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ECONOMIES OF SCALE IN RELATION TO INDUSTRIAL PROGRAMMING

Document prepared by the ECLA secretariat

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Introduction

The subject dealt with in the present document, within the framework of industrial programming, is the economic merits of alternative sizes for industrial plants. Accordingly, the term "economies of scale" is defined, together with several concepts to which reference will be made throughout the study.

On this basis, some account is given of characteristic aspects of the Latin American economies, with special attention to problems such as the various barriers to entry to markets, shortage of capital, versatile versus specialized uses of equipment, etc. Next follows an analysis of the usefulness of systematized knowledge of economies of scale with a view to the availability of precise information for the purposes of programming projects.

Methods of determining economies-of-scale curves are then discussed, with emphasis on the possible shapes which such curves may assume.

The study concludes with an annex summing up the findings of specific studies on economies of scale in several sectors of industry, with detailed comparative discussion of a number of characteristics in different sectors, such as production capacity, technology, etc.; and indices for several cost and investment items are included to show the trends followed by the economies-of-scale curves referred to above.

/1. Definition of

1. Definition of economies of scale

In establishing industrial profiles for use in programming, an exact idea must be given of the lines on which economies of scale should be studied, and the definition formulated should be as widely applicable as possible and should provide a basis for the establishment of a research methodology.

To confine the initial analysis to the use of a single factor: a suitable yardstick of economies and diseconomies of scale associated with a specific factor is the variation in the average cost of that factor per unit of output, on the assumption that installed capacity is being fully utilized and with due regard to changes in this latter.

In some exceptional cases, however, in which the producer unit is reduced to a single highly-specialized machine or piece of equipment, whose production capacity exceeds or is roughly commensurate with the size of the market, economies of scale on which data have to be assembled for the purposes of industrial programming do not fit in to the definition suggested above. They are attributable to variations in the level of utilization of a minimum capacity established on technological grounds, and not to changes in the capacity of the producer unit.

Varying degrees of importance may be attached to this exceptional case. Its significance will increase where markets are small, but will be negligible when the relations between market dimensions and available technologies permit of changes in production capacities. But for general purposes, where what matters is to work out a methodology regardless of individual cases, the suggested definition will be valid, and if care is taken to determine exactly what the minimum capacities are on technical grounds, it will still be possible to extend the analysis, should this prove necessary, to exceptional cases where demand is limited.

It must be borne in mind that a clear distinction should be drawn between this concept of economies of scale, and a second interpretation common in the literature of economics, which is consonant only with the exceptional case, and according to which economies of scale consist in changes in the total average cost per unit of output as a function of the level of utilization of a specific capacity. This other interpretation is also useful for programming, but at a later stage, when economic viability and the risks involved in given programmes or projects are being estimated, or when other possibilities, listed in paragraph 3 below, are being explored.

/To forestall

To forestall the danger of conceptual inconsistencies in the course of the research, the definition of economies of scale in the use of factors must be carefully evaluated.

One of the basic points to be considered is the need for data to be homogeneous, which means that the following ancillary definitions must be introduced.

(a) Product

The product is defined as the result of one or more operations whereby natural, processed or artificial materials are given market characteristics by virtue of which the result of these operations can be precisely identified, and different specifications can be obtained, or a different identification and different specifications simultaneously (the term different specifications implies characteristics that are sharply differentiated, by a change either in dimensions or in other properties, permitting different uses). Examples of products distinguished from each other by different identifications and specifications adapting them to different uses are cast-iron and steel. Examples of products distinguished from one another by different identifications, but whose specifications are such that they can be adapted to the same uses, are steel forgings and American malleable cast-iron. An example of products with the same identification but with specifications adapting them to different uses is constituted by hot rolled strips of different widths. In all three examples, the pig iron obtained by reduction of iron ore has acquired new market characteristics, which establish differences in identification, in specifications or in both, as the case may be.

(b) Industrial processes

By an industrial process is meant any transforming process deliberately applied to a given material to obtain a specific result by the use of facilities precisely defined in respect of their nature, of the agents employed and of the external physical media. The term "nature" implies a chemical reaction, a physical transformation, etc. By the "agent" is meant thermal energy, electric power, chemical affinities, etc. The "external physical media" are the production equipment proper.

One and the same product can be obtained by different processes; for instance, acetylene can be produced by the petrochemical process and on the basis of calcium carbide.

/ (c) Technique of

(c) Technique of operation

The term "technique of operation " is applied to the whole set of characteristics pertaining to the nature, agents and external physical media of industrial processes, differentiated by these factors or by the lay-out of the equipment, but not by such constructional details as do not depend upon this latter.

