apparent that a large-scale application will required them to be made by computer. Any program developed to handle this task should also be capable of taking traffic assignment results from one run and using them for adjusting the costs input to the next run, until examination of the output reveals that variations between successive runs are no longer significant.

/IV. INTERPRETATION

IV. INTERPRETATION OF RESULTS

One possible outcome of applying the model to trade flows between Japan and Latin America would be a recommendation to implement containerized services on some routes. From the standpoint of present-day shipping economics, container vessels offer several important advantages:

1. They permit economies of labor costs in port and on board ship;
2. Their productivity permits the release of as many as five or six conventional vessels per container ship introduced;
3. They offer the user a more rapid service, which may stimulate demand.

Because container ships are especially capital intensive, they are more easily afforded by the lines of capital-rich developed countries. Thus containerization gives these lines a relative advantage in comparison with those of developing countries. Also, since container vessels economize in terms of crew, they are a means of combating the high labor oosts of developed nations.

The advantages to the developing nations of being served by container ship services are less clear on the whole, even if the vessels were to be owned by nations of the developing world:

1. The port labor released may have no other utilization;
2. The mariners displaced from conventional ships likewise may have no other utilization;
3. The burden of capital-intensive ships is difficult for a capital-scarce country to bear;
4. There is no guarantee that any cost savings would be passed on in the form of lower freight rates, even though reductions should be possible;

/5. Were

5. Were freight rates to fall, the resulting lower prices for imported products might stimulate competition with local manufacturing industry and hence generate unemployment;
6. Container services are economical only if operated to a smaller number of ports than equivalent conventional services. This would act to the disadvantage of the countries or areas to which service was denied;
7. In the case of a developed country, a reduction in freight rates caused by the introduction of containerization would probably tend to benefit exports more than imports. In a developing country, on the other hand, containerization would be more likely to favor imports, while other new shipping technologies such as automobile carriers would increase this likelihood to virtual certainty.

The results of the model must therefore be interpreted with caution. It would be unwise to assume that they constitute definite answers to questions concerning policies that should be adopted to face the challenge of the new shipping technologies. Yet, if taken as guidelines indicative of areas in which to concentrate further studies, these results can be very useful indeed. It is thus definitely in the interest of Latin America to pursue development of the model with a view toward perfecting it as an analytical tool of considerable importance to the orderly implementation of new technologies in the region.

/Appendix A

Appendix A

PRIMAL FORMULATION

The model was tested in a hypothetical situation involving the shipment of timber and general cargo between two ports in Japan and two ports in Latin America via five different services, three of which handled only timber using specialized timber carriers, while the other two handled both timber and general gargo using nonspecialized cargo liners. Timber was shipped from one Latin American port to both Japanese ports. General cargo was shipped from both Latin American ports to both Japanese ports and vice versa, but not between the two Latin American ports or between the two Japanese ports.

Subscripts reflecting this situation were as follows:

$i = 1$: Latin American port LA
$= 2$: Latin American port LB
$j = 3$: Japanese port JA
$= 4$: Japanese port JB
$k = 1$: Cargo type timber
$= 2$: Cargo type general
$l = 1$: Service JA-LB-JA (specialized timber vessel)
$= 2$: Service JB-LB-JB (specialized timber vessel)
$= 3$: Service JA-LB-JB-JA (specialized timber vessel)
$= 4$: Service JA-JB-LA-LB-JB-JA (general cargo vessel)
$= 5$: Service JA-JB-LA-LB-LA-JB-JA (general cargo vessel)

Only a limited number of "ijkl" and "jikl" subscript sets were used in the equations, due to the exclusion from the model of intraregional trade and to the assumption that timber carriers cannot be used to ship general cargo. Nonadmissible sets might have appeared in the equations with arbitrarily high cost coefficients, so that they would not have been viable choices for traffic assignment, but for

/computational reasons

computational reasons it was deemed preferable to leave them out entirely. The admissible subscript sets were