An example of two techniques of operation for one and the same process is afforded by the reduction of iron ore in the classic type of blast furnace, and in the modern high-pressure blast furnace, the same agents being used, with variations in some of their characteristics and the indispensable changes in the equipment. Another case in point is constituted by discontinuous and continuous production, when the characteristics of the agents remain unchanged, and only the lay-out of the equipment and the constructional characteristics dependent thereon undergo alteration.

(d) Technology

Briefly, this term will be used to designate the combination formed by a process and a technique of operation.

Given this definition, a specification can be established: data must relate to a single product, obtained by the application of a single technology.

But since, for technical reasons, there are often limits to the capacities attainable by the application of a given technology, and these limits, moreover, are not always clearly defined, the curve corresponding to the average cost of a single factor per unit of output, and representing the trend of the pertinent economies of scale, should be taken as the envelope of the curves corresponding to the different technologies analysed. With reference to the discussion accompanying the definition, the starting-point of this curve should be the technology compatible with the lowest production capacity, and if necessary it should be extended to cover variations in the level of utilization of this minimum capacity.

Technologies where the limits to capacity are determined by the adoption of different processes are exemplified in the blooming of steel ingots by means of a hydraulic press and by rolling. Examples of technologies in which these limits are set by variations in techniques of operation within one and the same process are to be found in the use of the

/classic blast

classic blast furnace, whose production capacity does not at present exceed an annual maximum of 600,000 tons of pig iron, and of the high-pressure blast furnace, by means of which an output of about 1,500,000 tons can be attained.

Finally, one last requisite in respect of homogeneity is established by the need to determine the original product or products, so that the variation in the factor cost may not be influenced by the integration of different processes.

Hitherto it has been assumed, for the sake of simplicity, that the process or processes under consideration result in a single product. In the case of joint production, where a specific product is manufactured in association with another in fixed or approximately fixed proportions, as,, for example, electrolytically-produced caustic soda and chloride, the study of economies of scale in relation to the factors of production presents no difficulties, once it has been established which of the products determines the decision to manufacture.

The matter becomes more complex when different products are manufactured in variable proportions, as in the case of a commercial rolling mill for bars or shapes, whose production capacity varies with the type of finished product. In this instance factor inputs should be related either to a capacity determined by a composition of total output which is estimated to correspond to normal demand conditions, if the primary aim of the research is economy in the use of factors; or to the lowest-priced product, if the emphasis is to be laid on the economic viability of investment, in the face of possible changes in the composition of demand.

It may also happen that the product is obtained by the assembly of other products in fixed proportions (as in the capital goods and durable consumer goods industries), in which case the analysis will have to be carried out by stages, study being devoted to each of the manufactured parts that enter into the composition of the end product, with due regard, moreover, to the possible effect of integration of two or more of the processes involved within a single plant.

To sum up, when economies or diseconomies of scale are considered only in relation to the use of a single factor of production, once the product has been defined and the requisites in respect of homogeneity established - which also entails determining the degree of integration, as discussed above -, for each technology or combination of technologies, if their application implies limits to capacity, a clearly-defined curve will be obtained, which will represent the variation in the average factor cost per unit of output.

Given two technologies, or combinations of technologies, with a common range of capacities, they will either be equivalent to each other within that range as regards the average cost of the factor under consideration, or one of them will be preferable to the other, in the whole or part of the range.

But when it is important for the purposes of programming to study economies in the aggregate use of factors, relative prices will have to be introduced as a means of weighting inputs of each factor, and, given two technologies which use capital and labour in different proportions (calculated in physical terms, as set forth in section 4), variations in the level of relative prices may possibly lead to different results with respect to the aggregate economy, so that the problem of the choice between alternatives becomes more complex.

In this event, for each alternative technology a family of curves must be plotted as a function of the variations in relative price levels. From a comparison of these families, conclusions can be drawn as to how far the technologies concerned are sensitive to variations at each level, and the stability of results calculated on the basis of a single level can also be evaluated.

Clearly, in this case the choice between alternative technologies may be determined not so much by comparison of their total costs interms of one level of relative prices, as by the economy effected in the use of a factor which is considered to be in short supply, or by maximum use of a factor whose full utilization is aimed at. The availability of a family of curves plotted on the basis of various relative price levels will then facilitate evaluation of the influence exerted by the criterion of selection on the average combined factor cost.

/2. Discussion

2. Discussion of the problem of economies of scale in terms of economics

The existence of economies of scale is important from the standpoint of economics, in relation to both general equilibrium theory and the theory of industrialization in under-developed countries.[1/]

In general equilibrium theory, one of the central problems is that of the existence of a point of maximum production in the economic system. This problem is solved, under certain conditions, by a theorem leading up to the so-called Paretian optimum. But a situation of collective economic optimum, in Pareto's sense, may be modified. For instance, the existence of increasing returns may generate a monopolistic tendency, or, in other words, the possibility of a structure of supply such that the point of maximum profit does not coincide with the point of maximum production, but with a lower production level.

Another significant consequence of the existence of increasing returns is the possibility that an element of rigidity may be introduced into the structure of the market, inasmuch as other producers' chances of entry are reduced. There may also be other factors, of either an institutional or a structural type, which put barriers in the way of entry to the market and maintain a monopolistic or oligopolistic status quo; and in view of the interest attaching to these problems from the standpoint of developing economies, some account will be given of the four types of barriers to entry:[2/]

1. Economies-of-scale barrier
2. Product-differentiation barrier
3. Absolute-cost barrier
4. Capital-requirements barrier

1/ See Tibor Scitovsky, "Two concepts of external economies", Journal of Political Economy, Chicago, April 1954.

2/ Joe S. Bain, Barriers to new competition (their character and consequences in manufacturing industries), Harvard University Press, Cambridge, Mass., 1956.

/The first

The first of these barriers will be considered in the light of the cost-capacity ratio in section 4 of the present document. The second relates to problems of the differentiation of the product, with the corresponding market difficulties as regards competition, consumer preferences, patents, etc., and to questions of the cost of diversification and participation in the market under the so-called non price competition policy. The coverage of the third or absolute-cost barrier ranges from the cost of installing an industry on the selected site, to the organization and cost of distribution channels, areas of influence, etc.; the effects of these determine the classification of establishments as marginal, intra-marginal or extra-marginal with respect to the supply of industry, and, consequently, their ability to maintain or failure to secure a foothold in the market. Lastly, the fourth or capital-requirements barrier is of marked interest for under-developed countries (for which reason it was taken into consideration in section 1), linked as it is to the problems of capital accumulation, internal or external financing, capital-labour ratios, etc. which in the last analysis will combine with other factors to determine possibilities of entry.

To sum up, of the four barriers, the first is of a basically technical character, i.e., a matter of adjustment of the cost-capacity ratio; while the other three are influenced by a multiplicity of factors, including organization, publicity, know-how, exchange rates, etc. upon whose interaction will depend the opportunities open to new entrants to the market.

The experience of the industrialized countries has recently shown a trend towards

(a) decentralization,

(b) disintegration,

or, in other words, the practise adopted has reversed the direction envisaged by economic theory and formerly pursued. Typical of these countries is the distribution of industrial establishments in the light of rational location criteria, with a view to serving sales areas and minimizing transport problems. Again, a common feature is the existence of

/sub-contracts

sub-contracts with other firms for the separate manufacture of parts required in connexion with the production of what was previously an integrated establishment.[3/]

In under-developed countries the study of economies of scale must also take into account the effects of external economies, since they determine plant dimensions through the operation of a number of factors: size of market, capacity to import, etc. As the technologies applied are brought from industrialized countries, they have to be adapted to the environment. Consequently, problems arise such as those relating to the indivisibility of certain pieces of equipment, designed for different conditions, which make minimal scales impracticable. Moreover, the absence of the advantages deriving from external economies, which is a typical feature of under-developed countries, compels establishments to move in the direction of integration. If to this is added the shortage of capital which is another structural constant in these countries, the complexity of the problems of economies of scale in such circumstances becomes patent. And in under-developed countries all these difficulties, which in the aggregate serve to give an idea of the degree of under-development, constitute a barrier to entry to the market.[4/] The attempt to overcome them entails renewed consideration of the problem of adaptation of technologies, on account of two characteristics:

(a) costlines of capital equipment;

(b) impossibility of large-scale use of highly-specialized equipment.

As regards the first of these, in developed countries the relative prices of capital and labour inputs are not the same as in under-developed

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3/ For the experience of the United States, see Saul S. Sands, "Changes in scale of production in United States manufacturing industry (1904 - 1947)", Review of Economics and Statistics, Vol. XLIII, Harvard University Press, November 1961; and for the United Kingdom, E.A.G. Robinson, The Structure of Competitive Industry, Cambridge University Press, 1958.

4/ See ECLA, Examen Preliminar de las Posibilidades de Desarrollo Industrial Integrado en Centroamérica (E/CN.12/CCE/GT.Ind/10), November 1961.