Set ijkl	Set jikl
1324	3124
1325	3125
1424	3224
1425	3225
2311	4124
2313	4125
2314	4224
2315	4225
2324	
2325	
2412	
2413	
2414	
2415	
2424	
2425	

The objective function, with coefficients in US$/shipping ton, was

$$\begin{aligned} Z = {} & 21.10\, y_1 + 23.22\, y_2 + 23.74\, y_3 + 30.67\, y_4 + 31.70\, y_5 \\ & + 31.64\, x_{1324} + 31.80\, x_{1325} + 31.80\, x_{1424} + 31.96\, x_{1425} \\ & + 22.65\, x_{2311} + 22.74\, x_{2313} + 29.79\, x_{2314} + 29.79\, x_{2315} \\ & + 31.64\, x_{2324} + 31.64\, x_{2325} + 22.65\, x_{2412} + 22.74\, x_{2413} \\ & + 29.79\, x_{2414} + 29.79\, x_{2415} + 31.80\, x_{2424} + 31.80\, x_{2425} \\ & + 31.67\, x_{3124} + 31.83\, x_{3125} + 31.67\, x_{3224} + 31.67\, x_{3225} \\ & + 31.83\, x_{4124} + 32.00\, x_{4125} + 31.83\, x_{4224} + 31.83\, x_{4225} \end{aligned}$$

/which was

which was to be minimized subject to the following constraints, where the righthand side of each is expressed in shipping tons:

$$\begin{array}{ll}
-y_1 + x_{2311} \leq 0 & (1\text{-}1) \\
-y_2 + x_{2412} \leq 0 & (1\text{-}2) \\
-y_3 + x_{2313} + x_{2413} \leq 0 & (1\text{-}3) \\
-y_4 + x_{1324} + x_{1424} + x_{2314} + x_{2324} & \\
\quad + x_{2414} + x_{2424} \leq 0 & (1\text{-}4) \\
-y_5 + x_{1325} + x_{1425} + x_{2315} + x_{2325} & \\
\quad + x_{2415} + x_{2425} \leq 0 & (1\text{-}5) \\
-y_4 + x_{3124} + x_{3224} + x_{4124} + x_{4224} \leq 0 & (2\text{-}1) \\
-y_5 + x_{3125} + x_{3225} + x_{4125} + x_{4225} \leq 0 & (2\text{-}2) \\
x_{1324} + x_{1325} = 160{,}000 & (3\text{-}1) \\
x_{1424} + x_{1425} = 70{,}000 & (3\text{-}2) \\
x_{2324} + x_{2325} = 96{,}000 & (3\text{-}3) \\
x_{2424} + x_{2425} = 25{,}000 & (3\text{-}4) \\
x_{2311} + x_{2313} + x_{2314} + x_{2315} = 84{,}211 & (3\text{-}5) \\
x_{2412} + x_{2413} + x_{2414} + x_{2415} = 210{,}526 & (3\text{-}6) \\
x_{3124} + x_{3125} = 128{,}000 & (4\text{-}1) \\
x_{3224} + x_{3225} = 110{,}769 & (4\text{-}2) \\
x_{4124} + x_{4125} = 59{,}077 & (4\text{-}3) \\
x_{4224} + x_{4225} = 46{,}769 & (4\text{-}4) \\
y_4 + y_5 \geq 192{,}000 & (5\text{-}1)
\end{array}$$

/After 19

After 19 iterations, the optimal solution obtained gave a value for the objective function of US$46,157,800 and the following values, expressed in shipping tons, for the nonzero ordinary variables:

y_1 = 84,211

y_2 = 210,525

y_4 = 351,000

x_{1324} = 160,000

x_{1424} = 70,000

x_{2311} = 84,211

x_{2324} = 96,000

x_{2412} = 210,526

x_{2424} = 25,000

x_{3124} = 128,000

x_{3224} = 110,769

x_{4124} = 59,077

x_{4224} = 46,769

All other ordinary variables had a value of zero. One slack variable was nonzero with a value of 159,000 shipping tons, representing the amount by which the constraint on capacity of the service with minimum frequency was exceeded by the actual service level in the optimal solution. This constraint was thus redundant.