/countries; and

countries; and, secondly, the requirements of integration and amortization of equipment may necessitate more flexible rather than specialized use of machinery.

This brings us to the selection of alternative technologies, in which not only external economies will have to be taken into account, but also prospects such as those opened up by the Agreement on the Régime for Central American Integration Industries and by the Montevideo Treaty, instruments of regional integration whose application will permit of larger-sized markets and facilities for entry.

In face of these changing prospects, the alternatives might be set forth as follows:5/

1. Installation of sub-optimum equipment;
2. Installation of optimum equipment combined with partial utilization;
3. Postponement of the installation of optimum equipment until such time as an expansion of demand enables capacity to be more efficiently utilized.

Implicit in these alternatives is the importance attaching to a knowledge of economies of scale in the development of the Latin American countries. The necessary background material on which to base a decision will include not only criteria that take each country's special circumstances into consideration, but also concrete data on scales in several industrial sectors, the use of which for programming purposes will be discussed in the course of the present document.

3. Usefulness of systematized knowledge of economies of scale for industrial programming

In detailed programming for industrial sectors, where the determination of the over-all level of sectoral production precedes a breakdown by individual products, the need for systematized knowledge of economies of scale arises at various stages.

5/ Tibor Scitovsky, Economic Theory and Western European Integration, George Allen and Unwin Ltd., London, 1958.

/The transition

The transition from sectoral targets to individual product objectives may be effected simply through the incorporation of projects in the programme, these projects being selected in the light of their compatibility with the programme targets (which is established by means of evaluation coefficients for value added, employment, etc.); or from the standpoint of efficient utilization of resources in short supply (accounting prices of investment, of foreign exchange resources, etc.); or by the establishment of individual product targets before attention is turned to projects. Whichever procedure is adopted, knowledge of economies of scale is of considerable value for the perfecting of programming techniques.

During the initial stage, before the detailed criteria for project evaluation are applied, a preliminary analysis of the existing body of projects must be carried out, so that it can be decided whether these are in principle acceptable, or whether their evaluation should be suspended until a wider selection of suitable projects is available.

For this preliminary analysis, a basic requisite is the availability of profiles whereby, in the case of every product which is important enough to justify its separate treatment within the sector, the pattern of economies of scale can be appraised, as well as their possible influence on the successful attainment of the programme objectives.

For example, if one of these objectives - as will presumably be the case in most of the under-developed countries - is to reach certain production targets with a minimum use of capital resources, and if in respect of a specific product included in the programme substantial economies of scale have been proved to exist in relation to investment, it will be advisable to reject projects of which the scale implies inefficient utilization of capital, or to develop some as yet unavailable which would fall within the range of magnitudes permitting efficient utilization.

/Clearly, if

Clearly, if information on economies of scale covers other products in addition to those considered in the programme, not only will the problem of preliminary evaluation of the whole set of projects be simplified, but the possibility of alternatives to the programme can be evaluated, and their analysis carried a good deal further than over-all programming permits.

Where the production capacity required for attainment of the programme targets is below the levels at which investment reaches a minimum, a workable procedure will be to evaluate the cost in excessive use of capital resources which implementation of the programme would imply, and to estimate how long an interval is likely to elapse before the development of the market enables this cost to be eliminated.

Both the cost in excessive use of capital resources thus defined, and the length of the interval of waiting, afford additional criteria for the selection of alternative products to be included in the programme; reconsideration of the sectors to be developed may prove necessary, if differences are observable from one sector to another.

In the opposite case, where the production capacity required for the attainment of the programme targets is considerably higher than the level corresponding to minumum use of capital, consideration could be given to additional decentralization and area development objectives, which would not be consistent with the programme if the breaking-up of production into various units were to reduce scales unduly.

Clearly, to warrant the assertion, in this context, that a programme is optimal in relation to such criteria as minimum use of capital, the profiles previously referred to should completely cover all

/the most

the most important of the possible activities; they can then also be used for estimating total use of resources and production flows, from one sector to another and in relation to the total available supply of resources, at a higher level of approximation than can be reached with the average coefficients obtained in studies of inter-industrial relationships.

However, it must also be borne in mind that cases in which a single factor of production is the only element entering into a decision - as was assumed in respect of fixed investment in the foregoing discussion - constitute exceptions, and that, furthermore, it is always necessary to take the combination of factors into consideration for the preliminary estimate of costs which is indispensable in order to assess economic viability.

Assuming that relative price levels are known, in this case too, as in that of a single factor, the behaviour pattern of economies of scale enables programming methods to be supplemented by the introduction of additional criteria for the study of possible alternatives. Moreover, given specific international price levels, it can also be turned to account in estimating the economic policy implications of an industrial development plan, in so far as it establishes the need for protectionist measures and subsidies and their possible magnitude (according to the development mechanism adopted), and the length of time for which they will have to be maintained. Unquestionably, these last considerations may play a vital part in the final evaluation of a development plan, and in default of knowledge of economies of scale their significance cannot be estimated with any degree of accuracy.

/4. Methodology

4. Methodology of determination of economics-of-scale curves

(a) Fixed investment

The cost of fixed investment is understood to cover the following items:

(i) Cost of production equipment (estimated at place of origin);

(ii) Cost of transport of equipment to the place where the production unit is to be installed;

(iii) Costs of installing or assembling the equipment and putting it into operation (comprising honoraria in connexion with the industrial project, and advisory assistance in putting the equipment into operation,either from consultants or from the actual manufacturer of the equipment; but excluding possible consumption of raw materials and power in testing, etc.);

(iv) Building costs (evaluated by the application of a unit price to the unit of floor space required for the equipment, with due regard to the characteristics of each individual case, which may or may not require special structures for crane saddles, etc.);

(v) Project, and supervision of construction;

(vi) Interest payable during construction and installation (estimated at an annual rate of 12 per cent for half the sum total of items (i) to (v), during the estimated interval between the beginning of construction work and the putting of the equipment into operation, even if only partially).

Total fixed investment will be given by the sum of items (i) to (vi), and a detailed evaluation of item (i) is required, as far as possible with specifications of the cost of the various machines and sections. Items (ii) and (iii) can be evaluated in terms of a percentage of the cost of equipment which is considered sufficiently representative of their average cost in Latin America, and which will vary with each type of industry.

/(b) Labour

(b) Labour inputs

Labour inputs will be calculated in physical terms (hours worked or number of persons employed per unit of output), on the basis of the following classification:

(i) Direct labour (operatives tending production equipment);

(ii) Indirect labour (auxiliary personnel, maintenance and cleaning staff, and supervisors).

In every case, the labour productivity hypotheses on which the estimate of these inputs is based will be specified, so that variations from one country to another, or the influence of an improvement in the training of personnel, can be studied.

(c) Financial investment

For the purposes of estimating financial investment, a period of rotation will be fixed, during which the cost of the following items will be calculated: labour, raw material (specifying type and price), consumption of electric power and/or fuels, and maintenance.

In order to calculate labour costs, it will be necessary to estimate an average wage per hour worked, specifying whether it relates to a particular country, or whether it represents an average value for the region or for a group of countries in which industry is at a more advanced stage of development. This cost item will include an estimate of social security contributions.

To this direct expenditure will be added overheads (estimated as a percentage of the value of production).

(d) Estimate of cost

Production costs per unit of output will be estimated on the basis of variable items of expenditure, a rate of amortization of fixed investment estimated at 10 per cent per annum, and - on the assumption that in Latin America working capital is obtained in the form of credit - a 12-per-cent rate of interest on financial investment.

(e) Calculation of viability

If preliminary estimates of viability are required, a rate of return on capital (12 per cent) will have to be added to the cost, and if the information available is considered to be sufficiently detailed, the break-even point should be determined.

/(f) General

(f) General remarks

Given the definition of economies of scale formulated in the present document, production capacity will be interpreted in terms of maximum utilization of investment, which as a general rule implies three production shifts during a specified number of working days.

The need for three shifts means that in estimating labour inputs allowance must be made for a possible decrease in efficiency during the night shift, and in estimating labour costs, the higher wages payable in these conditions should be taken into account, as well as the reduction in the number of hours worked daily that may be imposed both by legislation and by maintenance requirements, etc., with the consequent additional repercussions on production capacity.

5. Typical shapes that may be assumed by investment cost curves

The classic way of treating economies of scale in the literature of economics as a function of variations in the capacity of production equipment is based on recognition of the existence of indivisible elements in the factors of production, which provides a first approach to discussion of the problem. This discussion, furthermore, will at first be restricted to investment in production equipment.

A range of production capacities having been established, together with the technologies to be applied, a certain composition of production equipment will correspond to each level of capacity. If the upper limit provisionally established is the capacity corresponding to a state of equilibrium, in which all sections of the production equipment work at full capacity, to lower capacities will correspond average costs in respect of the major items of equipment, owing to under-utilization of indivisible elements.