/Appendix B

Appendix B

DUAL FORMULATION

In order to understand the meaning of the dual, it had to be set up, run and then analyzed by quantitative comparison of its parameter values with the corresponding values from the primal. Once its significance was understood, however, it proved to be an extremely useful device for helping to interpret the results of the primal. The quantities "q" in the dual objective function were the port-to-port cargo movements from primal constraints (3) and (4). Since there was only one minimum service frequency in the primal, the dual contained but one such "pF" expression, for which F_{132} = 192,000 shipping tons. This would be the minimum combined level that would have to be offered by services 4 and 5, the only two available to carry such cargo.

The objective function, with coefficients in shipping tons, was

$$
\begin{aligned}
W = {} & 84{,}211\, t_{231} + 210{,}526\, t_{241} + 160{,}000\, t_{132} + 170{,}000\, t_{142} \\
& + 96{,}000\, t_{232} + 25{,}000\, t_{242} + 128{,}000\, t_{312} + 110{,}769\, t_{322} \\
& + 59{,}077\, t_{412} + 46{,}769\, t_{422} + 192{,}000\, p_{132}
\end{aligned}
$$

which was to be maximized subject to the following constraints, where the right-hand side of each is expressed in US$/shipping ton:

$$r_1 \le 21.10 \qquad (7\text{-}1)$$

$$r_2 \le 23.22 \qquad (7\text{-}2)$$

$$r_3 \le 23.74 \qquad (7\text{-}3)$$

$$r_4 + s_4 + p_{132} \le 30.67 \qquad (7\text{-}4)$$

$$r_5 + s_5 + p_{132} \le 31.70 \qquad (7\text{-}5)$$

/-$r_1 + t_{231}$

$$-r_1 + t_{231} \leq 22.65 \tag{8-1}$$

$$-r_2 + t_{241} \leq 22.65 \tag{8-2}$$

$$-r_3 + t_{231} \leq 22.74 \tag{8-3}$$

$$-r_3 + t_{241} \leq 22.74 \tag{8-4}$$

$$-r_4 + t_{132} \leq 31.64 \tag{8-5}$$

$$-r_4 + t_{142} \leq 31.80 \tag{8-6}$$

$$-r_4 + t_{231} \leq 29.79 \tag{8-7}$$

$$-r_4 + t_{232} \leq 31.64 \tag{8-8}$$

$$-r_4 + t_{241} \leq 29.79 \tag{8-9}$$

$$-r_4 + t_{242} \leq 31.80 \tag{8-10}$$

$$-r_5 + t_{132} \leq 31.80 \tag{8-11}$$

$$-r_5 + t_{142} \leq 31.96 \tag{8-12}$$

$$-r_5 + t_{231} \leq 29.79 \tag{8-13}$$

$$-r_5 + t_{232} \leq 31.64 \tag{8-14}$$

$$-r_5 + t_{241} \leq 29.79 \tag{8-15}$$

$$-r_5 + t_{242} \leq 31.80 \tag{8-16}$$

$$-s_4 + t_{312} \leq 31.67 \tag{9-1}$$

$$-s_4 + t_{322} \leq 31.67 \tag{9-2}$$

$$-s_4 + t_{412} \leq 31.83 \tag{9-3}$$

$$-s_4 + t_{422} \leq 31.83 \tag{9-4}$$

$$-s_5 + t_{312} \leq 31.83 \tag{9-5}$$

$$-s_5 + t_{322} \leq 31.67 \tag{9-6}$$

$$-s_5 + t_{412} \leq 32.00 \tag{9-7}$$

$$-s_5 + t_{422} \leq 31.83 \tag{9-8}$$

/After 18

After 18 iterations, the optimal solution obtained gave a value for the objective function of US$46,157,800. The values for the ordinary variables, together with their interpretations, are shown in table B-1. Table B-2 gives the same information for some of the slack variables.