If it is assumed, for the sake of simplicity, that there is only one indivisible (or fixed-cost) element, and that the remainder of the equipment is almost entirely divisible, the cost of investment and equipment can be broken down into two parts, one of these being the constant x, or, in other words, can be represented by the following equation:

$y = Kx + C$,

/in which

in which y = cost of investment in equipment

C = fixed cost

K = cost per unit of production capacity of remainder of equipment.

This expression is valid only for the segment corresponding to the capacity of the indivisible (or fixed-cost) element, nor will it be possible, as a general rule, to assume its continuity, since in fact it will be confined to points along the curve, once the existence of indivisible elements in the rest of the equipment has also to be assumed.

But its degree of approximation will be sufficient if there is a wide disparity between the capacity of the fixed-cost element and that of the most costly elements in the rest of the equipment. A case in point is that of the cotton spinning industry (see ST/ECLA/CONF.11/L.20), in which the ratio between the capacity of the indivisible (or fixed-cost) element (opener) and that of the continuous spinning frame is 1/11,000 for yarn count 10 (carded), and reaches 1/45,000 for yarn count 40 (combed).

If, instead of total cost, investment in equipment per unit of production capacity is considered, the following function will be obtained:

$$Y = \frac{C}{x} + K,$$

in which Y represents unit investment, or average cost per unit of output, on the assumption of full utilization.6/

This equation corresponds to that of one branch of an equilateral hyperbola, with asymptotes $x = 0$ and $Y = K$, which is completely determined by the constants C and K.

If C is known, it will be enough to determine one point - i.e., the investment required to attain a given production capacity - for K to be discovered, and, again, with two points C and K can be discovered simultaneously, but as the important thing will usually be to determine the type of curve rather than to assume it prior to research on costs, in practice three points will have to be established in order to ascertain whether the fitting is satisfactory.

6/ This equation corresponds to the "law of capacity" in the terminology proposed by I. Jantzen, "Laws of production and cost", Proceedings of the International Statistical Conferences, Vol. V, 1947, pp. 58-68.

/The first

The first derivative of the equilateral hyperbola equation shows the variations in equipment investment per unit of production capacity, in plants of different sizes:

$$\frac{dY}{dx} = - \frac{C}{x^2}$$

Consequently, the saving in investment measured by this derivative (owing to the negative sign), for each additional unit of production capacity, is in inverse proportion to the square of the size of the plant.

The significance of this saving from the standpoint of economies of scale depends upon the relation of magnitudes as between K, C and x. If C/x is in the same order of magnitude as K, there will be substantial economies of scale, but if C/x represents only a fraction of K, the economies of scale will be relatively limited.

To follow up the example previously adduced from the cotton spinning industry, if 1 000 kilogrammes per annum is taken as the unit of production capacity, the corresponding values will be as follows:

$$C = 53,500$$
$$K = 305.3$$
x falls between 1,094 and 2,295,

so that C/x fluctuates between 48.9 and 17.9, or, in other words, is small in relation to K, and economies of scale will be of little significance.

If production capacity rises to a level higher than the capacity of the fixed-cost element, a second indivisible element will have to be added. In the expression of total investment in equipment a new section will appear, represented by a curve with the same slope as before, but shifted upwards with the ordinates by a magnitude equivalent to C.

In the expression of the cost of equipment per unit of production capacity, a second hyperbola will be introduced, with an ordinate equivalent to

$$Y = \frac{2C}{x_1} + K,$$

at point x_1, representing the capacity of the indivisible or fixed-cost element, and valid between x_1 and $2x_1$, at which point will appear a hyperbola whose corresponding equation is

/Y =

$$Y = \frac{3C}{x} + K,$$

valid between $x = 2x_1$ and $x = 3x_1$, and so forth; it should be noted that all these hyperbolae have the same horizontal asymptote $Y = K$, and that the magnitude of the discontinuity decreases in proportion to the multiples of x_1.[7/]

A second approach to the discussion of economies of scale in relation to investment in production equipment is based on recognition of the fact that the mathematical ratio between surfaces and volumes is less than proportional. Thus, in any industry using a continuous process assimilable to circulation through tubing, and with a capacity proportional to the section of the tubing, the cost of equipment, at a first approximation, depends exclusively on the surface, which increases only as a function of the square root of the section, so that for two capacities, x_1 and x_2, the ratio between total costs would work out as follows:

$$\left(\frac{Y_1}{Y_2}\right) = \left(\frac{x_1}{x_2}\right)^{0.5}$$

This law will be fulfilled the more exactly, the less auxiliary equipment (pumps, etc.) there is, and within variations of diameter that do not necessitate an increase in the thickness of the sheet from which the tubing is made.