/Table B-1

Table B - 1

ORDINARY VARIABLES IN THE DUAL SOLUTION

(Values in US$/shipping ton)

Variable	Value	Interpretation
r_1	21.10	Shadow cost per ton of capacity on service 1, for timber moving from Latin America to Japan.
r_2	23.22	Idem for service 2.
r_3	23.13	Idem for service 3. Since no traffic moved via this service in the primal solution (which means that its actual cost was too high to attract traffic), the value here represents a threshold such that, if service 3 costs were reduced below this level, some traffic would be diverted to service 3.
r_4	30.67	Shadow cost per ton of capacity on service 4, for general cargo moving from Latin America to Japan.
r_5	30.67	Idem for service 5. The interpretation of variable r_3 applies here also.
s_4	0.00	Shadow cost per ton of capactity on service 4, for general cargo moving from Japan to Latin America. The value is zero because more traffic moves in the direction Latin America to Japan and so establishes the capacity level required on this service.
s_5	0.00	Idem for service 5. The interpretation of variable r_3 plus that of s_4 apply here.
t_{231}	43.75	Total shadow cost per shipping ton of timber transported from port LB to port JA using the optimal service, which was determined by the primal to be service 1.
t_{241}	45.87	Idem from port LB to port JB.

/Table B-1, continued

Table B-1, continued

Variable	Value	Interpretation
t_{132}	62.31	Total shadow cost per shipping ton of general cargo transported from port LA to port JA using the optimal service, which was determined by the primal to be service 4.
t_{142}	62.47	Idem from port LA to port JB.
t_{232}	62.31	Idem from port LB to port JA.
t_{242}	62.47	Idem from port LB to port JB.
t_{312}	31.67	Idem from port JA to port LA. Note that the cost of providing capacity at sea is excluded, because excess capacity is available in the direction Japan to Latin America.
t_{322}	31.67	Idem from port JA to port LB.
t_{412}	31.83	Idem from port JB to port LA.
t_{422}	31.83	Idem from port JB to port LB.
p_{132}	0.00	The shadow cost per shipping ton of requiring a minimum frequency of service for general cargo transported from port LA to port JA. The value is zero because the actual capacity provided by the primal is greater than the minimum.

/Table B-2

Table B-2

SOME SLACK VARIABLES IN THE DUAL SOLUTION

(Values in US$/shipping ton)

Variable	Value	Interpretation
u_3	0.61	Amount by which the cost per shipping ton of capacity on service 3 would have to be reduced in order for this service to attract timber traffic moving from port LB to port JB.
u_5	1.03	Amount by which the cost per shipping ton of capacity on service 5 would have to be reduced in order for this service to attract general cargo traffic moving from port LB to ports JA and JB, and from port JA to port LB.
v_{2313}	2.12	Extra amount (in addition to u_3) by which the cost per shipping ton of capacity on service 3 would have to be reduced in order to attract timber traffic moving from port LB to port JA.
v_{2314}	16.71	Amount by which the cost per shipping ton of capacity on service 4 would have to be reduced in order for this service to attract timber traffic moving from port LB to port JA. Note that this would constitute a shift from a specialized timber vessel to a general cargo vessel.
v_{2414}	14.59	Idem from port LB to port JB.
v_{2315}	16.71	Extra amount (in addition to u_5) by which the cost per shipping ton of capacity on service 5 would have to be reduced in order to attract timber traffic moving from port LB to port JA. Note that this would constitute a shift from a specialized timber vessel to a general cargo vessel.

/Table B-2, continued

Table B-2, continued

Variable	Value	Interpretation
v_{2415}	14.59	Idem from port LB to port JB.
v_{1325}	0.16	Extra amount (in addition to u_5) by which the cost per shipping ton of capacity on service 5 would have to be reduced in order for this service to attract general cargo traffic moving from port LA to port JA.
v_{1425}	0.16	Idem from port LA to port JB.
v_{3125}	0.16	Idem from port JA to port LA.
v_{4125}	0.17	Idem from port JB to port LA.