On the other hand, should the process be assimilable to treatment in vats, on the basis of the ratio between the area and the surface of the sphere an exponent of 0.67 would be obtained, the same remarks being applicable in relation to the requisites for a satisfactory approximation.

A large number of chemical industries (see examples in ST/ECLA/CONF.11/L.17) follow this behaviour pattern very closely as regards investment in equipment.

But the same expressions are applicable in other cases differing from the chemical industry, where they originated; their use is now becoming widespread, and is set forth in engineering handbooks as a rule for making rapid estimates of variations in investment corresponding to variations in capacity (the 0.6 rule).

7/ The "law of harmony", according to I. Jantzen (op.cit.).

/An important

An important example of the application of this rule outside the chemical industry is to be found in steel refining by the open-hearth (Siemens-Martin) method, which, within the range of 100 000 to 1 million tons, is well represented by the exponent 0.57; the reason no doubt is that in this special type of equipment the thickness of the walls is determined by the temperature of the process, (which is the same in all cases), and has no structural role, this being played by a metal casing whose cost in comparison with that of the refractory lining is very low.

This special type of behaviour, in contrast to the pattern previously discussed, corresponds to a hypothesis assuming unlimited divisibility of equipment, and would be analytically represented in a logarithmic graph by a curve parallel to the axis of the abcissae.

A combination of this extreme case with a case of complete divisibility, such as would occur, for example, if within the iron and steel industry the blast furnace stage (without coking) were isolated, would give in the logarithmic graph a horizontal curve extending from $x = 0$ tons to $x = 100{,}000$ tons per annum (which, for technical reasons, is the minimum limit that can nowadays be estimated for the blast furnace).

If, on the other hand, the case referred to were combined with one of complete indivisibility, for total investment a step function would result, with horizontal curves separated by jumps of a magnitude equal to the cost of the producer unit, and for investment per unit of production a graph of the same type as that of the law of harmony studied in the first place would be obtained, the sole difference being that the common horizontal asymptote of the hyperbolae would be the axis of the abcissae.

To sum up, in the logarithmic graph, on the assumption of a uniform technology, the total cost function will take the shape of a curve whose slope will define the evolution of economies of scale. If the corresponding exponent is equal to unity, there will be neither economies nor diseconomies. If it is lower than unity, there will be normal economies between 0.5 and 0.67, less than normal between 0.67 and unity, and more than normal if the exponent is lower than 0.5.

/The exceptional

The exceptional case will correspond to a horizontal curve touching another non-horizontal curve at a point representing the lower limit of capacity as determined by technical reasons.

In cases where equipment is completely divisible, total cost curves may be linear or non-linear. A linear curve represents an extreme case of what was described at the beginning of the present section 5, where average total cost and average investment curves show a marked downward trend (as in the manufacture of goods produced by the metal transforming industry on the basis of continuous lathing operations).

Where they are not linear,[8/] average total cost curves are U-shaped. In these circumstances, the maximum volume of production does not coincide with the minimum level of average total costs. Increases in scale, on such assumptions, merely reproduce the same phenomenon (U-shaped average total cost curves), and although a fairly flexible factor substitution ratio can be found, it can only be exploited at the price of increasing operational costs, and not even so unless the relative prices of factors remain stationary.

The way out of this problem is basically a question of industrial design; consequently, the activities that profit most by economies of scale are those which most directly enjoy the advantages of technological progress. Emphasis has recently been laid on complementarity between economies of scale and technical progress[9/], and also on the importance of an economy's capacity to absorb technical change with a view to its development.[10/]

8/ When the functional total-cost/production ratio takes the shape of a cubic parabola.

9/ W.E.G. Salter, Productivity and Technical Change, Cambridge University Press, 1960.

10/ Nicholas Kaldor, Essays on Economic Development, Centro de Estudios Monetarios Latinoamericano (CEMLA), Mexico, 1961.

/ANNEX

ANNEX

The attached table constitutes a simplified presentation of some of the findings of studies on economies of scale in a sample of industrial activities.

The transition from an over-all methodological approach to specific studies gives rise to innumerable difficulties which entail special treatment for each industrial activity, as in the case of the problem of the divisibility of equipment. Among the special features that emerged were sizes of plants in which the scale of variation was not always uniform, because real situations are sometimes lacking in uniformity; for instance, in the case of kraft manufactured in an integrated mill, the production indices ranged from 100 to 200 to 400, but in that of the manufacture of raw cotton textiles in an integrated mill processing its own yarn (yarn count Ne 10, carded), production indices ranged from 100 to 199 to 274.

In some branches of the chemical industry considerable variety was observable (18 cases were studied) and in the present brief outline the presentation of at least two cases has been contemplated, i.e., the production of sulphuric acid, and that of butadiene, to give some idea, however general, of the behaviour pattern of economies of scale in respect of investment. No attempt has been made to specify highly diversified characteristics affecting the various activities; for example, in the production of pulp and paper the processes are continuous and determine a specific level of output and quality, whereas in boiler-making the processes are discontinuous and the products obtained vary in accordance with the orders received.

In the manufacture of welded steel tubes, a characteristic and determining factor in the volume of production is the capacity of the welding equipment, which in turn allows of some degree of flexibility in the manufacturing programme (tubes with different nominal diameters). In the indices presented, the sample chosen was the case in which scale increases relate to larger sizes of welding equipment, always on the basis of a single line of production and a single work shift. By comparison with other working hypotheses, this case was seen to be the one corresponding to the use of the best industrial design available.

/Indices of

Indices of production, investment and costs are presented at the foot of each industrial activity column. Thus, under the head of "Pulp and paper", production scales vary in strict proportion, and the corresponding average total cost indices show fairly considerable economies of scale. In other activities, pertaining to the textile sector, the variations in the scale of production are not strictly proportional and average total cost indices do not reveal any very noteworthy economies of scale.

To sum up, even if complete uniformity is not to be found in each industrial activity, a comparison of indices sheds light on the phenomenon of the relations between production and average total cost, in which the problem of economies of scale is synthesized.

	Textiles	Boiler-making	Steel tubes	Pulp and paper	Chemicals
1. Divisibility of principal equipment	Three hypotheses are adopted for the cleaning section in the spinning mill and the sizing section in the weaving mill, the third assuming that all items of equipment are balanced	Owing to the discontinuity of the process, some equipment does not work to full capacity	Production with balanced processes in terms of speed of operation of welding equipment	Balanced continuous processes	Continuous processes. Elements designed for the capacity utilized.
2. Number of work shifts	According to hypotheses 1 to 3	1 and 2 shifts	According to technologies 1 to 3	3 shifts	3 shifts
3. Homogeneity of product	3 types of raw fabric of different qualities of cotton and different sizes	According to orders received	Only size and weight vary	Qualities and final product (pulp or paper) vary according to level of integration of treatment	Completely homogeneous in each sample process
4. Levels of productivity (especially productivity of labour)	Higher than the Brazilian standard, and considered attainable	In accordance with conventional European and United States standards	In accordance with conventional European and United States standards	In Accordance with conventional European and United States standards	European and United States standards, slightly modified to allow for the real situation in the region (± 20 per cent labour force in relation to United States standards). Low incidence of labour productivity.
5. Technology utilized	The same degree of mechanization, with emphasis alternatively on labour or on capital	In accordance with standards in the most highly mechanized Latin American countries	Same degree of mechanization, with emphasis alternatively on labour or on capital	Labour-intensive; all equipment items balanced	Conventional (modern). Capital-intensive
6. Percentage of amortization	10 per cent	10 per cent	10 per cent	15 per cent	Varying between 7 and 10 per cent
Production indices	100 - 199 - 274	100 - 200	100 - 178 - 447	100 - 200 - 400	100 - 276 - 830 (A) 100 - 200 - 400 (B)
Unit indices	100 - 88 - 85 (a)	100 - 77	100 - 72 - 44	100 - 59 - 46	100 - 83 - 66 (A) 100 - 75 - 56 (B)
Unit investment indices	100 - 88 - 86	100 - 76.2	100 - 76 - 44	100 - 67 - 52	100 - 83 - 67 (A) 100 - 75 - 56 (B)
Unit operational cost indices	100 - 95 - 93	100 - 100.9	100 - 95.8 - 91.2	100 - 73 - 59	100 - 86 - 77 (A) 100 - 89 - 84 (B)
Average total cost indices	100 - 90 - 88	100 - 88.9	100 - 85.8 - 67.8	100 - 71 - 57	100 - 84 - 71 (A) 100 - 79 - 65 (B)
	Case A (cotton, yarn count Ne 10c) (a) Unit = 100 metres of textile (see ST/ECLA/CONF.11/L.20)	Case A1 - A2 (see ST/ECLA/CONF.11/L.13)	Manufacturing programme IV (see ST/ECLA/CONF.11/L.14)	Kraft (integrated) (see ST/ECLA/CONF.11/L.19)	(A) Sulphuric acid (B) Butadiene (see ST/ECLA/CONF.11/L.17)